

NUMBER SYSTEMClass-16

REMAINDER THEOREM

शेषफल प्रमेय

Q) Find the remainder when $(585)^3 \times (1197)^{999} \times 2188$ is divided by 23.

जब $(585)^3 \times (1197)^{999} \times 2188$ को 23 से विभाजित किया जाए तो शेषफल ज्ञात कीजिए।

$$\frac{(585)^3 \times (1197)^{999} \times 2188}{23}$$

$$(10)^3 \times (1)^{999} \times 3$$

$$1000 \times 3 = 3000$$

$$\frac{3000}{23} R=10$$

* Cyclicity

Ex:- $\frac{2^n}{7}$

$$\text{Cyclicity} = \text{Power} = 2^3$$

$$\frac{2^1}{7} = R=2$$

$$\frac{2^4}{7} = \frac{16}{7} R=2$$

$$\frac{2^2}{7} = \frac{4}{7} R=4$$

$$\frac{2^5}{7} = \frac{32}{7} R=4$$

$$\frac{2^3}{7} = \frac{8}{7} \boxed{R=1}$$

$$\frac{2^6}{7} = \frac{64}{7} R=1$$

Ex:- $\frac{3^n}{4}$

$$\frac{3^1}{4} = R=3$$

$$\frac{3^2}{4} = \frac{9}{4} = \boxed{R=1}$$

$$\frac{3^3}{4} = \frac{27}{4} \quad R=3$$

$$\frac{a^p}{b} = \boxed{R=1}$$

↳ Euler Theorem

$a, b \rightarrow$ Coprime (सहअभाज्य)

$$\boxed{P = \text{Totient (टोटिएन्ट)}}$$

↓
Totient is nothing but cyclicity
Totient = Cyclicity

* How to find Totient (T)
Totient कैसे निकाले

$$\Downarrow$$

$$\frac{a^p}{b} \quad \boxed{\text{Rem}=1}$$

$b \rightarrow$ भाजक

• Case 1:-

b (भाजक) = अभाज्य संख्या (Prime)

$$\boxed{\text{Totient (T)} = (b-1)}$$

• Case 2:-

b (भाजक) = अभाज्य नहीं है।

$$b = p^x \times q^y \times r^z$$

$$T = b \times \left(1 - \frac{1}{p}\right) \times \left(1 - \frac{1}{q}\right) \times \left(1 - \frac{1}{r}\right)$$

$$\underline{\text{Ex:-}} \frac{2^4}{3} = \frac{2^2 \times 2^2}{3} = 1 \times 1 = 1$$

$$T = 3 - 1 = 2$$

$$\underline{\text{Ex:-}} \frac{(2)^{13}}{3} \boxed{R=1} \quad R=?$$

$$T = 3 - 1 = 2$$

$$\frac{2^1}{3} \quad R=2$$

$$\underline{\text{Ex:-}} \frac{2^{44}}{7} \quad R=?$$

$$T = 7 - 1 = 6$$

$$\frac{44}{6} \boxed{R=2}$$

$$\frac{2^2}{7} = \frac{4}{7} \quad R=4$$

$$\underline{\text{Ex:-}} \frac{5^{29}}{8} \quad R=?$$

$$8 = (2)^3$$

$$T = 8 \times \left(1 - \frac{1}{2}\right)$$

$$4 \times \frac{1}{2} = 4$$

$$\frac{29}{4} \quad R=1$$

$$\frac{5^1}{8} \quad R=5$$

$$\boxed{\frac{a^p}{b}}$$

- $b = 11$

$$T = 11 - 1 = 10$$

- $b = 13$

$$T = 13 - 1 = 12$$

- $b = 12$

$$12 = 4 \times 3$$

$$\downarrow$$

$$2^2 \times 3^1$$

$$T = 12 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right)$$

$$\overset{4}{\cancel{12}} \times \frac{1}{\cancel{2}} \times \frac{2}{\cancel{3}} = 4$$

- $b = 18$

$$18 = 9 \times 2$$

$$\downarrow$$

$$3^2$$

$$= 3^2 \times 2^1$$

$$T = 18 \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{2}\right)$$

$$\overset{6}{\cancel{18}} \times \frac{2}{\cancel{3}} \times \frac{1}{\cancel{2}} = 6$$

