



DSSSB TGT

PART (A+B)



MATHS

ADVANCE CALCULUS

GRADUATION LEVEL



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Partial Differentiation →

આંશિક અવકલન

$$y = x^2 + a^2$$

$\downarrow$ variable $\downarrow$ Constant

$$\frac{dy}{dx} = 2x + 0$$

$$y = x^2 + a^2$$

$\downarrow$ Variable $\rightarrow$ variable

$$\frac{\partial y}{\partial x} = 2x$$

$$\frac{\partial y}{\partial a} = 2a$$

$$u = ax^2 + 2hxy + by^2$$

$$\frac{\partial u}{\partial x} = 2ax + 2hy + 0$$

$$\frac{\partial^2 u}{\partial y \partial x} = 0 + 2h$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

$$\frac{\partial u}{\partial y} = 0 + 2hx + 2by$$

$$\frac{\partial^2 u}{\partial x \partial y} = 2h + 0$$

1. If $u = ax^2 + 2hxy + by^2$ then

यदि $u = ax^2 + 2hxy + by^2$ है तो

(A) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$

(B) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

(C) $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial y^2}$

(D) $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$

2. If $u = f\left(\frac{y}{x}\right)$ then what is the value of $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y}$?

यदि $u = f\left(\frac{y}{x}\right)$ है तो $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y}$ का मान क्या होगा?

(a) 1

(b) 2

(c) 3

(d) 0

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y}$$

$$-\frac{y}{x} f'\left(\frac{y}{x}\right) + \frac{y}{x} f'\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial x} = f'\left(\frac{y}{x}\right) \cdot \left(-\frac{y}{x^2}\right)$$

$$x \cdot \frac{\partial u}{\partial x} = -\frac{y}{x} \cdot f'\left(\frac{y}{x}\right)$$

$$= -\frac{y}{x} f'\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial y} = f'\left(\frac{y}{x}\right) \cdot \frac{1}{x}$$

$$y \cdot \frac{\partial u}{\partial y} = \frac{y}{x} f'\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial x} = \frac{1}{(x^3 + y^3 + z^3 - 3xyz)} (3x^2 - 3yz)$$

$$\frac{\partial u}{\partial x} = \frac{3x^2 - 3yz}{x^3 + y^3 + z^3 - 3xyz}$$

$$\frac{\partial u}{\partial y} = \frac{3y^2 - 3xz}{\text{same}}$$

$$\frac{\partial u}{\partial z} = \frac{3z^2 - 3xy}{\text{same}}$$

3. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then what is the value of $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$?

यदि $u = \log(x^3 + y^3 + z^3 - 3xyz)$ है तो $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$ का मान क्या होगा?

(a) $(x + y + z)$

(b) $(x^2 + y^2 + z^2)$

(c) $\frac{3}{x+y+z}$

(d) $\frac{-3}{x+y+z}$

$$\Rightarrow \frac{3[x^3 + y^3 + z^3 - xy - yz - zx]}{(x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)} = \frac{3}{x+y+z}$$

4. If $u = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$, then what is the value of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$?

यदि $u = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$ है, तो $\frac{\partial u}{\partial x}$ व $\frac{\partial u}{\partial y}$ का मान क्या होगा?

(a) $\frac{x}{a^2}, \frac{y}{b^2}$

(b) $\frac{2x}{a}, \frac{2y}{b}$

(c) $\frac{2x}{a^2}, \frac{2y}{b^2}$

(d) $\frac{1}{a}, \frac{1}{b}$

$$\frac{\partial u}{\partial x} = \frac{2x}{a^2} + 0 - 0$$

$$\frac{\partial u}{\partial y} = 0 + \frac{2y}{b^2} - 0$$

5. If $f(x, y) = x \cos y + y \sin x$, then –

यदि $f(x, y) = x \cos y + y \sin x$ है, तब-

$$\frac{\partial f}{\partial x} = \cos y + y \cos x$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\sin y + \cos x$$

~~(a) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$~~

~~(b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$~~

(c) $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$

(d) $\left(\frac{\partial f}{\partial x}\right)^2 = \left(\frac{\partial f}{\partial y}\right)^2$

$$\frac{\partial f}{\partial y} = -x \sin y + \sin x$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\sin y + \cos x$$

6. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then What is

$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \left(-\frac{x}{y^2}\right) + \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x}$$

the value of $\left(x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right)$?

