DSSSB TGT PART(A+B)

MATHS

QUADRATIC EQUATION

(द्विघात समीकरण) Part- 05

LIVE 12/04/2024 04:00PM

Quadratic Equations

Some Results on Roots of an Equation

(i) Every n -degree equation has n roots.

प्रत्येक n-डिग्री समीकरण के n मूल होते हैं।

जैसे - $x^2 - 9 = 0$, ⇒ (2)

$x^3 + x^2 + x + 1 = 0$ (3)

$x^8 + 3x^6 + 4x^4 + 9 = 0$

(8)

(ii) Every odd degree equation has at least one real root. If the coefficient of the highest degree term is positive, then the sign of this real root is opposite to the sign of the constant term.

प्रत्येक विषम डिग्री समीकरण का कम से कम एक वास्तविक मूल होता है। यदि उच्चतम डिग्री पद का गुणांक धनात्मक है, तो इस वास्तविक मूल का चिह्न अचर पद के चिह्न के विपरीत है।

जैसे → $x^3 + 3x^2 + 4x + 2 = 0$

$(-1)^3 + 3(-1)^2 + 4(-1) + 2 = 0$

$-1 + 3 - 4 + 2$

(iii) If the coefficient of the highest degree term of an even degree equation is positive and its constant term is negative, then this equation has at least two real roots: one positive and one negative.

यदि किसी सम अंश समीकरण के उच्चतम अंश पद का गुणांक धनात्मक है और इसका अचर पद ऋणात्मक है, तो इस समीकरण के कम से कम दो वास्तविक मूल हैं: एक धनात्मक और एक ऋणात्मक।

$2x^2 - 15x - 27 = 0$

$8x^2 - 4x - 4 = 0$

54 -15

$2x^2 - 18x + 3x - 27 = 0$

$2x(x-9) + 3(x-9) = 0$

$(x-9)(2x+3) = 0$

$x = 9, -\frac{3}{2}$

Quadratic Equations

Roots Under Particular Conditions

The nature of the roots of the equation $ax^2 + bx + c = 0$

Condition	Nature of roots
$a + b + c = 0$	Roots = 1, c/a
$b = 0$	roots are negative of each other
$c = 0$	One root = 0
$b = c = 0$	Both roots = 0
$a = c$	Roots are reciprocals of each other

Condition	Nature of roots
a, c are in opposite sign	Real roots in opposite sign
$b^2 - 4ac \geq 0$ and a, b, c are in same sign	Real roots, both negative
$b^2 - 4ac \geq 0$ and a, c are in same sign, b is in opposite sign	Real roots, both positive
$b = 0; a, c$ are in same sign	Imaginary

$x^2 - 5x - 6 = 0$ $3x^2 - 5x + 3 = 0$ $3x^2 - 2x - 1 = 0$

The nature of the roots of the equation $ax^2 + bx + c = 0$

If $a + b + c = 0$ Then Roots = $1, c/a$

जैसे → $3x^2 - 2x - 1 = 0$

$a=3, b=-2, c=-1$

$a+b+c=0$ ✓

$3-2-1=0$ ✓

$3x^2 - 2x - 1 = 0$

$3x^2 - 3x + x - 1 = 0$

$3x(x-1) + 1(x-1) = 0$

$(x-1)(3x+1) = 0$

$x = 1,$

$3x+1=0$

$x = -\frac{1}{3}$

$1, -\frac{1}{3}$

$-3 \quad -2$

$-3 \quad +1$

The nature of the roots of the equation $ax^2 + bx + c = 0$

α, β

If $b = 0$ then roots are negative of each other

$b = 0$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = 0$

$\alpha = -\beta$

$\beta = -\alpha$

जैसे- $x^2 - 9 = 0$

$x^2 + 11 = 0$

$\Downarrow$

$x^2 = -11$

$x = \sqrt{-11}$

$x = \sqrt{-1 \times 11}$

$= \sqrt{i^2 \times 11} \Rightarrow \pm i\sqrt{11}$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \sqrt{9} \Rightarrow \pm 3$