- $b = 101 = \text{Prime}$

$$T = 101 - 1 = 100$$

Ex: $\frac{2^{200}}{101} \quad R = ?$

$$T = 101 - 1 = 100$$

$$\frac{\cancel{200}}{\cancel{100}} \quad R = 0$$

$$\frac{2^0}{101} = 1$$

Ex:- $\frac{4^{2400}}{7} \quad R=?$

$$T = 7 - 1 = 6$$

$$\frac{\overset{400}{\cancel{2400}}}{6} \quad R=0$$

$$\frac{\overset{0}{4}}{7} = 1$$

Q) $\frac{2^{107}}{7}$ Find Remainder

$\frac{2^{107}}{7}$ शेषफल ज्ञात करें

$$T = 7 - 1 = 6$$

$$\frac{\cancel{107}}{6} \quad R=5$$

$$\frac{2^5}{7} = \frac{32}{7} \quad R=4$$

Q) $\frac{2^{1024}}{13}$ Find Remainder

$\frac{2^{1024}}{13}$ शेषफल ज्ञात करें

$$T = 13 - 1 = 12$$

$$\frac{2^{\cancel{1024}}}{13} \quad R=4$$

$$\frac{2^4}{13} = \frac{16}{13} \quad R=3$$

Q) $\frac{3^{39}}{19}$ Find Remainder

$\frac{3^{39}}{19}$ शेषफल ज्ञात करें

$$T = 19 - 1 = 18$$

$$\frac{3^{18}}{19} \quad R=3$$

$$\frac{3^3}{19} = \frac{27}{19} \quad R=8$$

Q1 $\frac{3^{34}}{17}$ Find Remainder

$\frac{3^{34}}{17}$ शेषफल जात करें

$$T = 17 - 1 = 16$$

$$\frac{3^{16}}{17} \quad R=2$$

$$\frac{3^2}{17} = \frac{9}{17} \quad R=9$$

Q2 $\frac{3^{152}}{15}$ Find Remainder

$\frac{3^{152}}{15}$ शेषफल जात करें

③ $\frac{2 \times 3^{151}}{15}$
 $\frac{2 \times 3^{151}}{5}$

$$T = 5 - 1 = 4$$

$$\frac{3^4}{5} \quad R=3$$

$$\frac{3^3}{5} = \frac{27}{5} \quad R=2$$

$$\text{Final Rem} = 2 \times 3 = 6$$

Q) $\frac{3^{16}}{16}$ Find Remainder

$\frac{3^{16}}{16}$ शेषफल ज्ञात करें

$16 = 2^4$

$T = 16 \times \left(1 - \frac{1}{2}\right)$

$\overset{8}{16} \times \frac{1}{2}$

$T = 8$

$\frac{3^{16}}{16} \quad R = 1$

$\frac{3^1}{16} = R = 3$

Q) $\frac{35^{24}}{16}$ Find Remainder

$\frac{35^{24}}{16}$ शेषफल ज्ञात करें

$16 = 2^4$

$T = 16 \times \left(1 - \frac{1}{2}\right)$

$\overset{8}{16} \times \frac{1}{2}$

$T = 8$

$\frac{35^{24}}{16} = \frac{35^{24}}{2^4} = \frac{35^{24}}{8^2}$

$= \frac{35^0}{16} = \frac{35^0}{16} = \frac{1}{16} \quad R = 1$

Q1 $\frac{23^{46^{60}}}{7}$ Find Remainder

