

NUMBER SYSTEMCLASS-17

Remainder Theorem

शेषफल प्रमेयType :- VII CRT (Chinese Remainder Theorem)Q) $(227)^{25}$ Find Remainder

$$\frac{(227)^{25}}{15} \text{ शेषफल ज्ञात करें}$$

$$\frac{(2)^{25}}{15}$$

$$3 \times 5$$

$$\frac{2^{25}}{3} \begin{matrix} R=1 \\ R=? \end{matrix}$$

$$T = 3 - 1 = 2$$

$$\frac{2^1}{3} \boxed{R=2}$$

$$\frac{2^{25}}{5} R=1$$

$$T = 5 - 1 = 4$$

$$\frac{2^1}{5} \boxed{R=2}$$

Q) $(719)^{240}$ Find Remainder

$$\frac{(719)^{240}}{143} \text{ शेषफल ज्ञात करें}$$

$$(719)^{240}$$

$$143$$

$$11 \times 13$$

$$\frac{(719)^{240}}{11} \Rightarrow \frac{(4)^{\frac{240}{10}}}{11} R=0$$

$$T = 11 - 1 = 10$$

$$\frac{4^0}{11} = \frac{1}{11} \boxed{R=1}$$

$$\frac{(719)^{240}}{13} = \frac{4^{\frac{240}{12}}}{13} R=0$$

$$T = 13 - 1 = 12$$

$$\frac{4^0}{13} = \frac{1}{13} \boxed{R=1}$$

Q | $\frac{2^{1201}}{105}$ Find Remainder

$\frac{2^{1201}}{105}$ शेषफल ज्ञात करें

$$(2)^{1201}$$

$$105$$

$$15 \times 7$$

$$3 \times 5$$

$$\frac{2^{1201}}{3} = R=1$$

$$T = 3 - 1 = 2$$

$$= \frac{2^2}{3} = \boxed{R=2}$$

$$\frac{2^{\frac{1201}{4}}}{5} R=1$$

$$T=5-1=4$$

$$= \frac{2^1}{5} \boxed{R=2}$$

$$\frac{2^{\frac{1201}{6}}}{7} R=1$$

$$T=7-1=6$$

$$\frac{2^1}{7} \boxed{R=2}$$

Q) $\frac{2^{200}}{35}$ Find Remainder

$\frac{2^{200}}{35}$ शेषफल ज्ञात करें
 $\swarrow \searrow$
 5×7

$$\frac{2^{\frac{200}{4}}}{5} R=0$$

$$T=5-1=4$$

$$\frac{2^0}{5} = \frac{1}{5} \boxed{R=1}$$

By option $\frac{11}{5} R=1$

$$\frac{2^{\frac{200}{6}}}{7} R=2$$

$$T=7-1=6$$

$$\frac{2^2}{7} = \frac{4}{7} = \boxed{R=4} \quad \frac{11}{7} R=4$$

Q) $\frac{2^{40}}{77}$ Find Remainder

$\frac{2^{40}}{77}$ शेषफल ज्ञात करें

$$\begin{array}{r} 2^{40} \\ \hline 77 \\ \swarrow \searrow \\ 7 \times 11 \end{array}$$

$$\frac{2^{40}}{7} \quad R=4 \quad T=7-1=6$$

$$\frac{2^4}{7} = \frac{16}{7} \quad \boxed{R=2} \quad \xrightarrow{\text{By option}} \frac{23}{7} \quad R=2$$

$$\frac{2^{40}}{11} \quad R=0 \quad T=11-1=10$$

$$\frac{2^0}{11} = \frac{1}{11} \quad \boxed{R=1} \quad \frac{23}{11} \quad R=1$$

Q) $\frac{3^{101}}{77}$ Find Remainder

$\frac{3^{101}}{77}$ शेषफल ज्ञात करें

$$\begin{array}{r} 3^{101} \\ \hline 77 \\ \swarrow \searrow \\ 7 \times 11 \end{array}$$

$$\frac{3^{101}}{7} \quad R=5 \quad T=7-1=6$$

$$\frac{3^5}{7} = \frac{243}{7} \quad \boxed{R=5} \quad \xrightarrow{\text{By option}} \frac{47}{7} \quad R=5$$

$$\frac{3^{101}}{11} \quad R=1 \quad T=11-1=10$$

$$\frac{3^1}{11} \quad \boxed{R=3} \quad \frac{47}{11} \quad R=3$$

Q) $\frac{2^{133}}{133}$ Find Remainder

$\frac{2^{133}}{133}$ शेषफल ज्ञात करें

$\frac{2^{133}}{133}$
 $\swarrow \searrow$
 19×7

$\frac{2^{133}}{19}$ $R=7$
 $T=19-1=18$

$\frac{2^7}{19} = \frac{128}{19}$ $R=14$ $\frac{128}{19}$ $R=14$ $\rightarrow$ By option