यदि $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ है, तो $\left(x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right)$

$$y \cdot \frac{\partial u}{\partial y} = \frac{y}{\sqrt{y^2-x^2}} \left(-\frac{x}{y}\right) + \frac{x^2 y}{x^2+y^2} \cdot \frac{1}{x} \frac{\partial u}{\partial y}$$

$$\Rightarrow \frac{-x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2}$$

(a) 1

(b) 0

(c) 5

(d) -2

$\therefore = 0$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \cdot \frac{1}{y} + \frac{1}{1+\frac{y^2}{x^2}} \cdot \left(-\frac{y}{x^2}\right)$$

$$x \cdot \frac{\partial u}{\partial x} \Rightarrow \frac{1}{y} \cdot \frac{xy}{\sqrt{y^2-x^2}} - \frac{x^2}{x^2+y^2} \cdot \frac{y}{x}$$

7. If $z = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$, then what is the value of $\frac{\partial^2 z}{\partial x \partial y}$?

यदि $z = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$ है, तो $\frac{\partial^2 z}{\partial x \partial y}$ का मान क्या होगा?

(a) $\frac{x+y}{x-y}$

(b) $\frac{x^2+y^2}{x^2-y^2}$

(c) $\frac{x}{y}$

(d) $\frac{x^2-y^2}{x^2+y^2}$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2y^2 - 2x^2 + 2x^2 - 2y^2}{(x^2 + y^2)^2} = 0$$

8. If $u = \log(x^2 + y^2)$, then what is the value of $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$?

यदि $u = \log(x^2 + y^2)$ है, तो $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ का मान

क्या होगा?

(a) 0

(b) 1

(c) 2

(d) 3

$$\frac{\partial u}{\partial x} = \frac{1}{x^2 + y^2} \cdot 2x$$

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2 + y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2(x^2 + y^2) - 2x(2x)}{(x^2 + y^2)^2} = \frac{2x^2 + 2y^2 - 4x^2}{(x^2 + y^2)^2}$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^2 + y^2} \cdot 2y$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{2(x^2 + y^2) - 2y(2y)}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{2x^2 + 2y^2 - 4y^2}{(x^2 + y^2)^2}$$

9. If $f(x, y) = x^4 + x^2y^2 + y^4$ then -

यदि $f(x, y) = x^4 + x^2y^2 + y^4$ है तब-

(a) $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 12$

(b) $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = \underline{14}$

~~(c)~~ $\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 4$

(d) इनमे से कोई नहीं

$$\frac{\partial f}{\partial x} = 4x^3 + 2xy^2 + 0$$

$$\frac{\partial^2 f}{\partial x^2} = 12x^2 + 2y^2$$

$$\left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1)} = 12(1)^2 + 2(1)^2 = 12 + 2 = 14$$

HOMOGENEOUS FUNCTIONS.

