



DSSSB TGT

PART (A+B)



MATHS

CALCULUS



28/05/2024 04:30PM

Calculus

- 
- Limits (सीमा)
 - Continuity
 - Differentiation
 - Integration

LIMITS

Function

Determinate
value

Indeterminate, અર્થહીન
≠ value

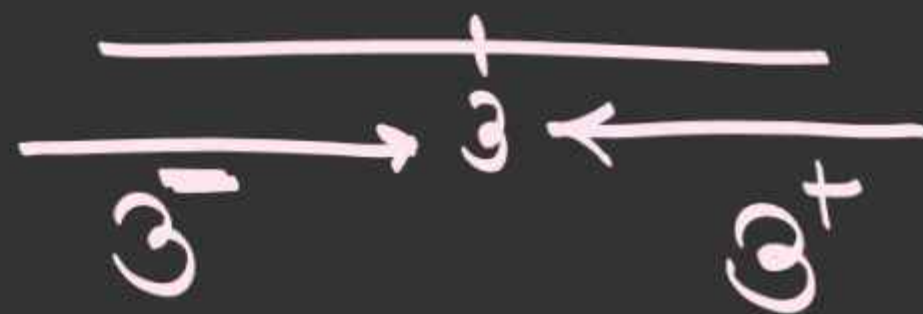
$\frac{0}{0}, \frac{\infty}{\infty}, \infty^0, 0^0, \infty - \infty,$

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$\boxed{x \neq 3}$$

$$\lim_{x \rightarrow 3} \frac{(x+3)(\cancel{x-3})}{\cancel{x-3}}$$

$$x^2 - 3^2$$



$$\lim_{x \rightarrow 3} (x+3)$$

$$3+3 \Rightarrow 6$$

(i) दो फलनों के योग की सीमा फलनों की सीमाओं का योग होता है, अर्थात्

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

(ii) दो फलनों के अंतर की सीमा फलनों की सीमाओं का अंतर होता है, अर्थात्

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

(iii) दो फलनों के गुणन की सीमा फलनों की सीमाओं का गुणन होता है, अर्थात्

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

(iv) दो फलनों के भागफल की सीमा फलनों की सीमाओं का भागफल होता है, (जबकि हर शून्येतर होता है), अर्थात्

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

Formule :-

$$\lim_{x \rightarrow 0} \frac{\sin 9x}{9x} = 1$$

$$1. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$2. \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\Rightarrow 3. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$4. \lim_{x \rightarrow 0} \frac{x^n - a^n}{x - a} = \underline{na^{n-1}}$$

$$5. \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$6. \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \underline{\log a}$$

$$7. \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = \underline{1}$$

$$8. \lim_{x \rightarrow 0} \log e^x = 0$$

1. सीमाएँ ज्ञात कीजिए:

Find the limits:

(a) $\lim_{x \rightarrow 1} [x^3 - x^2 + 1] \Rightarrow 1^3 - 1^2 + 1 \Rightarrow 1$

(b) $\lim_{x \rightarrow 3} [x(x + 1)] \Rightarrow 3(3 + 1) \Rightarrow 12$

(c) $\lim_{x \rightarrow -1} [1 + x + x^2 + \dots + x^{10}]$

$1 - 1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8 + (-1)^9 + (-1)^{10}$

$\Rightarrow 1$

2. सीमाएँ ज्ञात कीजिए:

Find the limits:-

$$(a) \lim_{x \rightarrow 1} \left[\frac{x^2 + 1}{x + 100} \right] \Rightarrow \frac{1^2 + 1}{1 + 100} = \frac{2}{101}$$

$$(b) \lim_{x \rightarrow 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 4} \right] \Rightarrow 0$$

$$(c) \lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{x^3 - 4x^2 + 4x} \right] \Rightarrow \frac{4}{0} = \infty \checkmark \text{ does not exist}$$

