DSSSB TGT PART(A+B)

MATHS

ADVANCE CALCULUS

CURVATURE

LIVE STREAMING

18/06/2024 04:30PM

1. $If\ u = ax^2 + 2hxy + by^2 then$

यदि $u = ax^2 + 2hxy + by^2$ है तो

(a) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$

(b) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

(c) $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^{2u}}{\partial y^2}$

(d) $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$

2. If $u = f\left(\frac{y}{x}\right)$ then what is the value of $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y}$?

यदि $u = f\left(\frac{y}{x}\right)$ है तो $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y}$ का मान क्या होगा?

(a) 1

(b) 2

(c) 3

(d) 0

3. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then what is the value of $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$?

यदि $u = log\,(x^3 + y^3 + z^3 - 3xyz)$ है तो $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$का मान क्या होगा?

(a) $(x + y + z)$

(b) $(x^2 + y^2 + z^2)$

(c) $\frac{3}{x+y+z}$

(d) $\frac{-3}{x+y+z}$

4. If $u = \frac{x^2}{a^2} + \frac{y^2}{b^2 is} - 1$, then what is the value of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$?

यदि $u = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$ है, तो $\frac{\partial u}{\partial x}$ व $\frac{\partial u}{\partial y}$ का मान क्या होगा?

(a) $\frac{x}{a^2}, \frac{y}{b^2}$

(b) $\frac{2x}{a}, \frac{2y}{b}$

(c) $\frac{2x}{a^2}, \frac{2y}{b^2}$

(d) $\frac{1}{a}, \frac{1}{b}$

5. $If\ f(x,y) = x\,cosy + y\,sinx, then -$

यदि $f(x,y) = x\,cosy + y\,sinx$ है, तब-

(a) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

(b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$

(c) $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$

(d) $\left(\frac{\partial f}{\partial x}\right)^2 = \left(\frac{\partial f}{\partial y}\right)^2$

6. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then What is the value of $\left(x.\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$?

यदि $u = sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$ है, तो $\left(x.\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$ का मान क्या होगा?

(a) 1

(b) 0

(c) 5

(d) -2

7. If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, then what is the value of $\frac{\partial^2 z}{\partial x \partial y}$?

यदि $z = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$ है, तो $\frac{\partial^2 z}{\partial x \partial y}$ का मान क्या होगा?

(a) $\frac{x+y}{x-y}$

(b) $\frac{x^2+y^2}{x^2-y^2}$

(c) $\frac{x}{y}$

(d) $\frac{x^2-y^2}{x^2+y^2}$

8. If $u = \log(x^2 + y^2)$, then what is the value of $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$?

यदि $u = log(x^2 + y^2)$ है, तो $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ का मान क्या होगा?

(a) 0

(b) 1

(c) 2

(d) 3

9. $If\ f(x,y) = x^4 + x^2y^2 + y^4 then -$

यदि $f(x,y) = x^4 + x^2y^2 + y^4$ है तब-

(a) $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 12$

(b) $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 14$

(c) $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 4$

(d) इनमे से कोई नहीं

HOMOGENEOUS FUNCTIONS.

10. x और y में एक व्यंजक जिसमें प्रत्येक पद समान घात का हो, x और y का समरूप फलन कहलाता है। द्वारा परिभाषित फलन पर विचार करें

An expression in x and y in which every term is of the same degree is called a homogeneous function of x and y. Consider the function defined by

$$f(x,y) = a_0x^2 + a_1x^{n-1}\,y + a_2\,x^{n-2}\,y^2 + \ldots + a_{n-1}\,xy^{n-1} + a_ny^n.$$

इस फ़ंक्शन में प्रत्येक पद n डिग्री का है। इसलिए यह x और y का n डिग्री का एक समरूप फ़ंक्शन है। इसके अलावा, (1) को इस प्रकार लिखा जा सकता है

In this function every term is of degree n. Therefore it is a homogeneous function of x and y of degree n. Moreover, (1) may be written as

$$f(x,y) = x^n\left\{a_0 + a_1\left(\frac{y}{x}\right) + a_2\left(\frac{y}{x}\right)^2 + \cdots\ldots + a_n\left(\frac{y}{x}\right)^n\right\}$$

or $f(x,y) = x^nF(y/x),$

EULER'S THEOREM ON HOMOGENEOUS FUNCTIONS.

