DSSSB TGT PART(A+B)

MATHS

CALCULUS
(REVISION CLASS)

LIVE

23/06/2024 10:00AM

1. If $u = x^1 y^1 f\left(\frac{y}{x}\right)$ then write the value of the expression $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

$\frac{\partial u}{\partial x} = y\left[1 \cdot f\left(\frac{y}{x}\right) + x f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)\right]$

$= yf\left(\frac{y}{x}\right) - \frac{xy^2 f'\left(\frac{y}{x}\right)}{x^2}$

यदि $u = xyf\left(\frac{y}{x}\right)$ तो व्यंजक $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान लिखिए।

(a) u (b) $2u$ (c) 0 (d) 4

$x\frac{\partial u}{\partial x} = xyf\left(\frac{y}{x}\right) - \frac{x^2 y^2 f'\left(\frac{y}{x}\right)}{x^2}$

$x\frac{\partial u}{\partial x} = u - y^2 f'\left(\frac{y}{x}\right)$

$\frac{\partial u}{\partial y} = x\left[1 \cdot f\left(\frac{y}{x}\right) + y f'\left(\frac{y}{x}\right) \cdot \frac{1}{x}\right]$

$y \cdot \frac{\partial u}{\partial y} = xyf\left(\frac{y}{x}\right) + \frac{xy^2 f'\left(\frac{y}{x}\right)}{x}$

$u + y^2 f'\left(\frac{y}{x}\right)$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u + u$

$= 2u$

$$\frac{xy\,f\left(\frac{y}{x}\right)}{(3x+2x+x)}$$

$$\frac{xy\,(3y+2y+y)}{(3x+2x+x)}$$

$$\frac{3}{1}$$

$3-1$

$\Rightarrow 24$

2. If $u = e^{my}\cos mx$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ का मान क्या होगा?

यदि $u = e^{my}\cos mx$, तो $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ का मान क्या होगा?

(a) 0

(b) 1

(c) u

(d) 4

$\frac{\partial u}{\partial x} = e^{my}(-\sin mx)\, m$

$\Rightarrow -me^{my}\sin mx$

$\frac{\partial^2 u}{\partial x^2} = -m^2 e^{my}\cos mx$

$-u$

$\frac{\partial u}{\partial y} = e^{my} \cdot m\cos mx$

$\Rightarrow me^{my}\cos mx$

$\frac{\partial^2 u}{\partial y^2} = m^2 e^{my}\cos mx$

u

3. If $u = log\,(x^2 + y^2)$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ का मान क्या होगा?

यदि $u = \log(x^2 + y^2)$ है, तो $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ का मान क्या होगा?

(a) 1

(b) 0

(c) 3

(d) 3u

$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2} \cdot 2x$

$\frac{\partial^2 u}{\partial x^2} = \frac{2(x^2+y^2) - 2x(2x)}{(x^2+y^2)^2}$

$\Rightarrow \frac{2x^2+2y^2-4x^2}{(x^2+y^2)^2}$

$\frac{\partial^2 u}{\partial x^2} \Rightarrow \frac{2y^2-2x^2}{(x^2+y^2)^2}$

$\frac{\partial u}{\partial y} \Rightarrow \frac{1}{x^2+y^2}(2y)$

$\frac{\partial^2 u}{\partial y^2} = \frac{2(x^2+y^2) - 2y(2y)}{(x^2+y^2)^2} \Rightarrow \frac{2x^2-2y^2}{(x^2+y^2)^2}$

$\Rightarrow -\frac{(2y^2-2x^2)}{(x^2+y^2)^2}$

4. If $u = x^2y + y^2z + z^2x$, तो $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ का मान क्या होगा?

$x^2y^2+z^2+2(xy+yz+zx)$

$(x+y+z)^2$

$\frac{\partial u}{\partial x} = 2xy + z^2$

$\frac{\partial u}{\partial y} = 2yz + x^2$

$\frac{\partial u}{\partial z} = 2zx + y^2$

(a) x^2

(b) y^2

(c) $(x + y + z)$

(d) $(x + y + z)^2$

5. Verify Euler's theorem for the function

$$u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}.$$

फलन $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$ के लिए यूलर प्रमेय का सत्यापन करें।

(a) u (b) $\frac{1}{20}u$ (c) $3u$ (d) $4u$

$\frac{1}{4} - \frac{1}{5}$

$\Rightarrow \frac{1}{20}u$

6. If $u = log\frac{x^2+y^2}{x+y}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान क्या होगा?

$\frac{\partial u}{\partial x} = \frac{(x+y)}{x^2+y^2} \cdot \frac{2x(x+y)-(x^2+y^2)}{(x+y)^2}$

यदि $u = \log\frac{(x^2+y^2)}{x+y}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान

$\frac{\partial u}{\partial x} = \frac{2x^2+2xy-x^2-y^2}{(x+y)(x^2+y^2)}$ क्या होगा?

