



DSSSB TGT

PART (A+B)



MATHS

ADVANCE CALCULUS (LAST PART)



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$$u = x^3 y^2 - x^4 y^2 - x^3 y^3$$

$$\frac{\partial u}{\partial x} = 3x^2 y^2 - 4x^3 y^2 - 3x^2 y^3 = 0$$

$$3 - 4x - 3y = 0$$

$$4x + 3y = 3 \quad \text{--- (i)}$$

$$\frac{\partial u}{\partial y} = 2x^3 y - 2x^4 y - 3x^3 y^2 = 0$$

$$(x^3 y) \rightarrow \text{common} -$$

$$2 - 2x - 3y = 0$$

$$2x + 3y = 2 \quad \text{--- (ii)}$$

following functions:

निम्नलिखित फलनों के उच्चिष्ठ और चर्चा कीजिए:

1. $u = xy + a^3 (1/x + 1/y).$

2. $u = x^3 y^2 (1 - x - y).$

3. $u = x^3 + y^3 - 3axy.$

$$u = x^3 + y^3 - 3axy$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3ay$$

$$3x^2 = 3ay$$

$$x^2 = ay$$

$$y = \frac{x^2}{a}$$

$$(y = a)$$

$$\frac{\partial u}{\partial y} = 3y^2 - 3ax = 0$$

$$y^2 = ax$$

$$\frac{x^3}{a^2} = ax$$

$$x^3 = a^3$$

$$(x = a)$$

$$r = 6a$$

$$r = \frac{\partial^2 u}{\partial x^2} = 6x$$

$$t = 6a$$

$$t = \frac{\partial^2 u}{\partial y^2} = 6y$$

$$s = \frac{\partial^2 u}{\partial x \partial y} = -3a$$

$$rt - s^2 = 27a^2 > 0$$

$$6a \times 6a - (-3a)^2$$

$$36a^2 - 9a^2 \Rightarrow$$

$$r = 6a$$

$$a^+ \quad r > 0$$

$$a^- \quad r < 0$$

$$\textcircled{8} \quad u = 2x^2y + x^2 - y^2 + 2y$$

$$\frac{\partial u}{\partial x} = 4xy + 2x = 0$$

$$24xy = -2x$$

$$y = -\frac{1}{2}$$

$$6x^2y$$

$$\frac{\partial u}{\partial y} = 2x^2 - 2y + 2 = 0$$

$$2x^2 - 2\left(-\frac{1}{2}\right) + 2 = 0$$

$$2x^2 + 1 + 2 = 0$$

$$2x^2 = -3$$

$$x = \sqrt{\frac{-3}{2}}$$

$$4. \quad u = x^2 + y^2 + 6x + 12.$$

$$5. \quad u = x^2 + y^2 + 2/x + 2/y.$$

$$6. \quad u = y^2 + 4xy + 3x^2 + x^3.$$

$$7. \quad u = 3x^2 - y^2 + x^3.$$

$$8. \quad u = 2x^2y + x^2 - y^2 + 2y.$$

ANSWERS:-

✓ 1. Minimum at $x = y = a$ (a, a)

2. Maximum at $\left(\frac{1}{2}, \frac{1}{3}\right)$

3. (a, a) $x = y = a$ gives ^{maximum} or a minimum according as a is negative or positive.

4 Minimum at $(-3, 0)$.

5. Minimum at (1,1)

6. minimum at $\left(\frac{2}{3}, \frac{-4}{3}\right)$

$$\begin{aligned} \eta f - S^2 &> 0 \\ \eta &> 0 \text{ min}^m \end{aligned}$$

7. Maximum at (-2,0)

$$\begin{aligned} \eta f - S^2 &> 0 \\ \eta &< 0 \text{ max}^m \end{aligned}$$

8. No Extreme value.

$$\eta f - S^2 < 0$$

Diff w.r.t. m

$$0 = 2mx - \frac{2}{m^3}$$

$$2mx = \frac{2}{m^3}$$

$$m^4 = \frac{1}{x}$$

$$m^2 = \frac{1}{\sqrt{x}}$$

2. Find The Envelope of the straight line

$y = m^2x + \frac{1}{m^2}$ parameter being m .