11. यदि u, x और y का घात n वाला एक समरूप फलन है, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$

If u be a homogeneous function of x and y of degree n, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$

This proves the theorem.

12. In general, if u be a homogeneous function of $x_1, x_2, \ldots, x_m$ of degree n, then.

सामान्यतः, यदि u $x_1, x_2, \ldots, x_m$ का घात n वाला एक समरूप फलन हो, तो।

$$x_1 \frac{\partial u}{\partial x_1} + x_2 \frac{\partial u}{\partial x_2} + \ldots\ldots + \partial_x \frac{\partial u}{\partial x_m} = nu.$$

13. *If* $u = f(x, y, z)$ **is a homogeneous function of degree** n **in variables** x, y **and** z**, then**

यदि $u = f(x, y, z)$ **चरों** x, y **तथा** z **में** n **घात का समघात फलन हो तो**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$$

14. If u be a homogeneous function of x and y of degree n, Then.

यदि u, x और y का घात n वाला एक समरूप फलन हो, तो.

(a) $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$.

(b) $x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$

(c) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$.

15. If $f(x, y) = 2x^2 + 2xy + 4y^2$ then what is the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$?

यदि $f(x, y) = 2x^2 + 2xy + 4y^2$ तो $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ का मान है?

(a) $2f(x, y)$

(b) $3f(x, y)$

(c) $4f(x, y)$

(d) $f(x, y)$

16. If $u = 2x^2 + xy + 4y^2$, then what is the value of $x.\frac{\partial^2 u}{\partial x^2} + y.\frac{\partial^2 u}{\partial x \partial y}$?

यदि $u = 2x^2 + xy + 4y^2$ है, तो $x.\frac{\partial^2 u}{\partial x^2} + y.\frac{\partial^2 u}{\partial x \partial y}$ का मान क्या होगा?

(a) u

(b) $\frac{\partial u}{\partial x}$

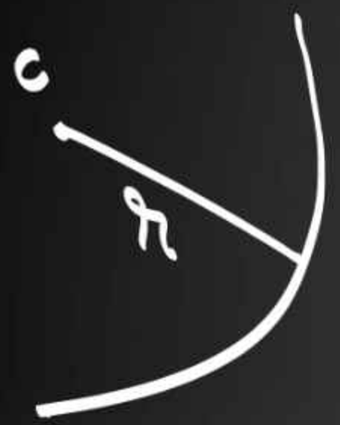
(c) $\frac{\partial u}{\partial y}$

(d) $\frac{\partial^2 u}{\partial y^2}$

वक्रता (Curvature)