(a) 1

(b) 0

(c) 2

(d) 4

$\Rightarrow \frac{x^2-y^2+2xy}{?}$

$x\frac{\partial u}{\partial x} = \frac{x^3-xy^2+2x^2y}{(x+y)(x^2+y^2)}$

$\frac{\partial u}{\partial y} = \frac{(x+y)}{x^2+y^2} \cdot \frac{2y(x+y)-(x^2+y^2)}{(x+y)^2}$

$\Rightarrow \frac{y^2-x^2+2xy}{(x+y)(x^2+y^2)}$

$y\frac{\partial u}{\partial y} = \frac{y^3-x^2y+2xy^2}{(x+y)(x^2+y^2)}$

7. यदि $u = \frac{\log(x^4+y^4)}{x+y}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान क्या होगा?

If $u = log\frac{x^4+y^4}{x+y}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान क्या होगा?

4-1=3

(a) 1

(b) 2

(c) 3

(d) 4

8. If $u = log \frac{x^3+y^3}{x+y}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ का मान क्या होगा?

(a) 1

(b) 2

(c) 3

(d) 4

3-1 = 2

24

9. If u is a homogeneous function of x and y of degree n, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to.

nu

यदि u, x और y का घात n वाला समरूप फलन है, तो $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ बराबर है।

(a) n

(b) n u

(c) $\frac{n}{u}$

(d) u

$\frac{1}{u} = \sqrt{x^2+y^2+z^2}$

10. If $\frac{1}{u} = \sqrt{(x^2 + y^2 + z^2)}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to.

यदि $\frac{1}{u} = \sqrt{(x^2 + y^2 + z^2)}$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ बराबर है।

(a) u

(b) $-u$

(c) u^2

(d) 0

$\frac{1}{u^2} = x^2+y^2+z^2$

$\frac{-2}{u^3}\frac{\partial u}{\partial x} = 2x$

$x\frac{\partial u}{\partial x} = -u^3x^2$

इसी तरह

$y\frac{\partial u}{\partial y} = -u^3y^2$

$z\frac{\partial u}{\partial z} = -u^3z^2$

$-u^3(x^2+y^2+z^2)$

$= -u^3 \times \frac{1}{u^2}$

$= -u$

11. If f(x, y) be a homogeneous function of x and y of degree n, then

यदि f(x, y) x और y का n घात वाला एक समरूप फलन है, तो

(a) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$

(b) $y\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = f$

(c) $y\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = nf$

(d) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

$\frac{\partial f}{\partial x} = 2x - \frac{2}{x^2} = 0$

$x = 1$

$\frac{\partial f}{\partial y} = 2y - \frac{2}{y^2} = 0$

$y = 1$

$r = \frac{\partial^2 f}{\partial x^2} = 2 + \frac{4}{x^3} = 6$ at $x=1, y=1$

$\frac{1}{x^2} = x^{-2}$

$-2x^{-3}$

12. The function $f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$.

$rt - s^2 > 0$ $r < 0$ max

फलन $f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$.

(A) (1,1) पर स्थानिय उच्चिष्ठ ma.

(B) (1,1) पर स्थानिय निम्निष्ठ min

(C) (0,0) पर स्थानिय अधिष्ठ max

(D) (0,0) पर स्थानिय चिम्निष्ठ min

13. $f(x,y) = 2x^4 + y^2 - x^2 - 2y$ holds.

$f(x,y) = 2x^4 + y^2 - x^2 - 2y$ रखता है।

(a) बिन्दु $(\frac{1}{2}, 1)$ पर स्थानीय निम्निष्ठ min

(b) बिन्दु $(1, \frac{1}{2})$ पर स्थानीय उच्चिष्ठ max

(c) बिन्दु (0, 1) पर स्थानीय उच्चिष्ठ max

(d) बिन्दु (0,1) पर स्थानीय निम्निष्ठ min

$\frac{\partial f}{\partial x} = 8x^3 - 2x = 0$

$x = \frac{1}{2}$, 0

$\frac{\partial f}{\partial y} = 2y - 2 = 0$

$y = 1$

$r = \frac{\partial^2 f}{\partial x^2} = 24x^2 - 2 \Rightarrow 4$

at $x = \frac{1}{2}$

$rt - s^2 > 0$

$r > 0$

$r > 0$

$\Rightarrow \boxed{2 = r}$

minimum at $x = y = a$

$\frac{\partial u}{\partial x} = y - \frac{a^3}{x^2} = 0$

$y = \frac{a^3}{x^2} \Rightarrow \boxed{y = a}$

$\frac{\partial u}{\partial y} = x - \frac{a^3}{y^2} = 0$

$x = \frac{a^3}{a^6} x^4$

$1 = \frac{x^3}{a^3}$

$\boxed{x = a}$

निम्नलिखित फलनों के
चर्चा कीजिए:

1. $u = xy + a^3$ (1

2. $u = x^3 y^2 (1 -$

3. $u = x^3 + y^3 -$

5. $u = x^2 + y^2 + 2/x + 2/y.$

6. $u = y^2 + 4xy + 3x^2 + x^3.$

7. $u = 3x^2 - y^2 + x^3.$

8. $u = 2x^2y + x^2 - y^2 + 2y.$

15. मान लीजिए $f(x, y)$ दो स्वतंत्र चर x और y का एक फलन है।

Let $f(x, y)$ be a function of two independent variables x and y Let.

$$r = \frac{\partial^2 f}{\partial x^2}, s = \frac{\partial^2 f}{\partial x \partial y} \text{ and } t = \frac{\partial^2 f}{\partial y^2}$$

If at the point (a, b), we have $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$, and $\frac{\partial f}{\partial y} =$ 0, then $f(x, y)$ is maximum at (a, b) if at (a,b) we have.