$\frac{2^{133}}{7}$ $R=1$
 $T=7-1=6$

$\frac{2^1}{7}$ $R=2$ $\frac{128}{7}$ $R=2$

• $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

• $7! = 7 \times \boxed{6 \times 5 \times 4 \times 3 \times 2 \times 1}$
 $6!$

$7! = 7 \times 6!$

• $5! = 5 \times 4 \times 3 \times 2 \times 1$

• $6! = 6 \times 5!$

• $7! = 7 \times 6 \times 5!$

• $12! = 12 \times 11!$
 $= 12 \times 11 \times 10!$

WILSON THEOREM

* Case-1 $N = \text{Prime No. (अभाज्य संख्या)}$

$$\frac{(N-1)!}{N} \Rightarrow \boxed{R = -1}$$

* Case-2 $N = \text{Composite Number (भाज्य संख्या)}$
 $\boxed{N > 4}$

$$\frac{(N-1)!}{N} \Rightarrow \boxed{R = 0}$$

Examples

• $\frac{4!}{5-N} \rightarrow (N-1)! = \boxed{R = -1}$

$$= 4 \times 3 \times 2 \times 1$$

$$\frac{24}{5} \Rightarrow \boxed{R = -1}$$

$$R = 5 - 1 = 4$$

• $\frac{28!}{29} \quad R = ?$

$$R = -1$$

$$R = 29 - 1 = 28$$

• $\frac{58!}{59-N} \rightarrow (N-1)!$

$$R = -1$$

$$R = 59 - 1 = 58$$

Q $\frac{9!}{11} R=?$

अगर $\frac{10!}{11} R=-1$
 $R=11-1=10$

$\frac{10 \times 9!}{11} = R=10$

$\frac{9!}{11} R=-10$
 $R=11-10=1$

II

$\frac{9!}{11} R=?$

अगर $\frac{10!}{11} R=-1$
 $R=11-1=10$

$\frac{10 \times 9!}{11} R=10$

$\frac{9!}{11} R=1$

Q $\frac{5!}{7} R=?$

$\frac{6!}{7} R=-1$
 $R=7-1=6$

$\frac{6 \times 5!}{7} R=6$

$\frac{5!}{7} R=1$

- $\frac{7!}{8} \rightarrow (N-1)!$
 $8 \rightarrow N$ $R=0$

- $\frac{17!}{18} \rightarrow (N-1)!$
 $18 \rightarrow N$ $R=0$

Q] $\frac{18!}{19}$ Find Remainder

$\frac{18!}{19} \rightarrow (N-1)!$ शेषफल आत करें
 $N \leftarrow 19 \rightarrow \text{Prime}$

$$R = -1$$

$$R = 19 - 1 = 18$$

Q] $\frac{28!}{29}$ Find Remainder

$\frac{28!}{29}$ शेषफल आत करें

$\frac{28!}{29} \rightarrow (N-1)!$
 $\leftarrow 29 \rightarrow N$
 Prime

$$R = -1$$

$$R = 29 - 1 = 28$$

Q] $\frac{100! + 6}{101}$ Find Remainder

$\frac{100! + 6}{101}$ शेषफल आत करें

$$\frac{\overset{-1}{100!} + \overset{+6}{6}}{101} = -1 + 6$$

$+5$

Q1 $\frac{9!}{10}$ Find Remainder

$\frac{9!}{10}$ शेषफल ज्ञात करें

$$\frac{9!}{10} \rightarrow (N-1)!$$

$$\leftarrow 10 \rightarrow N$$

Composite $R=0$

Q2 $\frac{21!}{22}$ Find Remainder

$\frac{21!}{22}$ शेषफल ज्ञात करें

$$\frac{21!}{22} \rightarrow (N-1)!$$

$$\leftarrow 22 \rightarrow N$$

Composite $R=0$

Q3 $\frac{5!}{7}$ Find Remainder

$\frac{5!}{7}$ शेषफल ज्ञात करें

$$\frac{5!}{7} \rightarrow (N-2)!$$

$$\leftarrow 7 \rightarrow N$$

Prime

According to Wilson
विलसन के अनुसार

$$\frac{6!}{7} \quad R=-1$$

$$R=7-1=6$$

$$= \frac{+6 \quad +1}{7} \frac{6 \times 5!}{7} = R=6$$

$$\frac{5!}{7} \quad R=1$$

Q $\frac{11!}{13}$ Find Remainder

$\frac{11!}{13}$ शेषफल ज्ञात करें

$$\frac{11!}{13} \rightarrow (N-2)$$

$$13 \rightarrow N$$

Prime

विलसन के अनुसार

$$\frac{12!}{13} R = -1$$

$$R = 13 - 1 = 12$$

$$\frac{12 \times 1}{12 \times 11!} R = 12$$

$$\frac{11!}{13} R = 1$$

Q $\frac{14!}{17}$ Find Remainder

$\frac{14!}{17}$ शेषफल ज्ञात करें

$$\frac{14!}{17} \rightarrow (N-3)!$$

$$17 \rightarrow N$$

विलसन के अनुसार

$$\frac{16!}{17} R = -1$$

$$R = 17 - 1 = 16$$

$$\frac{1 \times -2 \times \dots}{16 \times 15 \times 14!} R = 16$$

$$(+1) \times (+2) \times \dots = 16! \Rightarrow x = 8$$

$$\frac{14!}{17} R = 8$$

1. $\frac{(151)^{25}}{15}$ Find Remainder
 $\frac{(151)^{25}}{15}$ शेषफल ज्ञात करें

