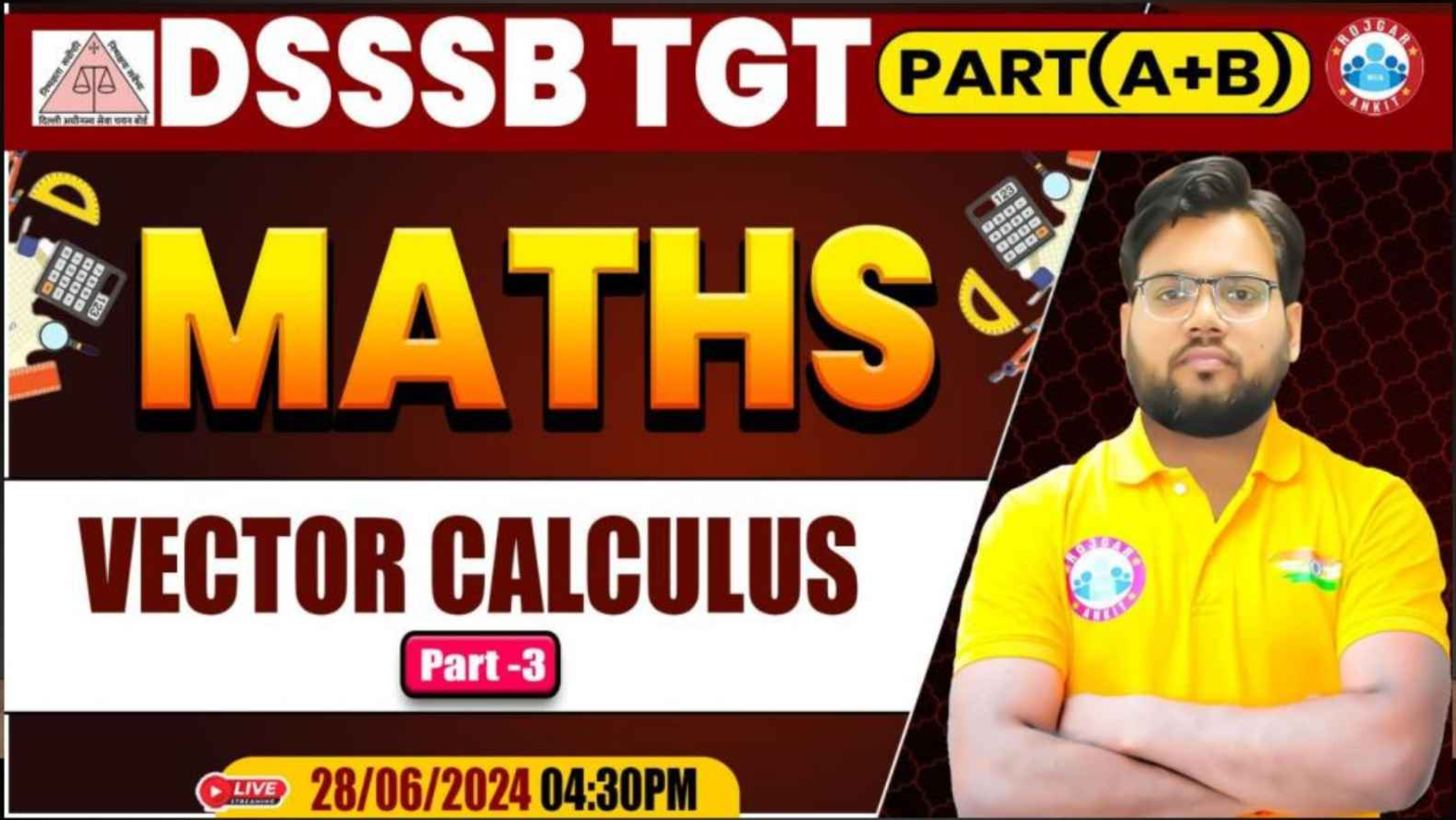
DSSSB TGT
PART(A+B)
MATHS
VECTOR CALCULUS
Part -3
LIVE
28/06/2024 04:30PM

Standard Vector Equation of a Circle
(वृत्त का मानक सदिश समीकरण) :

हम जानते हैं कि मूल बिन्दु पर केन्द्र तथा a त्रिज्या वाले वृत्त पर स्थित किसी बिन्दु के प्राचल निर्देशांक