$x^2 - 9 = 0$

$x^2 + 0 \cdot x - 9 = 0$

The nature of the roots of the equation $ax^2 + bx + c = 0$

α, β

If $c = 0$ *then* One root = 0

जैसे- $4x^2 + 4x = 0$ → $4x^2 = -4x$

$x = -1, 0$

$6x^2 + 3x = 0$

$6x^2 = -3x$

$x = -\frac{3}{6}, 0$

$\alpha\beta = \frac{c}{a}$ $\quad c = 0$

$\alpha\beta = 0$

$\alpha + \beta = \frac{b}{a}$

The nature of the roots of the equation $ax^2 + bx + c = 0$

If $b = c = 0$ then Both roots = 0

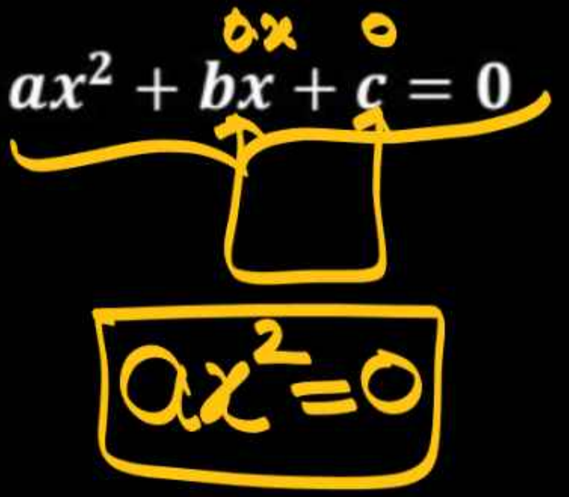
जैसे- $4x^2 = 0$

$4x^2 = 0$

$x^2 = \frac{0}{4}$

$x^2 = 0$

$x = \sqrt{0} \Rightarrow 0$



The nature of the roots of the equation $ax^2 + bx + c = 0$

If $a = c$ *then* Roots are reciprocals of each other

जैसे- $3x^2 - 5x + 3 = 0$

$a = c = 3$

$3x^2 - 5x + 3 = 0$

$\sqrt{-1 \times 9}$

$\alpha = \frac{-(-5) + \sqrt{25 - 4 \times 3 \times 3}}{2 \times 3} \Rightarrow \frac{+5 + \sqrt{-11}}{6}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\beta = \frac{5 - \sqrt{-11}}{6}$

$\frac{5 + \sqrt{-11}}{6} \times \frac{5 - \sqrt{-11}}{6}$

$\alpha\beta \Rightarrow \frac{c}{a}$

$\alpha\beta = 1$

$36 \times x = 1$

$x = \frac{1}{36}$

$\frac{5^2 - (\sqrt{-11})^2}{6 \times 6}$

$\Rightarrow \frac{25 - (-11)}{6 \times 6} \Rightarrow \frac{36}{36} = 1$

Quadratic Equations

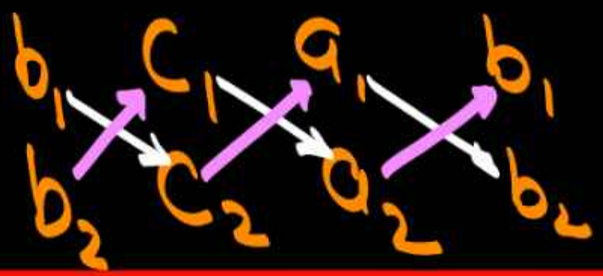
Condition for Common Roots

The equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

* Both roots are Common
दोनों मूल उभयनिष्ठ हो -

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Only one root is Common:



$$\frac{x^2}{b_1c_2 - b_2c_1} = \frac{x}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Quadratic Equations

Conditions for some Correlations between the Roots

(i) Roots in the same ratio: The roots of the equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are in the same ratio.