$$(d) \lim_{x \rightarrow 2} \left[\frac{x^3 - 2x^2}{x^2 - 5x + 6} \right] \Rightarrow \frac{x^2(x-2)}{x^2 - 3x - 2x + 6} = \frac{x^2(x-2)}{x(x-3) - 2(x-3)}$$
$$\Rightarrow \frac{x^2(x-2)}{(x-2)(x-3)} = \frac{x^2}{x-3} \Rightarrow \frac{4}{1} = 4$$

$$\frac{x(x^2 - 4x + 4)}{x^2 - 4}$$

$$\frac{x(x-2)^2}{(x-2)(x+2)}$$

$$\frac{x(x-2)}{(x+2)} \Rightarrow \frac{2(0)}{4}$$

$\Rightarrow 0$ Ans

3. $\lim_{x \rightarrow 3} x + 3$

6

4. $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

$\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

$\pi - \frac{22}{7} \neq \text{ans.}$

~~$\frac{22}{7} - \frac{22}{7} = 0$~~ ~~$\emptyset$~~

$$5. \lim_{r \rightarrow 1} \pi r^2$$

$$\lim_{r \rightarrow 1} \pi r^2$$

$$\Rightarrow \pi \times 1^2$$

$$\rightarrow \pi \text{ units}$$

$$6. \lim_{x \rightarrow 4} \frac{4x+3}{x-2}$$

$$\lim_{x \rightarrow 4} \frac{4x+3}{x-2}$$

$$\Rightarrow \frac{4 \times 4 + 3}{4 - 2} \Rightarrow \frac{19}{2} \text{ Ans.}$$

$$7. \lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x - 1}$$

$$\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x - 1}$$

$$\Rightarrow \frac{(-1)^{10} + (-1)^5 + 1}{-1 - 1} \Rightarrow \frac{1 - 1 + 1}{-2}$$

$$\Rightarrow -\frac{1}{2}$$

$$8. \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x} \quad] \quad na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{x^n - a^n}{x - a} \Rightarrow na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1^5}{(x+1) - 1}$$

$$\Rightarrow 5 \times 1^{(5-1)}$$

$$5 \times 1^4 \Rightarrow 5 \text{ ans.}$$

$$\frac{1-1+x}{x+1-1}$$

$$9. \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

$$\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

$$\frac{3x^2 - 6x + 5x - 10}{x^2 - 2^2} = \frac{3x(x-2) + 5(x-2)}{(x-2)(x+2)}$$

$$\Rightarrow \frac{\cancel{(x-2)}(3x+5)}{\cancel{(x-2)}(x+2)}$$

$$= \frac{3 \times 2 + 5}{2 + 2} = \frac{11}{4}$$

ગુણા $\Rightarrow -6 \times 1 = -6$
 જોડા $\Rightarrow -6 + 1 = -5$
 $-6 \quad +1$

$-2, -3$

જોડા $\Rightarrow -2 \cdot 3 = -6$
 ગુણા $\Rightarrow -2 \times 3 = -6$

10. $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

$\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

$\Rightarrow \frac{(x^2)^2 - 9^2}{2x^2 - 6x + x - 3} \Rightarrow \frac{(x^2 - 9)(x^2 + 9)}{2x(x-3) + 1(x-3)}$

$\Rightarrow \frac{(x+3)(x-3)(x^2+9)}{(x-3)(2x+1)}$

$= \frac{(3+3)(3^2+9)}{2 \times 3 + 1}$

$\Rightarrow \frac{108}{7}$

$$11. \lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$$

$$\lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$$

$$\Rightarrow \frac{a \times 0 + b}{c \times 0 + 1} \Rightarrow b \text{ ans.}$$

13. $\lim_{x \rightarrow 1} \frac{ax^2+bx+c}{cx^2+bx+a}, \underbrace{a+b+c} \neq \textcircled{0}$

$$\lim_{x \rightarrow 1} \frac{ax^2+bx+c}{cx^2+bx+a}$$

$$\Rightarrow \frac{a+b+c}{c+b+a} = \frac{a+b+c}{a+b+c} \Rightarrow 1 \text{ ans.}$$

14. Find the limit of $\frac{\sin Y}{X}$, where (x, y) approaches to $(0,0)$?

$\frac{\sin Y}{X}$ की सीमा ज्ञात करें, जहां (x, y) $(0,0)$ के करीब पहुंचता है?