$(a\cos\theta \, , a\sin\theta)$

$$x = a\cos\ t, y = a\sin\ t \ldots\ldots \qquad (1)$$

अत: $r = xi + yj + zk$ में (1) से प्रतिस्थापित करने पर वृत्त का अभीष्ट मानक सदिश समीकरण $r =$ $(a\cos\ t)i + (a\sin\ t)j + (0)k$,

जहाँ a एक अचर है तथा चर t अदिश है।

Standard Vector Equation of Parabola
(परवलय की मानक सदिश समीकरण) :

$r = (at^2)i + (2at)j + (0)k$, जहाँ a एक अचर है।

$at^2, 2at$

Standard Vector Equation of Ellipse
(दीर्घवृत्त की मानक सदिश समीकरण) :

$\boldsymbol{r} = (a\cos\ t)\boldsymbol{i} + (b\sin\ t)\boldsymbol{j} + (0)\boldsymbol{k}$, जहाँ a और b अचर है।

Circle → $a\cos\theta$, $a\sin\theta$

Ellipse ⇒ $\underline{a}\cos\theta$, $\underline{b}\sin\theta$

Derivative of the Dot Product of two vectors:

(दो सदिशों के बिन्दु गुणनफल का अवकलज):

$$\boxed{\frac{d}{dt}(a.b) = a.\frac{db}{dt} + \frac{da}{dt}.b}$$

Derivative of Scalar Triple Product :

(अदिश त्रिक गुणनफले का अवकलन) :

$$\frac{d}{dt}[a\,b\,c] = \left[\frac{da}{dt}\,b\,c\right] + \left[a\,\frac{db}{dt}\,c\right] + \left[a\,b\,\frac{dc}{dt}\right]$$

Theorem 1.2. The necessary and sufficient condition for any vector (t) to be a constant vector is that $\frac{d}{dt}(a) = 0$

(किसी सदिश a (t) के अचर सदिश होने का आवश्यक एवं पर्याप्त प्रतिबंध $\frac{da}{dt} = 0$)

Derivative of vector Triple Product:

(सदिश त्रिक गुणनफल का अवकलज) :

$$\frac{d}{dt}\{\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})\} = \frac{d\boldsymbol{a}}{dt} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{a} \times \left(\frac{d\boldsymbol{b}}{dt} \times \boldsymbol{c}\right) + \boldsymbol{a} \times \left(\boldsymbol{b} \times \frac{d\boldsymbol{c}}{dt}\right)$$

Derivative of Cross Product of two vectors:

(दो सदिशों के सदिश गुणनफल का अवकलज):

$$\left[\ \frac{d}{dt}(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times \frac{d\boldsymbol{b}}{dt} + \frac{d\boldsymbol{a}}{dt} \times \boldsymbol{b}\right.$$

$(3i + 4j + 5k) \cdot (3i + 9j + 4k)$

$9 + 36 + 20$

$\Rightarrow 65$

Derivative of the Sum and Difference of two vector functions:

(दी सदिशों के योग तथा अंतर का अवकलज) :

$$\boxed{\frac{d}{dt}(\boldsymbol{a} \pm \boldsymbol{b}) = \frac{d\boldsymbol{a}}{dt} \pm \frac{d\boldsymbol{b}}{dt}}$$

Some Theorems on limits (सीमाओं के कुछ प्रयेय):

सदिशों की सीमाओं से संबंधित कुछ प्रमेय निम्न है :

यदि $f(t)$ और $g(t)$ किसी अदिश चर t के दो सदिश फलन हों तथा $\phi(t), t$ का अदिश फ़ून हो, तो :

(i) $\lim_{t\to t_0}[f(t) \pm g(t)] = \lim_{t\to t_0} f(t) \pm \lim_{t\to t_0} g(t)$

(ii) $\lim_{t\to t_0}[f(t)g(t)] = [\lim_{t\to t_0} f(t)] \cdot [\lim_{t\to t_0} g(t)]$

Q. If $F = e^{xy}i + (x-2y)j + x\sin$

find the value of $\frac{\partial^2 F}{\partial x \partial y}$.