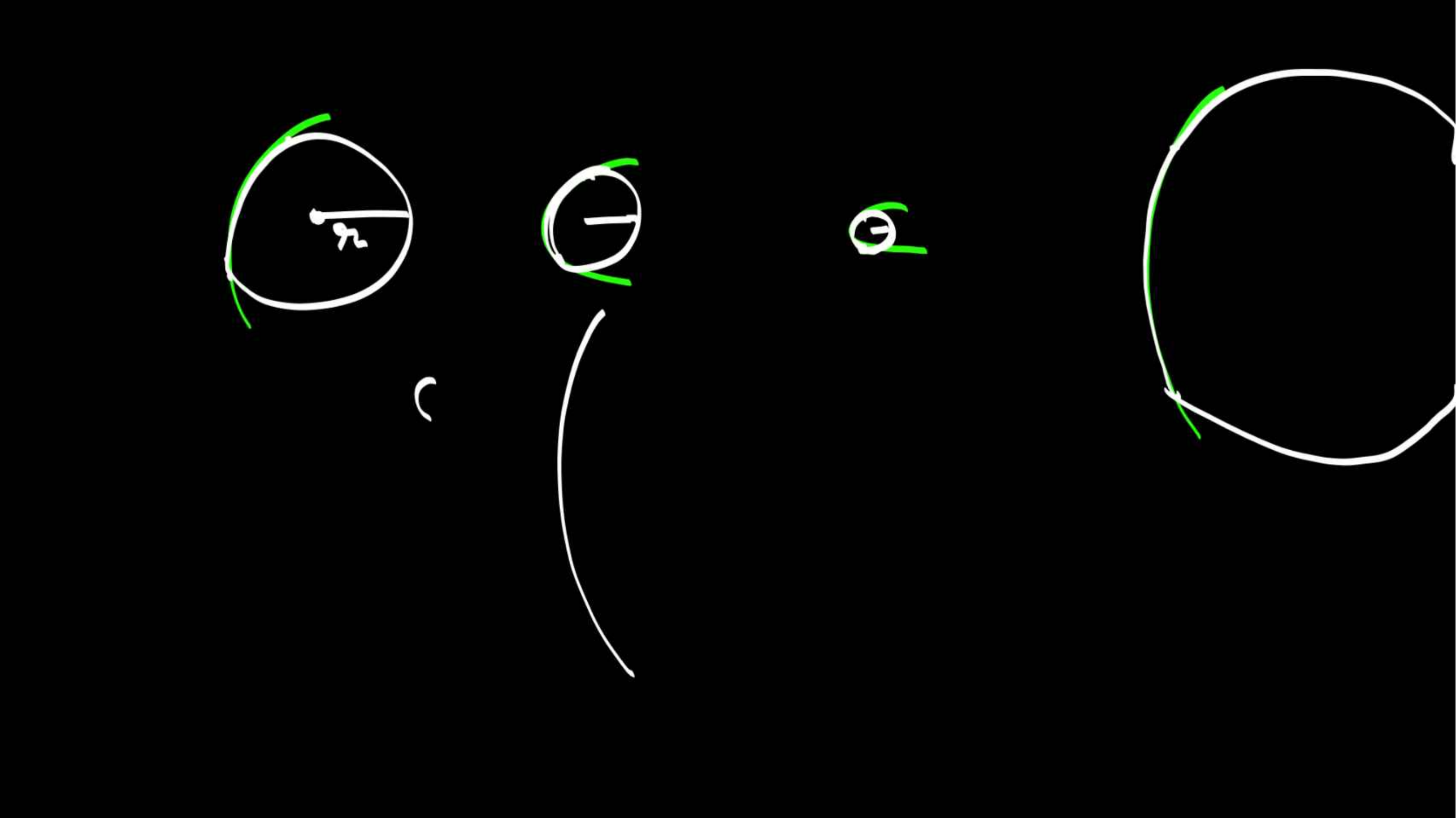
$$\text{वक्रता त्रिज्या} = \frac{1}{\text{वक्र की वक्रता}}$$

$$\text{वक्रता} = \frac{1}{\text{वक्रता त्रिज्या}}$$



Curvature → वक्रता
(घुमाव)

$$\text{Radius of curvature} = \frac{1}{\text{Curve of Curvature}}$$



17. *The curvature of the curve* $xy = a^2$ *at the point* (a, a) *is –*

वक्र $xy = a^2$ **की वक्रता, बिन्दु** (a, a) **पर है-**

(a) $\frac{1}{2^{5/2}.a}$

(b) $\frac{1}{2^{3/2}.a}$

(c) $\frac{1}{\sqrt{2}a}$

(d) $2^{5/2}a$

$xy = a^2$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$\left(\frac{dy}{dx}\right)_{(a,a)} = -\frac{a}{a} = -1$

$\frac{d^2y}{dx^2} = -\frac{\left[\frac{dy}{dx}.x - y.1\right]}{x^2}$

$= -\left[\frac{-x-y}{x^2}\right]$

$\left(\frac{d^2y}{dx^2}\right)_{(a,a)} = -\left(\frac{-a-a}{a^2}\right) \Rightarrow \frac{2a}{a^2} \Rightarrow \frac{2}{a}$

$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \Rightarrow \frac{(1+1)^{3/2}}{\frac{2}{a}} \Rightarrow \frac{2^{3/2}a}{2}$

$\Rightarrow 2^{\frac{3}{2}-1}a$

$\rho \Rightarrow \sqrt{2}.a$

वक्रता $= \frac{1}{\rho} \Rightarrow \frac{1}{\sqrt{2}.a}$

CARTESIAN FORMULA FOR RADIUS OF CURVATURE.