(a) 1

(b) 0

(c) infinite

(d) doesn't exist

$$\frac{\sin y}{x}$$

$$\Rightarrow \frac{\sin 0}{0} \Rightarrow \frac{0}{0}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}}$$

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15. Find the limit of the derivative of function $f(x, y) = \frac{xy^2}{(x^2 + y^4)}$ at the point u $(\sqrt{2}, \sqrt{2})$?

बिंदु u $(\sqrt{2}, \sqrt{2})$ पर फलन $f(x, y) = \frac{xy^2}{(x^2 + y^4)}$ के

अवकलज की सीमा ज्ञात कीजिए?

(a) 0

(b) $\frac{\sqrt{2}}{3}$

(c) 2

(d) $\sqrt{3}$

$$\Rightarrow \frac{\sqrt{2} \cdot (\sqrt{2})^2}{(\sqrt{2})^2 + (\sqrt{2})^4} \Rightarrow \frac{\sqrt{2}}{3}$$

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16. The correct value of $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ is.

$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ का सही मान है।

(a) $\frac{1}{\sqrt{2}}$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

(b) 0

(c) does not exist

(d) $\sqrt{2}$

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17. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = ? \quad a \neq 0, b \neq 0$

(a) $\frac{b}{a}$

(b) does not exist

(c) $\frac{a}{b}$

(d) $a.b$

$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = ?$

$\lim_{x \rightarrow 0} ax \times \frac{\sin ax}{ax} \div \lim_{x \rightarrow 0} \frac{\sin bx}{bx} \times bx$

$\frac{ax}{bx} \Rightarrow \frac{a}{b}$

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$\lim_{x \rightarrow 0} \frac{\sin ax}{ax} \cdot ax$
 $\frac{1 \cdot ax}{ax} \Rightarrow \frac{a}{b}$

18. Find the value of the limit, $\lim_{x \rightarrow 0} \frac{25}{x} = ?$

सीमा का मान ज्ञात कीजिए, $\lim_{x \rightarrow 0} \frac{25}{x} = ?$

(a) $3 \div 2$

(b) $1 \div 2$

(c) 0

(d) ∞

$$\lim_{x \rightarrow 0} \frac{25}{x}$$

$$\frac{25}{0} = \infty$$

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$$\frac{0}{5} = 0$$

$$\frac{0}{0} = \infty$$

$$\frac{5}{5} = 1$$
$$\frac{0}{0} \neq \infty$$

19. Find the limit of the function $f(x, y)$

$= x + y - 6$ at $x = 1$; $y = 2$?

$x = 1$ पर फलन $f(x, y) = x + y - 6$ की सीमा
ज्ञात कीजिए; $y = 2$?

$$1 + 2 - 6 \\ - 3$$

(a) -3

(b) -4

(c) 3

(d) 4

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20. The limit of the function

$$\frac{\log(1+x)}{x} \text{ as } x \rightarrow 0$$

फलन की सीमा $\frac{\log(1+x)}{x}$ as $x \rightarrow 0$ के रूप में

(a) 3

(b) 1

(c) 0

(d) 2

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = ? \quad \textcircled{1}$$

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21. Solve :- $\lim_{x \rightarrow 0} \log e^x = ?$

(a) -1

(b) e

(c) 0

(d) 1

$$\lim_{x \rightarrow 0} \log e^x = 0$$

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मान ज्ञात कीजिए:

Find the value:

(i) $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x} \Rightarrow$

$$\frac{4x \cdot \frac{\sin 4x}{4x}}{2x \cdot \frac{\sin 2x}{2x}} \Rightarrow \frac{4x}{2x} \Rightarrow 2$$

(ii) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ ✓
① ✓

$$\lim_{x \rightarrow 0} \frac{\sin x}{x \cos x}$$

$$\Rightarrow \frac{1}{\cos 0} \Rightarrow \frac{1}{1} \Rightarrow 1$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot 4x$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot 2x$$