



Foundation Batch

MATHS

Remainder Theorem

(शेषफल प्रमेय)

03

LIVE

13-03-2024 07:00PM





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$$\frac{(585)^3 \times (1197)^{999} \times 2188}{23}$$

$$(10)^3 \times (1)^{999} \times 3$$

$$1000 \times 3 = 3000$$

$$\frac{3000}{23}$$

$$R = 10$$

28. Find the remainder when $(585)^3 \times (1197)^{999} \times 2188$ is divided by 23.

जब $(585)^3 \times (1197)^{999} \times 2188$ को 23 से विभाजित किया जाए तो शेषफल ज्ञात कीजिए।

✓ 1. 10

2. 3

3. 2

4. 4



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TYPE – IV

Cyclicity

ex

$$\frac{2^n}{7}$$

$$\text{cyclicity} = \text{power} = 2^3$$

$$\frac{2^1}{7} = R = 2$$

$$\frac{2^2}{7} = \frac{4}{7} \quad R = 4$$

$$\frac{2^3}{7} = \frac{8}{7} \quad R = 1$$

$$\frac{2^4}{7} = \frac{16}{7} \quad R = 2$$

$$\frac{2^5}{7} = \frac{32}{7} = R = 4$$

$$\frac{2^6}{7} = \frac{64}{7} = R = 1$$

$$\frac{a^p}{b} = \boxed{R=1}$$

↳ Euler theorem. ✓

$a, b \rightarrow$ Coprime
સહઅભાજ્ય

$P = \text{Totient (ટોશિન્ટ)}$

↓
Totient is nothing
but cyclicity
Totient = cyclicity

ex

$$\frac{3^n}{4}$$

$$\frac{3^1}{4} = R=3$$

$$\frac{3^2}{4} = \frac{9}{4} = \boxed{R=1} \checkmark$$

$$\textcircled{=} \frac{3^3}{4} = \frac{27}{4} \quad R=3$$

How to find totient (T)
Totient कैसे निकालें

$$\begin{array}{c} \Downarrow \\ \frac{a^p}{b} \end{array} \quad \boxed{\text{Rem.} = 1}$$

भाजक

Case 1

$b(\text{भाजक}) \Rightarrow$ अभાજ્ય संख्या (Prime)

$$\boxed{\text{Totient}(T) = (b-1)}$$

Case-2

$b(\text{भाजक}) =$ अभાજ્ય नहीं है

$$b = p^x \times q^y \times r^z$$

$$T = b \times \left(1 - \frac{1}{p}\right) \times \left(1 - \frac{1}{q}\right) \times \left(1 - \frac{1}{r}\right)$$