कार्तीय

18. $\rho = \dfrac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

से

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

$x^2+y^2=a^2$ (Circle का equation)

$a \rightarrow$ त्रिज्या

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\frac{d^2y}{dx^2}=\frac{-\left[1\cdot y-x\cdot\frac{dy}{dx}\right]}{y^2}$$

$$\Rightarrow \frac{-\left[y-x\cdot\left(-\frac{x}{y}\right)\right]}{y^2} \Rightarrow \frac{-\left[y+\frac{x^2}{y}\right]}{y^2} \Rightarrow -\left[\frac{y^2+x^2}{y^3}\right] \Rightarrow -\frac{a^2}{y^3}$$

$$\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \frac{\left[1+\frac{x^2}{y^2}\right]^{3/2}}{\frac{a^2}{y^3}} \Rightarrow \frac{\left[\frac{y^2+x^2}{y^2}\right]^{3/2}y^3}{a^2}$$

$$\Rightarrow \frac{(a^2)^{3/2}}{(y^2)^{3/2}}\times\frac{y^3}{a^2}$$

$$\Rightarrow \frac{a^3}{a^2} \Rightarrow a$$

19. Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at the point $x = \frac{\pi}{2}$.

बिन्दु $x = \frac{\pi}{2}$ पर वक्र $y = 4\sin x - \sin 2x$ की वक्रता त्रिज्या ज्ञात कीजिए।

(a) $\frac{5}{4}$

(b) $\frac{5\sqrt{5}}{4}$

(c) $5\sqrt{5}$

(d) $-\frac{5\sqrt{5}}{4}$

$y = 4\sin x - \sin 2x$

$\frac{dy}{dx} = 4\cos x - 2\cos 2x$

$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 4\times 0 - 2(-1) = 2$

$\frac{d^2y}{dx^2} = -4\sin x + 4\sin 2x$

$\left(\frac{d^2y}{dx^2}\right)_{x=\frac{\pi}{2}} = -4 + 0 \Rightarrow -4.$

$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \Rightarrow \frac{[1+4]^{3/2}}{-4} \Rightarrow \frac{(5^3)^{1/2}}{-4}$

$\Rightarrow -\frac{\sqrt{5\times5\times5}}{4}$

$= -\frac{5\sqrt{5}}{4}$

$(a^n)^m = a^{n\times m}$

20. Find the radius of curvature of the curve $y^2 = 4ax$.

वक्र $y^2 = 4ax$ की वक्रता त्रिज्या ज्ञात कीजिए।

(a) $\frac{2(x+a)}{\sqrt{a}}$

(b) $\frac{2(x+a)^{3/2}}{\sqrt{a}}$

(c) $\frac{(x+a)^{3/2}}{2\sqrt{a}}$

(d) $\frac{2(x+a)}{5a}$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{y}$ ✓ $\frac{\sqrt{a}}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{2a}{2\sqrt{ax}} \Rightarrow \sqrt{\frac{a}{x}}$

$\frac{d^2y}{dx^2} = -\frac{\sqrt{a}}{2x\sqrt{x}}$

$\Rightarrow \frac{\sqrt{a}}{2x^{3/2}}$

$y = \sqrt{4ax} \Rightarrow 2\sqrt{ax}$

$\rho = \frac{\left\{1+\frac{a}{x}\right\}^{3/2}}{\frac{\sqrt{a}}{2x^{3/2}}}$

$\Rightarrow \frac{2x^{3/2}(a+x)^{3/2}}{\sqrt{a}\,x^{3/2}} \Rightarrow \frac{2(a+x)^{3/2}}{\sqrt{a}}$