$\frac{23^{46^{60}}}{7}$ शेषफल ज्ञात करें

$$T = 7 - 1 = 6$$

$$\frac{23^{\frac{46^{60}}{6}}}{7}$$

$$\frac{2^4}{7} = \frac{2^1}{7} = 2$$

Q2 $\frac{2^{32^{32}}}{9}$ Find Remainder

$\frac{2^{32^{32}}}{9}$ शेषफल ज्ञात करें

$$9 = 3^2$$

$$T = 9 \times \left(1 - \frac{1}{3}\right)$$

$$3 \times \frac{2}{3} = 6$$

$$\boxed{T = 6}$$

$$\frac{2^{\frac{32^{32}}{6}}}{9} = \frac{2^{2^2}}{9} = \frac{2^4}{9}$$

$$= \frac{16}{9} \quad R = 7$$

Q3 $\frac{13^{22^{42}}}{9}$ Find the remainder

$\frac{13^{22^{42}}}{9}$ शेषफल ज्ञात करें

$$9 = 3^2$$

$$T = 9 \times \left(1 - \frac{1}{3}\right)$$

$$3 \times \frac{2}{3} = 6$$

$$\boxed{T=6}$$

$$\frac{13 \frac{22}{6}}{9} = \frac{4^0}{9} = \frac{4^1}{9} \quad R=4$$

Q $\frac{32^{32}}{7}$ Find Remainder

$\frac{32^{32}}{7}$ शेषफल ज्ञात करें

$$T = 7 - 1 = 6$$

$$\frac{32 \frac{32^2}{6}}{7} = \frac{4^2}{7} = \frac{4^4}{7}$$

$$= \frac{256}{7} \quad R=4$$

Q $\frac{(32^{32})^{32}}{7}$ Find Remainder

$\frac{(32^{32})^{32}}{7}$ शेषफल ज्ञात करें

$$T = 7 - 1 = 6$$

$$\frac{32 \frac{32^2 \times 32^2}{6}}{7}$$

$$\frac{4^4}{7} = \frac{256}{7} \quad R=4$$

Q) $\frac{5^{795}}{18}$ Find Remainder

$\frac{5^{795}}{18}$ शेषफल ज्ञात करें

$$18 = 9 \times 2$$

$$\downarrow$$

$$3^2 \times 2^1$$

$$T = 18 \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{2}\right)$$

$$18 \times \frac{2}{3} \times \frac{1}{2}$$

$$T = 6$$

$$\frac{5^{795}}{18} = \frac{5^{\overbrace{795}^{3 \times 265}}}{18} = \frac{5^1}{18} = \frac{5^1}{18} \quad R = 5$$

Q) $\frac{11^{17^{15}} + 13^{11^{15}}}{7}$ Find Remainder

$\frac{11^{17^{15}} + 13^{11^{15}}}{7}$ शेषफल ज्ञात करें

$$T = 7 - 1 = 6$$

$$\frac{11^{\overbrace{17^{15}}^{5 \times 3}} + 13^{\overbrace{11^{15}}^{5 \times 3}}}{7}$$

$$\frac{11^{5^3} + 13^{5^3}}{7} = \frac{4^{125} + 6^{125}}{7}$$

$$\frac{4^{\overbrace{125}^{5 \times 25}} + 6^{\overbrace{125}^{5 \times 25}}}{7} = \frac{4^5 + 6^5}{7}$$

