DSSSB TGT PART(A+B)

MATHS

VECTOR CALCULUS

Part -4

LIVE STREAMING 01/07/2024 04:30PM

Q. obtein the DD of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector ~~$i+j+2k$~~. $i+2j+2k$.

(a) 3

(b) 2

(c) -3

(d) -5

$\text{grad}\phi =$

$y^2 i + (2xy + z^2) j + 2yz k$

at $(2, -1, 1)$

$= i + (-4+1) j + (-2) k$

$= i - 3j - 2k$

DD $\Rightarrow \nabla f \cdot \frac{(i+2j+2k)}{|i+2j+2k|}$

$= (i-3j-2k) \cdot \frac{(i+2j+2k)}{\sqrt{1+4+4}}$

$\Rightarrow \frac{1-6-4}{3} \Rightarrow \frac{-10}{3} \Rightarrow -3$

Q. Find the maximum value of the directional derivative of $\phi = x^2yz$ at the point $(1, 4, 1)$

(a) 4

(b) 6

(c) 8

(d) 9

$\text{grad } f \Rightarrow 2xyi + x^2j + x^2yk$

at pt $(1,4,1)$

$\Rightarrow 8i + j + 4k$

Max^m value DD $\Rightarrow |8i + j + 4k|$

Q. Find the greatest value of the directional derivative of the function $2x^2 - y - z^4$ at the point $(2, -1, 1)$:

(a) 9

(b) 6

(c) $\sqrt{9}$

(d) 3

grad $F \Rightarrow 4xi - j - 4z^3k$

at pt $(2, -1, 1)$

$\Rightarrow 8i - j - 4k$

greatest value $DD \Rightarrow |8i - j - 4k| = \sqrt{8^2 + 1^2 + 4^2}$

$\Rightarrow \sqrt{81} \Rightarrow 9$

Q. Find the value of div $\hat{r}$?

(a) 1

(b) 0

(c) 3

(d) 4

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\text{div}\,\vec{r}$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$$

$$1 + 1 + 1 \Rightarrow 3$$

Q. Find the value of curl $\hat{r}$.

(a) 1

(b) 0

(c) 3

(d) 4

$i \cdot i = 1$

$j \cdot j = 1$

$k \cdot k = 1$

$i \times i = 0$

$j \times j = 0$

$k \times k = 0$

$\underline{\text{Curl } \hat{r}}$: $r = xi + yj + zk$

$$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times (xi + yj + zk)$$

$$i \times i\left(\frac{\partial}{\partial x}(x)\right) + j \times j\left(\frac{\partial}{\partial y}(y)\right) + k \times k\,\frac{\partial}{\partial z}(z)$$

$$0 \quad + \quad 0 \quad + \quad 0$$

Q. If $F = x^2yi - 2xzj + 2yzk$ है

मान ज्ञात कीजिए।

(a) $x + y + z$

(b) $x^2 + y^2 + z^2$

(c) $2y(x + 1)$

(d) $2x(y + 1)$

$$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\left(x^2yi - 2xzj + 2yzk\right)$$

$$\frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(-2xz) + \frac{\partial}{\partial z}(2yz)$$

$$2xy + 0 + 2y$$

$$2y(x+1)$$

Curl f → $\nabla \times f$

→ $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times$

Q. यदि $F = x^2y\,i - 2xz\,j + 2yz\,k$ है तो Curl f का मान ज्ञात कीजिए।

(a) $(2x + 2z)i$

(b) $(2x + 2z)i - (x^2 + 2z)k$

(c) $xi + yj + zk$

(d) $(2x + 2z)j - (x^2 + 2z)k$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -2xz & 2yz \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(2yz) - \frac{\partial}{\partial z}(-2xz)\right) - j\left(\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial z}(x^2y)\right) + k\left(\frac{\partial}{\partial x}(-2xz) - \frac{\partial}{\partial y}(x^2y)\right)$$

$$(2z + 2x)i - (0 - 0)j + (-2z - x^2)k$$

$$(2x + 2z)i - (x^2 + 2z)k$$

Curl Curl F

$\nabla \times (\nabla \times F)$

$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times ($

$\nabla \times (\nabla \times F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x+2z & 0 & -(x^2+2z) \end{vmatrix}$