यदि बिंदु (a,b) पर हमारे पास $\frac{\partial f}{\partial x} = 0$ और $\frac{\partial f}{\partial y} = 0$, और $\frac{\partial f}{\partial y} = 0$ है, तो f(x,y) (a,b) पर अधिकतम है यदि (a,b) पर हमारे पास है।

(a) $m - s^2 < 0$

(b) $rt - s^2 > 0 \text{ and } r < 0$

(c) $rt - s^2 > 0 \text{ and } r > 0$

(d) $rt - s^2 < 0 \text{ and } r < 0$

$rt - s^2 > 0$

$r < 0$

No Extreme points

16. फलन $u = x^2 + y^2 + 6x + 12$ किस [redacted] बिन्दु पर न्यूनतम [redacted] है।

The function $u = x^2 + y^2 + 6x + 12$ is minimum at no

(a) (3,0)

(b) (0,3)

(c) (-3,0)

(d) (3,3)

$\frac{\partial u}{\partial x} = 2x + 6 = 0$

$x = -3$

$r = \frac{\partial^2 u}{\partial x^2} = 2 > 0$

17. The function $u = 3x^2-y^2 + x^3$ is maximum at

फलन $u = 3x^2-y^2 + x^3$ अधिकतम है ?

$\frac{\partial u}{\partial x} = 6x+3x^2 = 0$

$3x^2 = -6x$

$x = -2$

(a) (-2,0)

(b) (2,0)

(c) (0,-2)

(d) (0,2)

$\frac{\partial u}{\partial y} = -2y = 0$

$y = 0$

18. Find the radius of curvature at origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$.

वक्र $x^3 + y^3 - 2x^2 + 6y = 0$ के लिए मूल बिंदु पर वक्रता त्रिज्या ज्ञात कीजिए।

Diff. w.r.t. x

$$3x^2 + 3y^2\frac{dy}{dx} - 4x + 6\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4x - 3x^2}{3y^2 + 6}$$

at $(0,0)$ $= 0$

$$\frac{d^2y}{dx^2} = \frac{(4 - 6x)(3y^2 + 6) - (4x - 3x^2)(- - -)}{(3y^2 + 6)^2}$$

at $(0,0)$

$$\frac{d^2y}{dx^2} = \frac{24}{36} = \frac{2}{3}$$

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{1}{\frac{2}{3}}$$

$$= \frac{3}{2}$$

$\frac{d^2y}{dx^2} = \frac{-1\sqrt{a}}{2x\sqrt{x}}$

19. Find the radius of curvature at the point (x, y) on the following curves:

निम्नलिखित वक्रों पर बिंदु (x, y) पर वक्रता त्रिज्या ज्ञात कीजिए:

1. $a^2y = x^3 - a^3$
2. $y^2 = 4ax$ $\Rightarrow y = \sqrt{4ax} \Rightarrow 2\sqrt{ax}$
3. $xy = c^2$
4. $ay^2 = x^3$

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{y} \Rightarrow \frac{2a}{2\sqrt{ax}}$

$\frac{dy}{dx} \Rightarrow \frac{\sqrt{a}}{\sqrt{x}}$

20. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$.

बिन्दु $\left(\frac{1}{4}, \frac{1}{4}\right)$ पर वक्र $\sqrt{x} + \sqrt{y} = 1$ की वक्रता त्रिज्या ज्ञात कीजिए।

$\sqrt{x} + \sqrt{y} = 1$

Diff w.r.t. x

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \times 2\sqrt{y} \Rightarrow -\frac{\sqrt{y}}{\sqrt{x}}$

$\frac{dy}{dx} \Rightarrow -1$

$\frac{d^2y}{dx^2} = -\left[\frac{\frac{1}{2\sqrt{y}}\frac{dy}{dx}\sqrt{x} - \sqrt{y}\cdot\frac{1}{2\sqrt{x}}}{x}\right]$

$= -\left[\frac{\frac{1}{2}(-1) - \frac{1}{2}}{\frac{1}{4}}\right]$

$= -\left[\frac{-4}{}\right]$

$\frac{d^2y}{dx^2} = 4$

$\rho = \frac{\left\{1 + (-1)^2\right\}^{3/2}}{4}$

$\Rightarrow \frac{2^{3/2}}{2^2} \Rightarrow \frac{1}{\sqrt{2}}$