$$\frac{(2)^2}{3} \quad R = ?$$

$$T = 3 - 1 = \textcircled{2}$$

$$\frac{2^1}{3} \quad \textcircled{R=2} \quad \underline{\text{Ans}}$$

$$\frac{2^0}{3} \quad \textcircled{R=1}$$

ex

$$\frac{2^4}{3} = \frac{2^2 \times 2^2}{3} = 1 \times 1 = \textcircled{2}$$

$$T = 3 - 1 = \textcircled{2}$$

$$\frac{5^{29}}{8}$$

$$R=?$$

$$8 = (2)^3$$

$$T = 8 \times (1 - \frac{1}{2})$$

$$8 \times \frac{1}{2} = \textcircled{4}$$

$$\frac{29}{4} \textcircled{R=1}$$

$$\frac{5^1}{8} \underline{\underline{R=5}}$$

$$\frac{2^{44}}{7}$$

$$R=?$$

$$T = 7 - 1 = 6$$

$$\frac{44}{6} \textcircled{R=2}$$

$$\frac{2^2}{7} = \textcircled{4} \text{ --- Rem.}$$

$$b = 12$$

$$T = \cancel{12 - 1 = 11}$$

$$12 = 4 \times 3$$

$$\downarrow$$

$$2^2 \times 3^1$$

$$T = 12 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right)$$

$$4 \times 12 \times \frac{1}{2} \times \frac{2}{3} = \textcircled{4}$$

$$\boxed{\frac{a^p}{b}}$$

$$b = 11$$

$$T = 11 - 1 = 10$$

$$b = 13$$

$$T = 13 - 1 = 12$$

$$b = 101 = \underline{\underline{\text{Prime}}}$$

$$T = 101 - 1 \\ = \underline{\underline{100}}$$

$$b = 18$$

$$T =$$

$$18 = 9 \times 2$$

$$\begin{array}{c} \downarrow \\ 3^2 \\ = 3^2 \times 2^1 \end{array}$$

$$T = 18 \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{2}\right)$$

$$18 \times \frac{2}{3} \times \frac{1}{2} \\ \textcircled{6}$$

ex

$$\frac{200}{101}$$

$$R = ?$$

$$\frac{2^0}{100} = \underline{\underline{\textcircled{1}}}$$

$$T = 101 - 1 = \textcircled{100}$$

$$\frac{200}{100} \underline{\underline{R = 0}}$$

ex

$$\frac{4^{2400}}{7}$$

$$R = ?$$

$$T = 7 - 1 = \textcircled{6}$$

$$\frac{\overset{400}{\cancel{2400}}}{\cancel{6}}$$

$$\boxed{R = 0}$$

$$\frac{4^0}{7} = \textcircled{1}$$



$$T = 7 - 1 = \underline{\underline{6}}$$

29. $\frac{2^{107}}{7}$ Find Remainder

$$\frac{107}{6} \quad \boxed{R=5}$$

$$\frac{2^{107}}{7} = \frac{32}{7} \quad \boxed{R=4}$$

$\frac{2^{107}}{7}$ शेषफल ज्ञात करें

a. 5

b. 3

☒ c. 4

d. None



$$T = 13 - 1 = \underline{\underline{12}}$$

30. $\frac{2^{1024}}{13}$ Find Remainder

$$\frac{2^{1024}}{13}$$

$$R = 4$$

$$\frac{2^{1024}}{13}$$

शेषफल ज्ञात करें

a. 2

b. 3

c. 1

d. None

$$= \frac{2^4}{13} = \frac{16}{13}$$

$$R = 3$$



$$T = 19 - 1 \\ = 18$$

$$3 \overline{) 3^{39}}_{19} \quad (R=3)$$

$$\frac{3^3}{19} = \frac{27}{19} \quad R=8$$

31. $\frac{3^{39}}{19}$ Find Remainder

$\frac{3^{39}}{19}$ शेषफल ज्ञात करें

a. 3

b. 5

c. 8

d. 11



$$T = 17 - 1 = \underline{\underline{16}}$$

32. $\frac{3^{34}}{17}$ Find Remainder

$$\frac{3^{34}}{17} \quad R = \underline{\underline{2}}$$

$\frac{3^{34}}{17}$ शेषफल ज्ञात करें

a. 8

b. 9

c. 1

d. 16

$$\frac{3^2}{17} = \frac{9}{17} \quad R = 9$$



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③

$$\frac{3 \times 3^{151}}{15}$$

33. $\frac{3^{152}}{15}$ Find Remainder

$$\Rightarrow \frac{3^{151}}{5}$$

$\frac{3^{152}}{15}$ शेषफल ज्ञात करें

$$T = S - 1 = 4$$

a. 3

b. 6

c. 2

d. 4

$$\frac{3^3}{5} R = 3$$

$$\therefore \frac{3^3}{5} = \frac{27}{5} R = 2$$

final Rem = $2 \times 3 = 6$



$$16 = 2^4$$

$$T = 16 \times (1 - \frac{1}{2})$$

$$8 \mid 16 \times \frac{1}{2}$$

$$T = 8$$

$$3 \overline{) 161} \quad (R=1)$$
$$\underline{16}$$

$$\frac{3^1}{16} = (R=3)$$

34. $\frac{3^{161}}{16}$ Find Remainder