- a. 1
b. 2
c. 3
d. 5

2. $\frac{(312)^{120}}{77}$ Find Remainder
 $\frac{(312)^{120}}{77}$ शेषफल ज्ञात करें

- a. 13
b. 12
c. 5
d. 1

3. $\frac{(149)^{600}}{21}$ Find Remainder
 $\frac{(149)^{600}}{21}$ शेषफल ज्ञात करें

- a. 1
b. 2
c. 3
d. 5

4. $\frac{10!}{11}$ Find Remainder
 $\frac{10!}{11}$ शेषफल ज्ञात करें

- a. 1
b. 2
c. 10
d. 9

5. $\frac{36!}{37}$ Find Remainder
 $\frac{36!}{37}$ शेषफल ज्ञात करें

- a. 1
b. 2
c. 35
d. 36

6. $\frac{11!}{13}$ Find Remainder
 $\frac{11!}{13}$ शेषफल ज्ञात करें

- a. 1
b. 11
c. 12
d. 2

7. $\frac{15!}{17}$ Find Remainder
 $\frac{15!}{17}$ शेषफल ज्ञात करें

- a. 16
b. 15
c. 2
d. 1

8. $\frac{22!}{23}$ Find Remainder
 $\frac{22!}{23}$ शेषफल ज्ञात करें

- a. 2
b. 1
c. 21
d. 22

ANSWER SHEET

1	2	3	4	5	6	7	8
A	D	A	C	D	A	D	D

Sol.1 $R=1$

$$\frac{(151)^{25}}{15}$$

$$\sqrt[3 \times 5]{15}$$

$$\frac{1}{3}^{\frac{25}{2}}$$

$$\frac{1}{5}^{\frac{25}{4}}$$

$$T = 3-1 \Rightarrow 2$$

$$T = 5-1 \Rightarrow 4$$

$$\frac{1}{3}$$

$$\frac{1}{5}$$

$$\boxed{R=1}$$

$$\boxed{R=1}$$

Sol.2 $R=4$

$$\frac{(312)^{120}}{77}$$

$$\sqrt[7 \times 11]{77}$$

$$4^{120/6}$$

$$4^{120/10}$$

$$T = 7-1 \Rightarrow 6$$

$$T = 11-1 \Rightarrow 10$$

$$\frac{4^0}{7} = \frac{1}{7}$$

$$\frac{4^0}{11} = \frac{1}{11}$$

$$\boxed{R=1}$$

$$\boxed{R=1}$$

Sol.3 $R=2$

$$\frac{(149)^{600}}{21}$$

$$\sqrt[3 \times 7]{21}$$

$$2^{600/2}$$

$$2^{600/6}$$

$$T = 3-1 \Rightarrow 2$$

$$T = 7-1 \Rightarrow 6$$

$$\frac{2^0}{3} \Rightarrow \frac{1}{3}$$

$$\frac{2^0}{7} \Rightarrow \frac{1}{7}$$

$$\boxed{R=1}$$

$$\boxed{R=1}$$

Sol.4 $\frac{10!}{11} (n-1)!$

$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11}$$

$$R = -1$$

$$R \Rightarrow 11-1$$

$$\boxed{R \Rightarrow 10}$$

Sol.5

$$\frac{36!}{37} \rightarrow (n-1)!$$

$$\frac{36 \times 35 \times 34 \times \dots \times 2 \times 1}{37}$$

$$\boxed{R = -1}$$

$$R = 37-1$$

$$\boxed{R = 36}$$

Sol.6

$$\frac{11!}{13}$$

$$\text{अथवा } \frac{12!}{13}$$

$$\boxed{R = -1}$$

$$R = 13-1 \Rightarrow 12$$

$$\frac{12 \times 11!}{13}$$

$$R \Rightarrow 12$$

$$\frac{11!}{13} \Rightarrow R = -12$$

$$R = 13-12$$

$$\boxed{\Rightarrow 1}$$

Sol. 7

$$\frac{15!}{17}$$

37512

$$\frac{16!}{17}$$

$$R = -1$$
$$R \Rightarrow 16$$

$$\frac{16 \times 15!}{17}$$

$$R = -16$$
$$R \Rightarrow 17 - 16$$

$$\boxed{R \Rightarrow 1}$$

Sol. 8

$$\frac{22!}{23}$$

$$\frac{22 \times 21 \times 20 \times \dots \times 3 \times 2 \times 1}{23}$$

$$R = -1$$
$$R = 23 - 1$$
$$\boxed{R = 22}$$