(a) $e^{xy}(xy+1)i + \cos\ yk.$

(b) $e^{xy}(x+y+1)$

(c) $e^{xy}(xy+1)i + \sin\ yk$

(d) $xi + yj + zk$

$$F = e^{xy}i + (x-2y)j + x\sin y\,k$$

$$\frac{\partial F}{\partial x} = y e^{xy} i + j + \sin y\,k$$

$$\frac{\partial}{\partial y}\left[\frac{\partial F}{\partial x}\right] \Rightarrow \frac{\partial}{\partial y}\left(\quad \right)$$

$$\Rightarrow \frac{\partial^2 F}{\partial x \partial y} = (1\cdot e^{xy} + y\cdot e^{xy}\cdot x)i + 0 + \cos y\,k$$

$$\Rightarrow (e^{xy} + xy e^{xy})i + \cos y\,k$$

$$e^{xy}[1+xy]i + \cos y\,k$$

$$\frac{\partial F}{\partial y} = x e^{xy} i - 2j + x\cos y\,k$$

$$\frac{\partial^2 F}{\partial x \partial y} = [1\cdot e^{xy} + x e^{xy} y]i - 0 + \cos y\,k$$

$$e^{xy}[1+xy]i + \cos y\,k$$

$$\frac{\partial^3 F}{\partial x \, \partial y \, \partial z}$$

Q. If $\phi(x, y, z) = xy^2z$ and $f = xzi - xyj + yz^2k$, then $(2, -1, 1)$ पर $\frac{\partial^3}{\partial x^2 \partial \cdot z}(\phi f)$ का मान ज्ञात कीजिए।

(a) $4i$

(b) $4i + 2j + 3k$

(c) $2i + 4j$

(d) $4i + 2j$

$(\phi f) = xy^2z(xzi - xyj + yz^2k)$

$(\phi f) = x^2y^2z^2i - x^2y^3zj + xy^3z^3k$

$\frac{\partial}{\partial x}(\phi f) = 2xy^2z^2i - 2xy^3zj + y^3z^3k$

$\frac{\partial^2}{\partial x^2}(\phi f) = 2y^2z^2i - 2y^3zj + 0$

$\frac{\partial^3(\phi f)}{\partial x^2 \partial z} = 4y^2zi - 2y^3j$

at $(2, -1, 1)$

$= 4(-1)^2(1)i - 2(-1)^3j$

$4i + 2j$

Q. यदि $u = xyzi + xz^2j - y^3k$ और $v = x^3i - xyzj + x^2zk$ है तो बिन्दु $(1, 1, 0)$ पर $\frac{\partial^2 u}{\partial y^2} \times \frac{\partial^2 v}{\partial x^2}$ का मान ज्ञात करो।

(a) $36j$

(b) $-36j$

(c) 48

(d) 36

$\frac{\partial u}{\partial y} = xzi + 0 - 3y^2k$

$\frac{\partial^2 u}{\partial y^2} = 0 + 0 - 6yk$

at $(1,1,0)$

$= -6(1)k$

$\Rightarrow -6k$

$\frac{\partial v}{\partial x} = 3x^2i - yzj + 2xzk$

$\frac{\partial^2 v}{\partial x^2} = 6xi - 0 + 2zk$

at $(1,1,0)$

$\frac{\partial^2 v}{\partial x^2} = 6 \times 1 i - 0 + 0$

$\frac{\partial^2 v}{\partial x^2} = 6i$

$-6k \times 6i \Rightarrow -36\, k \times i$

$\Rightarrow -36j$

Q. यदि $A = x^2yzi - 2xz^3j + xz^2k$ तथा $B = 2zi + yj - x^2k$ है तो बिन्दु $(1, 0, -2)$ पर $\frac{\partial^2}{\partial x \partial y}(A \times B)$ का मान ज्ञात कीजिए

(a) $4i + 8j + 5k$

(b) $-4i - 8j$

(c) $2i + 4j + 5k$

(d) $6i + 4j + 3k$

Q. यदि $f(x,y,z) = 3x^2y - y^3z^2$ है तो बिन्दु $(1, -2, -1)$ पर (gradf f का मान ज्ञात कीजिए।?

(a) $-4i - 12j + 9k$

(b) $-12i - 16j - 9k$.

(c) $-12i - 9j - 16k$

(d) $12i + 9j + 16k$

$\nabla f \Rightarrow 6xy\, i + (3x^2 - 3y^2z^2)\, j - 2y^3 z\, k$

$\Rightarrow 6(1)(-2)\, i + (3(1)^2 - 3(-2)^2(-1)^2)\, j - 2(-2)^3 \times (-1)\, k$

$\Rightarrow -12i - 9j - 2 \times (-8)(-1)\, k$

$-12i - 9j - 16k$

grad $f \Rightarrow \nabla f = \left(i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}\right)$

$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) f$

$\Rightarrow i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

Q. Gradient of $e^{(x^2+y^2+z^2)}$ at $(1,1,1)$

(a) $2c^3(x+y+z)$

(b) $e^3(x+y+z)$

(c) $2e^3(i+j+k)$

(d) $2(i+j+k)$

$\nabla f \Rightarrow e^{(x^2+y^2+z^2)} \cdot 2x\, i + e^{x^2+y^2+z^2} \cdot 2y\, j + e^{x^2+y^2+z^2} \cdot 2z \cdot k$