Q. यदि $F = x^2yi - 2xzj + 2yzk$ है तो Curl Curl F का मान = ?

(a) $(x+2)j$

(b) $(x+4)k$

(c) $(2x+2)j$

(d) $(x+5)k$

$\text{Curl}\ F = (2x+2z)i - (x^2+2z)k$

$(\nabla \times F)$

$\Rightarrow -j(-2x-2)$

$= (2x+2)j$

$i\left(\frac{\partial}{\partial y}(-x^2-2z) - 0\right) - j\left(\frac{\partial}{\partial x}(-x^2-2z) - \frac{\partial}{\partial z}(2x+2z)\right) + k\left(0 - \frac{\partial}{\partial y}(2x+2z)\right)$

Q. यदि $F = xy^2i + 2x^2yzj - 3yz^2k$ है तो div $\hat{F}$ का मान क्या होगा। बिन्दु $(1, -1, 1)$ पर।

(a) 4

(b) 6

(c) 8

(d) 9

$\text{div } F = y^2 + 2x^2z - 6yz$

$\downarrow \text{at } (1, -1, 1)$

$= 1 + 2 + 6$

$= 9$

Q. यदि $F = xy^2i + 2x^2yzj - 3yz^2k$ है तो Curl F का मान ज्ञात कीजिए, बिन्दु $(1, -1, 1)$ पर।

(a) $i + 2k$

(b) $-i - 2k$

(c) $2k$

(d) ~~$3j + 2k$~~

$\nabla \times F =$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & 2x^2yz & -3yz^2 \end{vmatrix}$$

$i(-3z^2 - 2x^2y) - j(0 - 0) + k(4xyz - 2xy)$

at pt $(1, -1, 1)$

$i(-3+2) + k(-4+2)$

$-i - 2k$

Q. यदि $f = x^2zi - 2y^3z^2j + xy^2zk$ है तो बिन्दु $(1, -1, 1)$ पर div f तथा Curl F ज्ञात कीजिए।

(a) $-3, -6i$

(b) $3, 6i$

(c) $-3, -5i$

(d) $-5, -3i$

$\nabla f \rightarrow 2xz - 6y^2z^2 + xy^2$

at Pt $(1,-1,1)$

$2 - 6 + 1$

-3

$$\nabla \times f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2z & -2y^3z^2 & xy^2 \end{vmatrix}$$

$= i(2xy - 4y^3z) - j(y^2 - x^2) + k(0 - 0)$

at Pt $(1,-1,1)$

$= (-2-4)i - (1-1)j$

$= -6i$

Q. If the vector $\mathbf{V} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + az)\mathbf{k}$ is solenoidal, then the constant a is.

(a) 0

(b) 1

(c) -2

(d) 2

Solenoidal vector

$\nabla \cdot f \Rightarrow$

$\text{div } f = 0$

$1 + 1 + a = 0$

$2 + a = 0$

$a = -2$

Q. Show that the vector $\mathbf{V} = (\sin y + z)\mathbf{i} + (x\cos y - z)\mathbf{j} + (x - y)\mathbf{k}$ is irrotational.

Curl F = 0

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin y + z & x\cos y - z & (x-y) \end{vmatrix}$$

$$= i\left(-1-(-1)\right) - j\left(1-1\right) + k\left(\cos y - \cos y\right)$$

$$(-1+1)i - (1-1)j + (\cos y - \cos y)k$$

$$0 - 0 + 0$$

Q. Find the constants a, b, c so that the vector $\mathbf{F} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)j + (4x + cy + 2z)\mathbf{k}$ is irrotational.

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+2y+az) & (bx-3y-z) & (4x+cy+2z) \end{vmatrix} = 0$$

$$i(c-1) - j(4-a) + k(b-2) = 0$$

$c = 1$ $\quad a = 4$ $\quad b = 2$