$x^n \pm \frac{1}{x^n}$

Concept:

$a+b=$

$ab=$

$a+b=\checkmark$ $ab=\checkmark$ $a^n+b^n=?$

$a-b=\checkmark$ $ab=\checkmark$ $a^n-b^n=?$

$a+b$, $a-b$, ab

$a^2+b^2=$	$a^2-b^2=$
$a^3+b^3=$	$a^3-b^3=$
$a^4+b^4=$	$a^4-b^4=$
$a^5+b^5=$	$a^5-b^5=$
$a^6+b^6=$	$a^6-b^6=$
$a^7+b^7=$	$a^7-b^7=$
$a^8+b^8=$	$a^8-b^8=$

$a+b=5$ $ab=3$

$a^2+b^2=(a+b)^2-2ab$

$a^3+b^3=(a+b)^3-3ab(a+b)$

$a=x$, $b=\frac{1}{x}$

$x+\frac{1}{x}=5$

$x\times\frac{1}{x}=1$ → No need to mention

$x^n+\frac{1}{x^n}$

$x+\frac{1}{x} \rightarrow x^n+\frac{1}{x^n}$

$x-\frac{1}{x} \rightarrow x^n-\frac{1}{x^n}$

$x+\frac{1}{x}=5$

$a^2+b^2=(a+b)^2-2ab$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2x\cdot\frac{1}{x}$

$=25-2=23$

$a^3+b^3=(a+b)^3-3ab(a+b)$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$=125-3\times5$

$=110$

① $(a+b)^2 - (a-b)^2 = 4ab$

$$\left(x+\frac{1}{x}\right)^2 - \left(x-\frac{1}{x}\right)^2 = 4x\frac{1}{x}$$

$x+\frac{1}{x}$ → $x-\frac{1}{x}$

$x^n + \frac{1}{x^n}$

$x+\frac{1}{x}=5 \rightarrow x-\frac{1}{x}=\sqrt{21}$ (25 → 21)

$x-\frac{1}{x}=10 \quad x+\frac{1}{x}=\sqrt{104}$ (100 → 104)