(a) $y = 4ax$

(b) $y^2 = 4ax$

(c) $y^2 = 4x$

(d) $x^2 + y^2 = a^2$

$$y = \frac{1}{\sqrt{x}}x + \frac{1}{\sqrt{x}}$$

$$y = \sqrt{x} + \sqrt{x}$$

$$y = 2\sqrt{x}$$

$$y^2 = 4x$$

3. Find the Envelope of the straight line

$y = mx \pm a\sqrt{1+m^2}$ parameter being m .

$$y - mx = a\sqrt{1+m^2}$$

$$(y - mx)^2 = a^2(1+m^2)$$

$$y^2 + m^2x^2 - 2mxy = a^2 + a^2m^2$$

$$(x^2 - a^2)m^2 - 2xym + y^2 - a^2 = 0$$

$$B^2 - 4AC = 0$$

$$4x^2y^2 = 4(x^2 - a^2)(y^2 - a^2)$$

$$x^2y^2 = x^2y^2 - a^2x^2 - a^2y^2 + a^4$$

$$(a) \ x^2 + y^2 = a^2$$

$$(b) \ x^2 + y^2$$

$$(c) \ x^2 = a^2$$

$$(d) \ y^2 = a^2 + x^2$$

$$y = mx \pm a\sqrt{1+m^2}$$

$$0 = -a^2x^2 - a^2y^2 + a^4$$

$$x^2 + y^2 = a^2$$

$$x^2 + \alpha^2 - 2x\alpha + y^2 - 4\alpha = 0$$

$$\alpha^2 - \alpha(2x + 4) + y^2 + x^2 = 0$$

$$B^2 - 4AC = 0$$

$$(2x + 4)^2 - 4 \times 1 \times (x^2 + y^2) = 0$$

$$4x^2 + 16 + 16x - 4x^2 - 4y^2 = 0$$

$$-4y^2 + 16x + 16 = 0$$

$$y^2 - 4x - 4 = 0$$

4. Find the Envelope of the Circle $(x - \alpha)^2 + y^2 = 4\alpha$, α parameter.

(a) $x^2 + 4x = 4$

(b) $y^2 + 4x = 4$

(c) $y^2 - 4x = 0$

(d) $y^2 - 4x - 4 = 0$

$$x^2 + \alpha^2 - 2x\alpha + y^2 + \alpha^2 - 2y\alpha - 2\alpha = 0$$

$$2\alpha^2 - \alpha(2x + 2y + 2) + x^2 + y^2 = 0$$

$$B^2 - 4AC = 0$$

$$(2x + 2y + 2)^2 - 4 \times 2(x^2 + y^2) = 0$$

$$\sqrt{(x+y+1)^2} = \sqrt{4(x^2 + y^2)}$$

$$(x+y+1)^2 = 2(x^2 + y^2)$$

5. Find the envelope of the Circle

$$(x - \alpha)^2 + (y - \alpha)^2 = 2\alpha, \alpha \quad \mathbf{b}$$

parameter.

$$\mathbf{(a)} \quad (x + y + 1)^2 = x^2 + y^2$$

$$\mathbf{(b)} \quad (x + y + 1)^2 = 2(x^2 + y^2)$$

$$\mathbf{(c)} \quad (x + y)^2 = x^2 + y^2 + 2$$

$$\mathbf{(d)} \quad (x + y + 2)^2 = 2(x^2 + y^2)$$



6. Find the envelope of the family of parabolas $y^2 = m^2 (x - m)$ m being the parameter.

7. option पिछले question के

(a) $x^3 = y^2$

(b) $4x^3 = y^2$

(c) $4x^3 = 27y^2$

(d) $x^3 = 4y^2$

8. Find the envelope of the family of parabolas $tx^3 + t^2y = a$ the parameter being t .

9. Option पिछले question के

(a) $x^2 + 4ay = 0$

(b) $x^6 + 4ay = 0$

(c) $x^6 + 40x = 0$

(d) $x^4 + 4ay = 0$

10. Find all the asymptotes of the following curves:

1. $a^2/x^2 + b^2/y^2 = 1.$

→ 2. $y^2(x^2 - a^2) = x.$

$x=0$

3. $xy^2 = 4a^2(2a - x).$

4. $x^2y^2 = a^2(x^2 + y^2).$

→ 5. $y^2(x^2 - a^2) = x^2(x^2 - 4a^2).$

6. $y^3 = x^2 + 3.$

$$\begin{aligned} xy^2 &= -4a^2x \\ xy^2 + x4a^2 &= 0 \\ y^2 + 4a^2 &= 0 \\ y^2 &= -4a^2 \\ y &= \pm i2a \end{aligned}$$

$$2. \quad y^2(x^2 - a^2) = x$$

$$y^2 x^2 - y^2 a^2 - x = 0$$

y -अक्ष के समानांतर

$$x^2 - a^2 = 0$$

$$x^2 = a^2$$

$$x = \pm a$$

x -अक्ष के समानांतर

$$y^2 = 0$$

$$y = 0$$

11. ANSWERS

1 $x = \pm a, y = \pm b.$

2 $x = \pm a, y = 0.$

3 $x = 0.$

4 $x = \pm a, y = \pm a.$

5 $x = \pm a, y = \pm x.$

6 No asymptotes.