$(i.e., \frac{\alpha_1}{\beta_1} = \alpha_2/\beta_2)$ then

$$\frac{b_1^2}{b_2^2} = \frac{a_1c_1}{a_2c_2}$$

$5x^2 + 10x + 6 = 0 \Rightarrow -6, -4$

$x^2 + 20x + 96 = 0 \Rightarrow -12, -8$

Example. If the roots of the equation $2x^2 + 3x + 1 = 0$ and $x^2 - x + \lambda = 0$ are in the same ratio then λ

$a_1=2, b_1=3, c_1=1$ $\quad a_2=1, b_2=-1, c_2=\lambda$

$$\frac{(-1)^2}{3^2} = \frac{2\times1}{1\times\lambda} \Rightarrow \frac{9}{1} = \frac{2}{\lambda} \Rightarrow \lambda = \frac{2}{9}$$

(ii) **One root k times the other:** If one roots of the equation $ax^2 + bx + c = 0$ is k times the other root, then $\frac{(k+1)^2}{k} = \frac{b^2}{ac}$

$x^2 + 8x + 12 = 0$

$-6 \quad -2$

$-6 \times -2 = 12$

$-6 + -2 = -8$

Example. If one root of the equation $x^2 + 3x + \lambda = 0$ is three times the other root, then λ.

$\frac{(3+1)^2}{3} = \frac{3^2}{1 \times \lambda} \Rightarrow \frac{16}{3} = \frac{9}{\lambda} \Rightarrow \lambda = \frac{27}{16}$

$b^2 - 4ac < 0$

$x^2 - bx + 1 = 0$

$(-b)^2 - 4 \times 1 \times 1 < 0$

$b^2 - 4 < 0$

$b^2 < 4$

$-3 < 0$

$-4 < 0$

$-3 < 4$

1. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then which one of the following is correct?

   यदि समीकरण $x^2 - bx + 1 = 0$ के वास्तविक मूल नहीं हैं, तो निम्नलिखित में से कौन सा सही है?

   $D < 0$

   (a) $-3 < b < 3$, $-2, -1, 0, 1, 2$

   (b) $-2 < b < 2$, $-1, 0, 1$

   (c) $b > 2$, $\infty, 8, 7, 4, 3,$

   (d) $b < -2$,

2. If α and β are the roots of $ax^2 + bx + b = 0$ then what is $\frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}}$ equal to?

यदि α और β $ax^2 + bx + b = 0$ के मूल है, तो $\frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}}$ किसके बराबर है?

(a) 0

(b) 1

(c) 2

(d) 3

$\sin\alpha \cdot \cos\alpha = \frac{c}{a}$

$\sin\alpha + \cos\alpha = -\frac{b}{a}$ ←

$(\sin\alpha + \cos\alpha)^2 = \left(-\frac{b}{a}\right)^2$

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cdot\cos\alpha = \frac{b^2}{a^2}$

$1 + 2 \times \frac{c}{a} = \frac{b^2}{a^2}$

$\frac{a+2c}{a} = \frac{b^2}{a \times a}$

$a^2 + 2ac = b^2$

3. If the roots of $ax^2 + bx + c = 0$ are $\sin\alpha$ and $\cos\alpha$, then which one of the following is correct?

यदि $ax^2 + bx + c = 0$ की जड़ें $\sin\alpha$ और $\cos\alpha$ हैं, तो निम्नलिखित में से कौन सा सही है?

$2ac = b^2 - a^2$

(a) $a^2 + b^2 = 2ac$

(b) $b^2 - c^2 = 2ab$

(c) $b^2 - a^2 = 2ac$

(d) $b^2 + c^2 = 2ab$

4. One root of the equation $x^2 = px + q$ is reciprocal of the other and $p \neq \pm 1$. What is the value of q?

समीकरण $x^2 = px + q$ का एक मूल दूसरे और $p \neq \pm 1$ का व्युत्क्रम है। Q का मान क्या है?

(1) $q = -1$

(2) $q = 1$

(3) $q = 0$

(4) $q = \frac{1}{2}$