② Power doubling (+ve): Square-2

If $x+\frac{1}{x}=a \longrightarrow x^2+\frac{1}{x^2}=a^2-2$

If $x^2+\frac{1}{x^2}=b \longrightarrow x^4+\frac{1}{x^4}=b^2-2$

If $x^3+\frac{1}{x^3}=c \longrightarrow x^6+\frac{1}{x^6}=c^2-2$

If $x^5+\frac{1}{x^5}=d \longrightarrow x^{10}+\frac{1}{x^{10}}=d^2-2$

If $x^n+\frac{1}{x^n}=f \longrightarrow x^{2n}+\frac{1}{x^{2n}}=f^2-2$

eg: $x+\frac{1}{x}=7$

$x^2+\frac{1}{x^2}=7^2-2=47$

$x^{36}+\frac{1}{x^{36}}=10$

$\rightarrow x^{72}+\frac{1}{x^{72}}=10^2-2=98$

③ Power tripple (+ve): Cube – 3 times

If $x+\frac{1}{x}=a \longrightarrow x^3+\frac{1}{x^3}=a^3-3a$

If $x^2+\frac{1}{x^2}=b \longrightarrow x^6+\frac{1}{x^6}=b^3-3b$

If $x^5+\frac{1}{x^5}=c \longrightarrow x^{15}+\frac{1}{x^{15}}=c^3-3c$

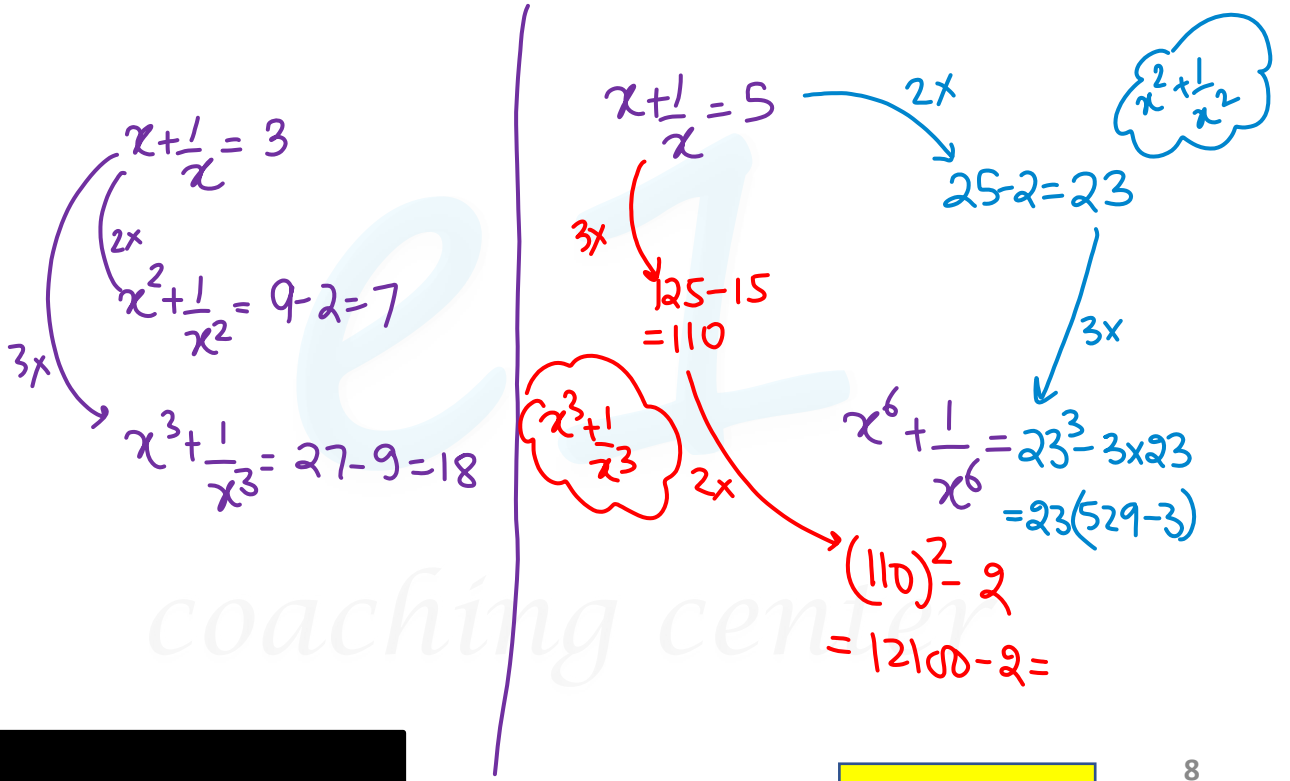
fx' $x+\frac{1}{x}=5$

$\rightarrow x^3+\frac{1}{x^3}=125-3(5)$

$=110$

$x^{1000}+\frac{1}{x^{1000}}=10$

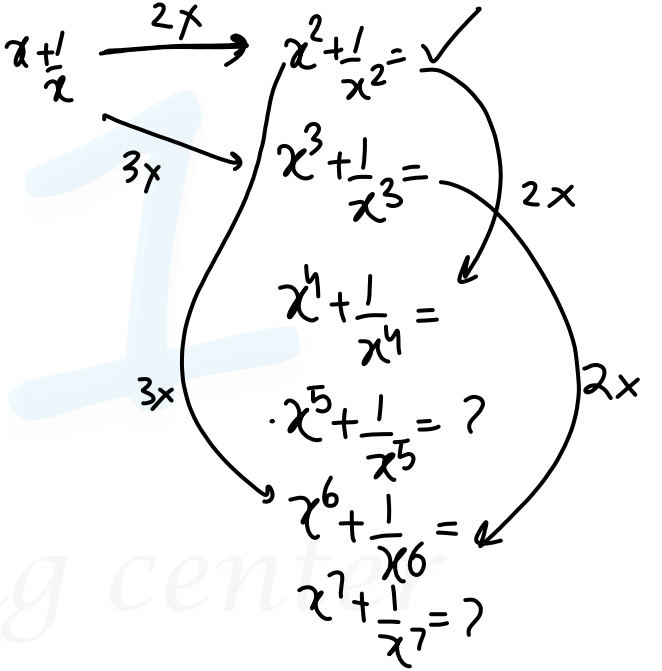
$\rightarrow x^{3000}+\frac{1}{x^{3000}}=10^3-30$