10. x और y में एक व्यंजक जिसमें प्रत्येक पद समान घात का हो, x और y का समरूप फलन कहलाता है। द्वारा परिभाषित फलन पर विचार करें

An expression in x and y in which every term is of the same degree is called a homogeneous function of x and y . Consider the function defined by

$$f(x, y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \dots + a_{n-1} x y^{n-1} + a_n y^n.$$

इस फंक्शन में प्रत्येक पद n डिग्री का है। इसलिए यह x और y का n डिग्री का एक समरूप फंक्शन है। इसके अलावा, (1) को इस प्रकार लिखा जा सकता है

In this function every term is of degree n . Therefore it is a homogeneous function of x and y of degree n . Moreover, (1) may be written as

$$f(x, y) = x^n \left\{ a_0 + a_1 \left(\frac{y}{x} \right) + a_2 \left(\frac{y}{x} \right)^2 + \dots + a_n \left(\frac{y}{x} \right)^n \right\}$$

$$\text{or } f(x, y) = x^n F(y/x),$$

$$x^2 + y^2 + xy$$

✓ ✓ ✓
1+1=2 ✓

EULER'S THEOREM ON HOMOGENEOUS FUNCTIONS.

11. यदि u, x और y का घात n वाला एक समरूप फलन है, तो $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu.$

If u be a homogeneous function of x and y of degree n , then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu.$

$n \rightarrow \text{Degree}$

$$x^2 \rightarrow 2$$

$$x^2 + y^2 \rightarrow 4$$

$$\frac{x^2}{y^2} \left. \begin{matrix} 2 \\ 2 \end{matrix} \right\}$$

This proves the theorem.

12. In general, if u be a homogeneous function of $x_1, x_2, \dots, x_m$ of degree n , then.

सामान्यतः, यदि $u, x_1, x_2, \dots, x_m$ का घात n वाला एक समरूप फलन हो, तो।

$$x_1 \frac{\partial u}{\partial x_1} + x_2 \frac{\partial u}{\partial x_2} + \dots + x_m \frac{\partial u}{\partial x_m} = \underline{nu}.$$

13. If $u = f(x, y, z)$ is a homogeneous function of degree n in variables x, y and z , then

यदि $u = f(x, y, z)$ चरों x, y तथा z में n घात का समघात फलन हो तो

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu$$

 If u be a homogeneous function of x and y of degree n , Then.

यदि u, x और y का घात n वाला एक समरूप फलन हो, तो.

समघात

$$(a) \quad x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial x \partial y} = \underline{(n-1)} \frac{\partial u}{\partial x}.$$

$$(b) \quad x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = (n-1) \frac{\partial u}{\partial y}$$

$$(c) \quad x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

15. If $f(x, y) = 2x^2 + 2xy + 4y^2$ then what is the value of $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$?

यदि $f(x, y) = 2x^2 + 2xy + 4y^2$ तो $\boxed{x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}} \Rightarrow n f$
 $n = 2$ (degree)

का मान है?

(a) $2f(x, y)$

(b) $3f(x, y)$

(c) $4f(x, y)$

(d) $f(x, y)$

$x \frac{\partial f}{\partial x} = 4x^2 + 2y^2$

$y \frac{\partial f}{\partial y} = 2xy + 8y^2$

$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$

$\Rightarrow 4x^2 + 2xy + 2xy + 8y^2$

$\Rightarrow 4x^2 + 4xy + 8y^2$

$2[2x^2 + 2xy + 4y^2]$

$2f(x, y)$

16. If $u = 2x^2 + xy + 4y^2$, then what is the value of $x \cdot \frac{\partial^2 u}{\partial x^2} + y \cdot \frac{\partial^2 u}{\partial x \partial y}$?

यदि $u = 2x^2 + xy + 4y^2$ है, तो $x \cdot \frac{\partial^2 u}{\partial x^2} + y \cdot \frac{\partial^2 u}{\partial x \partial y}$ का मान क्या होगा? $n=2$

(a) u

(b) $\frac{\partial u}{\partial x}$

(c) $\frac{\partial u}{\partial y}$

(d) $\frac{\partial^2 u}{\partial y^2}$

$$\Rightarrow (n-1) \frac{\partial u}{\partial x}$$

$$(2-1) \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x}$$

वक्रता (Curvature)

$$\text{वक्रता त्रिज्या} = \frac{1}{\text{वक्र की वक्रता}}$$

CURVATURE



वक्रता