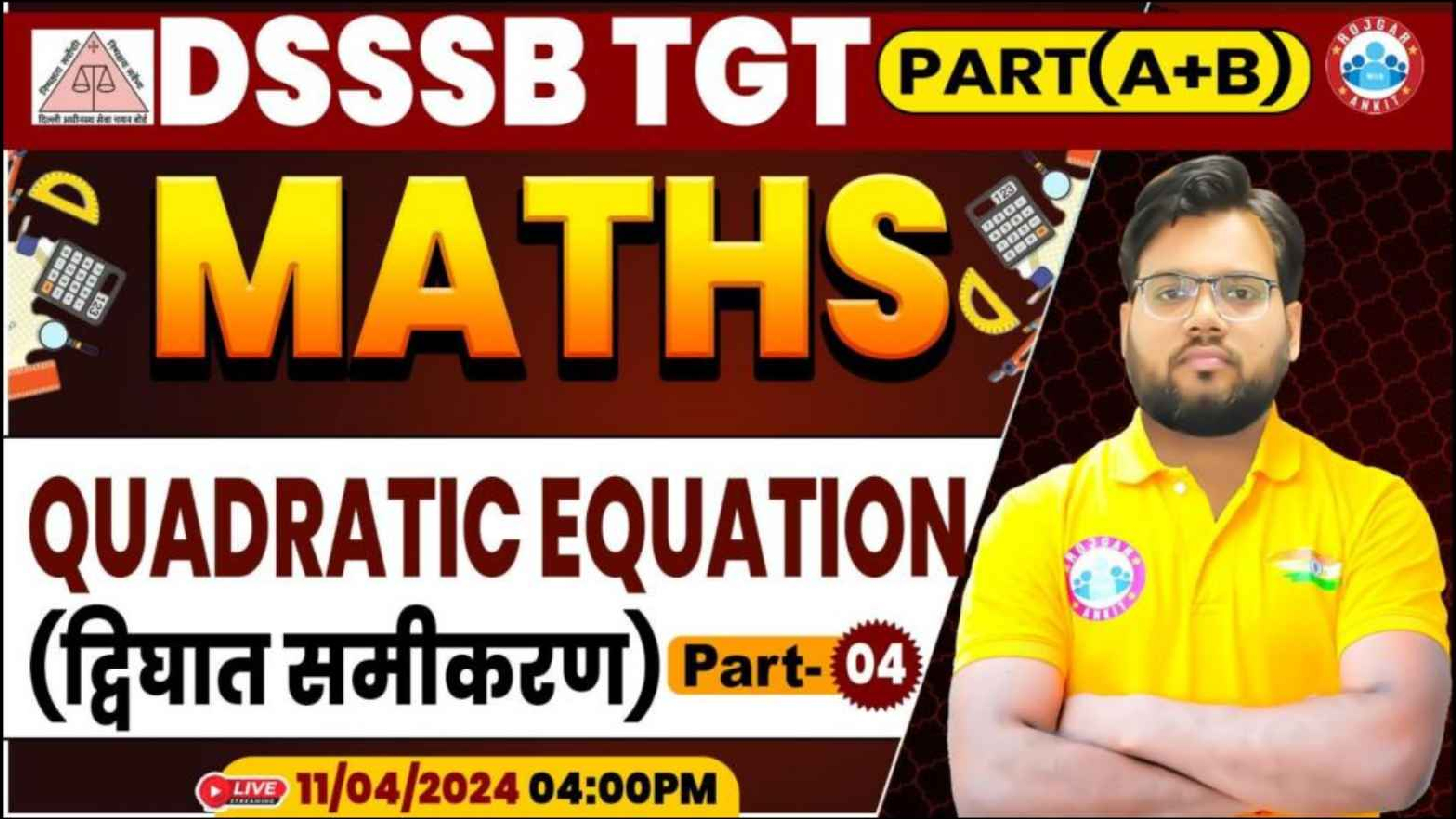
DSSSB TGT
PART(A+B)
MATHS
QUADRATIC EQUATION
(द्विघात समीकरण)
Part- 04
LIVE
11/04/2024 04:00PM

Quadratic Equations

Some Results on Roots of an Equation

$+x^3+3x+2x-1=0$

(i) Every n -degree equation has n roots.

(ii) Every odd degree equation has at least one real root. If the coefficient of the highest degree term is positive, then the sign of this real root is opposite to the sign of the constant term.

(iii) If the coefficient of the highest degree term of an even degree equation is positive and its constant term is negative, then this equation has at least two real roots: one positive and one negative.

$x^2 - 5x - 24 = 0$

$-8 \quad +3$

$8, -3$

(iv) If there is only one change in sign in an equation then it has only one positive root.

(v) If all terms of an equation are positive and it has no odd degree term, then it's all roots are imaginary.

(vi) Between any two real roots of every polynomial equation $f(x) = 0$ there always exists a real root of the equation $f(x) = 0$

(vii) If $f(x)$ is a real polynomial such that for two real numbers α, β; f(alpha) and f(beta) have opposite signs, then equation $f(x) = 0$ will have at least one root lying between a and β.