$\frac{3^{161}}{16}$ शेषफल ज्ञात करें

a. 3

b. 8

c. 5

d. 1



$$16 = 2^4$$

$$T = 8$$

$$\begin{array}{r} 35 \overline{) 24} \\ 16 \end{array}$$

$$= \frac{35}{16} = \frac{35^0}{16}$$

$$35. \frac{35^{24^{26}}}{16} \text{ Find Remainder}$$

$$\frac{35^{24^{26}}}{16} \text{ शेषफल ज्ञात करें}$$

a. 8

b. 1

c. 15

d. 0

$$= \frac{1}{16} \quad R = 1$$



$$T = 7 - 1 = 6$$

36. $\frac{2346^{60}}{7}$ Find Remainder

$$\begin{array}{r} 2346^{60} \\ 7 \end{array}$$

$$\frac{2346^0}{7} = \frac{2346^1}{7} = 2$$

$\frac{2346^{60}}{7}$ शेषफल ज्ञात करें

a. 6

b. 2

c. 7

d. 0



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$$9 = 3^2$$

$$T = 9 \times \left(1 - \frac{1}{3}\right)$$

$$9 \times \frac{2}{3} = 6$$

$$\boxed{T=6}$$

$$\frac{2^{32}}{9} = \frac{2^{32}}{3^2}$$

$$\frac{2^2}{9} = \frac{2^4}{9}$$

$$\frac{2^4}{9} = \frac{2^4}{3^2}$$

$$\frac{2^4}{3^2} = \frac{2^4}{3^2}$$

$$\frac{2^4}{3^2} = \frac{2^4}{3^2}$$

$$\frac{2^4}{3^2} = \frac{2^4}{3^2}$$

$$37. \frac{2^{32} 3^{32}}{9}$$

Find Remainder

$$\frac{2^{32} 3^{32}}{9}$$

शेषफल ज्ञात करें

a. 2

b. 7

c. 8

d. 0

$$\frac{16}{9} \quad R=7$$



$T = 6$ → as per previous ques.

38. $\frac{13^{22^{42}}}{9}$ Find Remainder

$$\frac{13^{22^{42}}}{9} = \frac{4^{4^{4^0}}}{9}$$

$$= \frac{4^1}{9} = R = 4$$

$$\frac{13^{22^{42}}}{9} \text{ शेषफल ज्ञात करें}$$

a. 0

b. 5

c. 4

d. 1



$$T = 7 - 1 = 6$$

$$\begin{array}{r} 32 \\ 7 \end{array}$$

$$\begin{array}{r} 4 \\ 7 \end{array}$$

$$= \frac{4^4}{7} \Rightarrow \frac{256}{7} R=4$$

42. $\frac{32^{32^{32}}}{7}$ Find Remainder

$\frac{32^{32^{32}}}{7}$ शेषफल ज्ञात करें

a. 6

b. 3

c. 2

d. 4



$$\frac{32}{7} \times \frac{32}{7}$$

$$T = 7 - 1 = 6$$

$$\frac{4^4}{7} = \frac{256}{7}$$

$$\Rightarrow R = 4$$

44. $\frac{(32^{32})^{32}}{7}$ Find Remainder

$\frac{(32^{32})^{32}}{7}$ शेषफल ज्ञात करें

a. 4

b. 3

c. 1

d. None



$$\frac{5^{\frac{1}{6}} \times 3^{\frac{9}{6}} \times 9^{\frac{5}{6}}}{10}$$

$$= \frac{5^{\frac{1}{6}} \times 3^{\frac{1}{6}} \times 3^{\frac{5}{2}}}{10} = \frac{5^{\frac{1}{6}} \times 3^{\frac{1}{6}} \times 3^{\frac{5}{2}}}{10}$$

$$= R = S$$

47. $\frac{5^{795}}{18}$ Find Remainder

$\frac{5^{795}}{18}$ शेषफल ज्ञात करें

a. 5

b. 7

c. 9

d. 8

$$18 = 9 \times 2 = 3^2 \times 2^1$$

$$T = 18 \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{2}\right)$$

$$18 \times \frac{2}{3} \times \frac{1}{2}$$

$T = 6$



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$$T = 7 - 1 = 6$$

$$4^{\frac{5}{6}} + 6^{\frac{5}{6}}$$

$$4^5 + 6^5$$

$$1024 + 6 \times 36 \times 36$$

$$2 + 6 = \frac{8}{7} \quad R=1$$

51. $\frac{11^{17^{15}} + 13^{11^{15}}}{7}$ Find Remainder

$\frac{11^{\frac{5}{6} \cdot 17^{\frac{3}{6}}} + 13^{\frac{5}{6} \cdot 11^{\frac{3}{6}}}}{7}$ शेषफल ज्ञात करें

a. 1

b. 0

c. 2

d. 6

$$= 11^5 + 13^5$$

$$= 4^{125} + 6^{125}$$



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$$\begin{array}{r} 2 \\ 222 \\ 3 \overline{) 4} + 2 \overline{) 888} \end{array}$$

5

$$T = 5 - 1 = 4$$

$$\begin{array}{r} 2 \\ 3 + 2^0 \\ \hline 5 \end{array}$$

$$= \frac{9+1}{5} = \frac{10}{5} \quad R=0$$

53. $\frac{888^{222} + 222^{888}}{5}$ Find Remainder

$\frac{888^{222} + 222^{888}}{5}$ शेषफल ज्ञात करें

a. 0

b. 1

c. 4

d. 3