$= 2e^{(x^2+y^2+z^2)}(xi+yj+zk)$ at point $(1,1,1) \Rightarrow 2e^3(i+j+k)$

Q Find the directional derivative of $f = xy + yz + zx$ in the direction of the vector $i + 2j + 2k$ at the point $(1, 2, 0)$

(a) 5

(b) $\frac{5}{3}$

(c) $\frac{10}{3}$

(d) $\frac{20}{3}$

$f = xy + yz + zx$

$\nabla f \Rightarrow (y+z)i + (x+z)j + (y+x)k$

at $(1,2,0)$

$\nabla f = 2i + j + 3k$

$DD \Rightarrow \nabla f \cdot \hat{N}$

$(2i + j + 3k) \cdot \frac{(i + 2j + 2k)}{3}$

$\Rightarrow \frac{2+2+6}{3} \Rightarrow \frac{10}{3}$

grad $f \cdot \hat{a}$

$\vec{N} \Rightarrow i + 2j + 2k \quad \Rightarrow \nabla f \cdot \frac{\vec{N}}{|\vec{N}|}$

$|\vec{N}| = \sqrt{1^2 + 2^2 + 2^2} \Rightarrow 3$

Q Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2i - j - 2k.$

(a) $\frac{37}{3}$ (b) $\frac{38}{3}$

(c) $\frac{16}{3}$ (d) $\frac{23}{9}$

$DD \Rightarrow \nabla f \cdot \frac{\vec{N}}{|\vec{N}|}$

$\nabla f \Rightarrow (2xyz + 4z^2)i + (x^2z)j + (x^2y + 8xz)k$

at $(1, -2, -1)$

$\nabla f \Rightarrow (4+4)i - j + (-2-8)k$

$8i - j - 10k$

$\vec{N} = 2i - j - 2k$

$|\vec{N}| \Rightarrow \sqrt{4+1+4} \Rightarrow 3$

$DD \Rightarrow \nabla f \cdot \frac{\vec{N}}{|\vec{N}|} \Rightarrow \frac{(8i - j - 10k)\cdot(2i - j - 2k)}{3}$

$\Rightarrow \frac{16+1+20}{3} \Rightarrow \frac{37}{3}$

Q. Find the directional derivative of $f(x,y,z) = x^2 - 2y^2 + 4z^2$ at the point $(1,1,-1)$ in the direction of $2i + j - k$.

(a) $\frac{8}{\sqrt{6}}$

(b) $\frac{4}{\sqrt{6}}$

(c) $\frac{33}{7}$

(d) $\frac{49}{\sqrt{8}}$

$\vec{N} = 2i + j - k$

$|\vec{N}| = \sqrt{4+1+1} = \sqrt{6}$

$\nabla f = 2xi - 4yj + 8zk$

at point $(1,1,-1)$

$\nabla f = 2i - 4j - 8k$

$DD = \nabla f \cdot \frac{\vec{N}}{|\vec{N}|}$

$= \frac{\cancel{4} - \cancel{4} + 8}{\sqrt{6}}$

$= \frac{8}{\sqrt{6}}$

Q. Find the DD of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, 1)$ in the direction of $2i - j - 2k$.

(a) $\frac{13}{3}$

(b) $-\frac{13}{3}$

(c) $\frac{26}{3}$

(d) $\frac{-26}{3}$

$|\vec{N}| \Rightarrow 3$

$\nabla f = (2xyz + 4z^2)i + (x^2z)j + (x^2y + 8xz)k$

at $(1, -2, 1)$

$= (-4+4)i + j + 6k$

$\nabla f \Rightarrow 0i + j + 6k$

$DD \Rightarrow \frac{0 - 1 - 12}{3}$

$\Rightarrow \frac{-13}{3}$

Q. obtain the DD of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\cancel{i+j+2k}$.

$i+2j+2k$.

(a) 3

(b) 2

(c) -3

(d) -5

Q. Find the maximum value of the directional derivative of $\phi = x^2 y =$ at the point $(1, 4, 1)$

(a) 4

(b) 6

(c) 8

(d) 9

Q. Find the greatest value of the directional derivative of the function $2x^2 - y - z^4$ at the point $(2, -1, 1)$:

(a) 9

(b) 6

(c) $\sqrt{9}$

(d) 3