Symmetric Expressions of the Roots

(i) $\alpha^2 + \beta^2 =$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} =$

कल कर चुके हो

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} =$

(iv) $\alpha^2 + \alpha\beta + \beta^2 =$

(v) $\alpha^2\beta + \beta^2\alpha =$

(vi) $\alpha^3 + \beta^3 =$

(vii) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 =$

(viii) $\alpha^4 + \beta^4 =$

Transformation of an Equation

If a, β are roots of the equation ax^2 + bx + c = 0

Roots $\frac{1}{\alpha} = x$ $-\alpha = x \Rightarrow \alpha = -x$	Transformation on x	Transformed Equation
$-\alpha, -\beta$	$x \to -x$	$ax^2 - bx + c = 0$
$1/\alpha, 1/\beta$	$x \to 1/x$	$cx^2 + bx + a = 0$
$k\alpha, k\beta$	$x \to x/k$	$ax^2 + bkx + ck^2 = 0$
α^2, β^2	$x \to \sqrt{x}$	$a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
α^2, β^3	$x \to x^{1/3}$	$a^3x^2 + (b^2 - 3abc)x + c^3 = 0$
$\alpha + k, \beta + k$	$x \to (x - k)$	$ax^2 + (b - 2ak)x$ $+ (ak^2 - bk + c) = 0$

α, β -

$ax^2+bx+c=0$

अब वह equation ज्ञात कीजिए जिसके Roots $\alpha+2, \beta+2$ हो?

$\beta+2=x$

$\boxed{\beta=x-2}$

$a(x-2)^2+b(x-2)+c=0$

$a(x^2+4-4x)+bx-2b+c=0$

$ax^2+4a-4ax+bx-2b+c=0$

$ax^2-x(4a-b)+4a-2b+c=0$

$ax^2+bx+c=0$, $\underline{\alpha, \beta}$

$a(\sqrt{x})^2+b(\sqrt{x})+c=0$ वह eq^n ज्ञात कीजिए जिसके Roots $\underline{\alpha^2}$ व $\underline{\beta^2}$ हो?

$$\boxed{\underline{ax}+\underline{b\sqrt{x}}+\underline{c}=0}$$

$a^2x^2+b^2x+c^2+2axb\sqrt{x}+2bc\sqrt{x}$
$+2acx=0$

$\alpha^2=x$

$$\boxed{\alpha=\sqrt{x}}$$

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc$
$+2ca$

Roots Under Particular Conditions

$\alpha+\beta=-\frac{b}{a}$ $\alpha\beta=1 \Rightarrow \alpha=\frac{1}{\beta}$

$\beta=\frac{1}{\alpha}$

The nature of the roots of the equation $ax^2+bx+c=0$

Condition	Nature of roots
$a+b+c=0$	Roots = 1, c/a
$b=0$	roots are negative of each other
$c=0$	One root = 0
$b=c=0$	Both roots = 0
$a=c$	Roots are reciprocals of each other

Condition	Nature of roots
a, c are in opposite sign	Real roots in opposite sign
$b^2-4ac \geq 0$ and a, b, c are in same sign	Real roots, both negative
$b^2-4ac \geq 0$ and a, c are in same sign, b is in opposite sign	Real roots, both positive
$b=0; a, c$ are in same sign	Imaginary

$x^2-5x-6=0$ $3x^2-5x+3=0$ $3x^2-2x-1=0$

$$x^2 - 12x + 20 = 0$$

$-10 \quad -2$

$$\boxed{+10 \,,\, +2}$$

Real & positive

$b^2 - 4ac$

$12^2 - 4 \times 1 \times 20$

$144 - 80 > 0$

यदि $b^2 - 4ac \geq 0$

a, b, c

Same

Sign

$-13, -2$

$+13 \quad +2$

$x^2 + 15x + 26 = 0$ के की Roots होंगे?

(a) Real & positive

(b) imaginary & positive

(c) Real & Negative

(d) None of these

Quadratic Equations

Condition for Common Roots

The equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

Quadratic Equations

Conditions for some Correlations between the Roots

(i) Roots in the same ratio: The roots of the equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x^2 + b_2x + c_2 = 0$ are in the same ratio.

$(i.e., \frac{\alpha_1}{\beta_1} = \alpha_2/\beta_2)$ **then** $\boxed{\frac{b_1^2}{b_2^2} = \frac{a_1c_1}{a_2c_2}}$

Example. If the roots of the equation $2x^2 + 3x + 1 = 0$ and $x^2 - x + \lambda = 0$ are in the same ratio then λ

(ii) One root k times the other: If one roots of the equation $ax^2 + bx + c = 0$ is k times the other root, then $\boxed{\frac{(k+1)^2}{k} = \frac{b^2}{ac}}$

Example. If one root of the equation $x^2 + 3x + \lambda = 0$ is three times the other root, then λ.

2. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then which one of the following is correct?

यदि समीकरण $x^2 - bx + 1 = 0$ के वास्तविक मूल नहीं हैं, तो निम्नलिखित में से कौन सा सही है?

(a) $-3 < b < 3$ (b) $-2 < b < 2$

(c) $b > 2$ (d) $b < -2$