DSSSB TGT PART(A+B)

MATHS

CALCULUS

(LIMITS & CONTINUITY)

LIVE STREAMING

29/05/2024 04:30PM

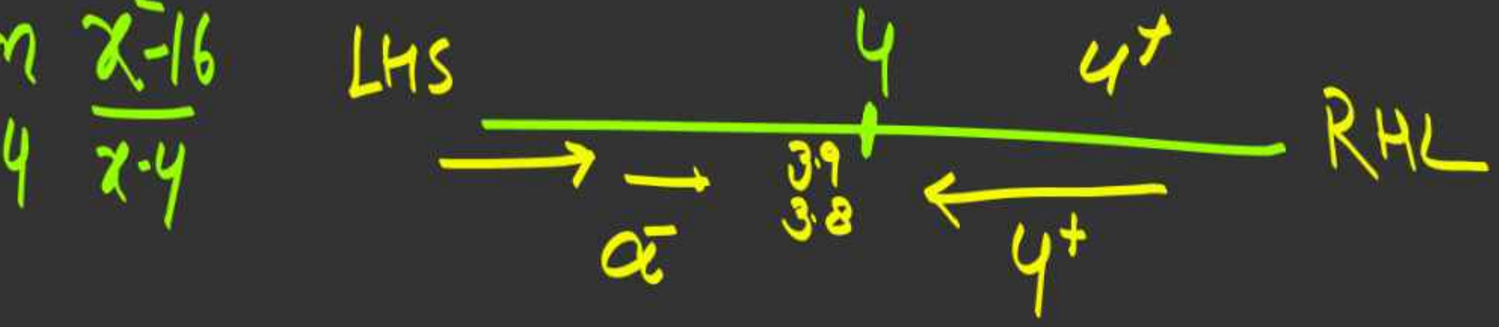
Calculus :-

- Limit
- Continuity
- Differentiation
- Integration

$LHL = f(a-0)$ or $\lim_{x \to a^-} f(x)$ or $\lim_{x \to a-0} f(x)$

$RHL = f(a+0)$ or $\lim_{x \to a^+} f(x)$ or $\lim_{x \to a+0} f(x)$

$\lim_{x \to 4} \frac{x^2-16}{x-4}$



Methods of Evaluation of Limits:

Case-1 → When $x \to \infty$ ✓ ⇒ Divide by Higher power

Ex → $\lim_{x\to\infty} \frac{1^2+2^2+3^2+4^2+\cdots+x^2}{x^3} \Rightarrow \frac{x(x+1)(2x+1)}{6\cdot x^3}$

$\frac{1}{0} = \infty$

$\frac{1}{\infty} = 0$

$$\frac{x\left(1+\frac{1}{x}\right)x\left(2+\frac{1}{x}\right)}{6\cdot x^x}$$

$$\lim_{x\to\infty} \frac{\left(1+\frac{1}{x}\right)\left(2+\frac{1}{x}\right)}{6} \Rightarrow \frac{(1+0)(2+0)}{6} = \frac{1}{3}$$

$$\lim_{x \to \infty} \frac{x^3 + x^2 + 4x}{x^3}$$

$$\frac{x^3\left(1 + \frac{1}{x} + \frac{4}{x^2}\right)}{x^3}$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x} + \frac{4}{x^2}\right)$$

$$\Rightarrow 1 + 0 + 0 \Rightarrow \boxed{1}$$

Case-II →

First method → (Method of Simplification) →

$$\lim_{x \to 1} \cos^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$\cos^{-1}\left(\frac{1-\sqrt{x}}{1^2-(\sqrt{x})^2}\right)$$

$$\cos^{-1}\left(\frac{(1-\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}\right)$$

$$\lim_{x \to 1} \cos^{-1}\left(\frac{1}{1+\sqrt{x}}\right)$$

$$\cos^{-1}\left(\frac{1}{1+1}\right)$$

$$\Rightarrow \cos^{-1}\frac{1}{2} \Rightarrow \frac{\pi}{3}$$

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$

0/0

$x^2 - 5^2$

$$\lim_{x \to 5} \frac{(x-5)(x+5)}{(x-5)}$$

$\Rightarrow 5 + 5 \Rightarrow 10$

Third method → L'Hospital Rule → $\frac{0}{0}, \frac{\infty}{\infty}$

$\frac{d}{dx} x^2$

$\Rightarrow \lim_{x \to 2} \frac{x^2-4}{x-2}$

$\Rightarrow \frac{(x-2)(x+2)}{(x-2)}$

$\lim_{x \to 2} (x+2)$

$\Rightarrow 2+2 \Rightarrow 4$

$\lim_{x \to 2} \frac{x^2-4}{x-2} \Rightarrow \frac{\frac{d}{dx}(x^2-4)}{\frac{d}{dx}(x-2)} \Rightarrow \frac{2x-0}{1-0}$

$\Rightarrow 2x$

$\lim_{x \to 2} 2x$

$\Rightarrow 2 \times 2 \Rightarrow 4$

$$\rightarrow \frac{\sin 0}{0} \Rightarrow \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sin 0}{0} \Rightarrow 1$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$$

$$\frac{\sin y}{x}$$

$(x, y) \; (0, 0)$

$x \to 0$

$y \to 0$

a 1

b 0

c infinite

d does not exist

$$\frac{\sin y}{x}$$

$$\frac{\frac{d}{dy}\sin y}{\frac{d}{dy}x} \rightarrow \frac{\cos y}{0} \qquad \infty$$

$x \to 0$, $y \to 0$

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

$$\frac{\frac{d}{dx}(x^4 - 81)}{\frac{d}{dx}(2x^2 - 5x - 3)} \Rightarrow \frac{4x^3 - 0}{4x - 5 - 0}$$

$$\Rightarrow \lim_{x \to 3} \frac{4x^3}{4x - 5} \Rightarrow \frac{108}{7}$$