21. The radius of curvature at $(0, 1)$ for the curve $y = e^x$ is –

वक्र $y = e^x$ के लिए (0,1) पर वक्रता - त्रिज्या है-

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{2\sqrt{2}}$

$y = e^x$

$\frac{dy}{dx} = e^x$

$\left(\frac{dy}{dx}\right)_{(0,1)} = e^0 = 1$

$\frac{d^2y}{dx^2} = e^x \Rightarrow e^0 \Rightarrow 1$ at (0,1)

$\rho = \frac{(1+1)^{3/2}}{1} \Rightarrow 2^{\frac{3}{2}} \Rightarrow 2^1 \times 2^{\frac{1}{2}}$

$\Rightarrow 2\sqrt{2}$

22. The radius of curvature of the curve $y = x^4 - 4x^3 - 18x^2 at\ (0,0) is -$

वक्र $y = x^4 - 4x^3 - 18x^2$ की (0,0) पर वक्रता त्रिज्या है-

(a) $\frac{1}{12}$

(b) $\frac{1}{24}$

(c) $\frac{1}{36}$

(d) $\frac{1}{48}$

$\frac{dy}{dx} = 4x^3 - 12x^2 - 36x$

$at(0,0) = 0$

$\frac{d^2y}{dx^2} = 12x^2 - 24x - 36$

$(at\ (0,0)) \Rightarrow -36$

$$\rho = \frac{\{1+0\}^{3/2}}{-36} \Rightarrow \frac{1}{36}$$

23. *The curvature of the curve* $x^2 + y^2 - 6x - 8y = 0$ *at the origin is –*

वक्र $x^2 + y^2 - 6x - 8y = 0$ की मूल बिन्दु की वक्रता है-

(a) 5

(b) 25

(c) $\frac{1}{5}$

(d) शून्य (Zero)

$2x + 2y\frac{dy}{dx} - 6 - 8\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{6-2x}{2y-8}$

at (0,0) $\Rightarrow \frac{6}{-8} = -\frac{3}{4}$

$\frac{d^2y}{dx^2} = \frac{-2(2y-8) - (6-2x)2\frac{dy}{dx}}{(2y-8)^2}$

at (0,0)

$\Rightarrow \frac{-2(-8) - (6)2\cdot(-\frac{3}{4})}{(-8)^3} = \frac{16+9}{64} = \frac{25}{64}$

$\rho = \frac{\left(1 + \frac{9}{16}\right)^{3/2}}{\frac{25}{64}}$

$\Rightarrow \frac{25^{3/2}}{16^{3/2}} \times \frac{64}{25}$

$\Rightarrow \frac{5^3}{4^3} \times \frac{4^3}{5^2} \Rightarrow 5$

(0,0) वक्रता $= \frac{1}{\rho}$

$\Rightarrow \frac{1}{5}$

$$6 + x - 6 = 0 + 6$$

$$x = 6$$

24. Find the radius of curvature at origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$

वक्र $x^3 + y^3 - 2x^2 + 6y = 0$ के लिए मूल बिन्दु पर वक्रता त्रिज्या ज्ञात कीजिए।

(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) 0

(d) $\frac{4}{3}$

$3x^2 + 3y^2\frac{dy}{dx} - 4x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{4x - 3x^2}{(3y^2+6)}$

at (0,0)

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = \frac{(4-6x)(3y^2+6) - (4x-3x^2)(\quad)}{(3y^2+6)^2}$

$\Rightarrow \frac{24}{36} \Rightarrow \frac{2}{3}$

$\rho = \frac{(1+0)^{3/2}}{2/3} = \frac{3}{2}$

ASYMPTOTES

Definition:- A straight line at a finite distance from the origin to which a tangent to a curve tends, as the distance from the origin of the point of contact tends to infinity, is called an asymptote of the curve.

परिभाषा:- मूल बिंदु से एक निश्चित दूरी पर एक सीधी रेखा जिस पर एक वक्र की स्पर्श रेखा जाती है, क्योंकि संपर्क बिंदु के मूल बिंदु से दूरी अनंत तक जाती है, उसे वक्र की अनंतस्पर्शी रेखा कहा जाता है।

अनन्त स्पर्शी ↳ YT Book B.Sc.