$$\frac{1024 + 6 \times 36 \times 36}{7}$$

$$2 + 6 = 8 \quad \frac{8}{7} \quad R = 1$$

Q1 $\frac{888^{222} + 222^{888}}{5}$ Find Remainder

$\frac{888^{222} + 222^{888}}{5}$ शेषफल ज्ञात करें

$$\frac{3^2 + 2^0}{5}$$

$$T = 5 - 1 = 4$$

$$\frac{3^2 + 2^0}{5}$$

$$= \frac{9 + 1}{5} = \frac{10}{5} \quad R = 0$$

1. $\frac{25^{25^{25}}}{9}$ Find Remainder

$\frac{25^{25^{25}}}{9}$ शेषफल ज्ञात करें

a. 6

b. 7

c. 1

d. 0

2. $\frac{15^{30^{45}}}{8}$ Find Remainder

$\frac{15^{30^{45}}}{8}$ शेषफल ज्ञात करें

a. 1

b. 4

c. 3

d. None

3. $\frac{35^{24^{26}}}{16}$ Find Remainder

$\frac{35^{24^{26}}}{16}$ शेषफल ज्ञात करें

a. 4

b. 1

c. 15

d. 8

4. $\frac{32^{32^{32}}}{9}$ Find Remainder

$\frac{32^{32^{32}}}{9}$ शेषफल ज्ञात करें

a. 4

b. 6

c. 1

d. 3

5. $\frac{34^{31^{30}}}{9}$ Find Remainder

$\frac{34^{31^{30}}}{9}$ शेषफल ज्ञात करें

a. 5

b. 3

c. 1

d. None

6. $\frac{7^{21^{22^{23}}}}{25}$ Find Remainder

$\frac{7^{21^{22^{23}}}}{25}$ शेषफल ज्ञात करें

a. 7

b. 21

c. 22

d. 23

7. $\frac{11^{17^{15}}}{7}$ Find Remainder

$\frac{11^{17^{15}}}{7}$ शेषफल ज्ञात करें

a. 2

b. 7

c. 3

d. None

8. $\frac{5^{1024^{1024}}}{9}$ Find Remainder

$\frac{5^{1024^{1024}}}{9}$ शेषफल ज्ञात करें

a. 2

b. 9

c. 4

d. 1

9. $\frac{5555^{2222} + 2222^{5555}}{7}$ Find

Remainder

$\frac{5555^{2222} + 2222^{5555}}{7}$ शेषफल

ज्ञात करें

a. 0

b. 1

c. 4

d. 4

ANSWER SHEET

1	2	3	4	5	6	7	8	9
B	A	B	A	D	A	A	C	A

Sol. 1

$$\frac{25^{25^{25}}}{9}$$

$$9 = 3^2$$

$$T = 9 \times (1 - \frac{1}{3})$$

$$\Rightarrow 9 \times \frac{2}{3} \Rightarrow 6$$

$$\frac{25^{\frac{25}{6}^{\frac{25}{6}}}}{9} \Rightarrow \frac{7^1}{9} \Rightarrow \frac{7}{9}$$

$$\Rightarrow 7$$

Sol. 2

$$\frac{15^{30^{45}}}{8}$$

$$8 \Rightarrow 2^3$$

$$T = 8 \times (1 - \frac{1}{2})$$

$$\Rightarrow 8 \times \frac{1}{2} \Rightarrow 4$$

$$\frac{15^{\frac{30}{4}^{\frac{45}{4}}}}{8} = \frac{7^2}{8}$$

$$\Rightarrow \frac{49}{8} = 12 \frac{1}{4}$$

$$\Rightarrow 1$$

Sol. 3

$$16 \Rightarrow 2^4$$

$$\frac{35^{24^{26}}}{16}$$

$$T \Rightarrow 16 \times (1 - \frac{1}{2})$$

$$\Rightarrow 8$$

$$\frac{35^{\frac{24}{8}^{\frac{26}{8}}}}{16} \Rightarrow \frac{3^0}{16} \Rightarrow \frac{1}{16}$$

$$\Rightarrow 1$$

Sol. 4

$$\frac{32^{32^{32}}}{9}$$

$$9 = 3^2$$

$$T = 9 \times (1 - \frac{1}{3})$$

$$9 \times \frac{2}{3} \Rightarrow 6$$

$$\frac{32^{\frac{32}{6}^{\frac{32}{6}}}}{9} = \frac{5^2}{9} \Rightarrow \frac{5^4}{9} \Rightarrow \frac{625}{9}$$

$$\Rightarrow 4 \frac{21}{4}$$

Sol. 5

$$\frac{34^{31^{30}}}{9}$$

$$T = 9 \times (1 - \frac{1}{3})$$

$$T = 6$$

$$\frac{34^{\frac{31}{6}^{\frac{30}{6}}}}{9} = \frac{7^1}{9}$$

$$\Rightarrow \frac{7}{9}$$

$$\Rightarrow 7 \frac{7}{9}$$

Sol. 6

$$\frac{7^{21} 2^3}{25}$$

$$25 = 5^2$$

$$T = 25 \left(1 - \frac{1}{5}\right)$$

$$\boxed{T = 20}$$

$$\frac{7^{\frac{21}{20}} 2^{\frac{3}{20}}}{25} = \frac{7^1 2^3}{25} = \frac{7}{25}$$

$$\Rightarrow 7 \text{ शी}$$

Sol. 7

$$\frac{11^{17} 15}{7}$$

$$T = 7 - 1$$

$$T \Rightarrow 6$$

$$\frac{11^{\frac{17}{6}} 15^{\frac{15}{6}}}{7} \Rightarrow \frac{4^5 3}{7} \Rightarrow \frac{4^{125}}{6} \quad \boxed{R=5}$$

$$\frac{4^5}{7} \Rightarrow \frac{1024}{7} \Rightarrow 2 \text{ शी}$$

Sol. 8

$$\frac{5^{1024}}{9}$$

$$T \Rightarrow 6$$

$$\frac{5^{\frac{1024}{6}}}{9} \Rightarrow \frac{5^4}{9} \Rightarrow 5^{\frac{256}{6}} \Rightarrow \frac{5^1}{9} = \frac{625}{9}$$

$$\Rightarrow 4 \text{ शी}$$

Sol. 9

$$\frac{5555^{2222} + 2222^{5555}}{7}$$

$$\Rightarrow \frac{4^{\frac{2222}{6}} + 3^{\frac{5555}{6}}}{7}$$

$$T \Rightarrow 7 - 1$$

$$\Rightarrow 6$$

$$\Rightarrow \frac{4^2 + 3^5}{7}$$

$$\Rightarrow \frac{16 + 243}{7} = \frac{7}{7}$$

$$\Rightarrow 0 \text{ शी}$$