$x + \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$

$x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$



(4) Big odd powers' (+ve)

$$a^5+b^5=(a^2+b^2)(a^3+b^3)-a^2b^2(a+b)$$

$ab=x\cdot\frac{1}{x}=1$

$$x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$x^7+\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$x^{11}+\frac{1}{x^{11}}=\left(x^6+\frac{1}{x^6}\right)\left(x^5+\frac{1}{x^5}\right)-\left(x+\frac{1}{x}\right)$$

(5). Power doubling (-ve)' Square+2

$$a^2+b^2 = (a-b)^2+2ab$$

but +ve (even power)

If $x-\frac{1}{x}=a \longrightarrow x^2+\frac{1}{x^2} = a^2+2$

If $x^2-\frac{1}{x^2}=b \longrightarrow x^4+\frac{1}{x^4} = b^2+2$

If $x^3-\frac{1}{x^3}=c \longrightarrow x^6+\frac{1}{x^6} = c^2+2$

Ex If $x-\frac{1}{x}=5$

$\downarrow$

$x^2+\frac{1}{x^2} = 5^2+2$

$=27$

⑥ Power trippling (-ve): Cube +3 times

If $x-\frac{1}{x}=a \longrightarrow x^3-\frac{1}{x^3}=a^3+3a$ (-ve odd power)

If $x^2-\frac{1}{x^2}=b \longrightarrow x^6-\frac{1}{x^6}=b^3+3b$

If $x^{15}-\frac{1}{x^{15}}=c \longrightarrow x^{45}-\frac{1}{x^{45}}=c^3+3c$

Ex $x-\frac{1}{x}=5$

$x^3-\frac{1}{x^3}=5^3+3\times5$

$=140$

(7) Big odd powers (-ve)'

$$a^5 - b^5 = (a^2 - b^2)(a^3 + b^3) \oplus a^2b^2(a - b)$$

$$x^5 - \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 - \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right)$$

$$x^7 - \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right)\left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$$

(8.) Even powers with -ve sign'

$$a^2-b^2=(a+b)(a-b)$$

$$a^4-b^4=(a^2-b^2)(a^2+b^2)$$

$$a^4-b^4=(a^2+b^2)(a+b)(a-b)$$

$$a^8-b^8=(a^4+b^4)(a^2+b^2)(a+b)(a-b)$$

$$x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$x^4-\frac{1}{x^4}=\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$x - \frac{1}{x} = 3$ $\qquad$ $x^8 - \frac{1}{x^8} = ?$

$3^2 + 2 = 11$ $\qquad$ $\sqrt{9+4}$

$2x$

$$= \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= 119 \times 11 \times \sqrt{13} \times 3$$

Concept:

$$x + \frac{1}{x} = 3$$

2x

3x

$$7 \times 18 - 3$$

$$\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$x^2 + \frac{1}{x^2} = 3^2-2=7$$

$$x^3 + \frac{1}{x^3} = 27-9=18$$

$$x^4 + \frac{1}{x^4} = 49-2=47$$

$$x^5 + \frac{1}{x^5} = 123$$

$$x^6 + \frac{1}{x^6} = 18^2-2=322$$

formula

$$x^7 + \frac{1}{x^7} =$$

$$x^8 + \frac{1}{x^8} =$$

Concept:

$x + \frac{1}{x} = \sqrt{13}$

$x - \frac{1}{x} = 3$

$()^3 - 3x$

$\left(x^3 + \frac{1}{x^3}\right)\left(x^2 - \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right)$

$x^2 + \frac{1}{x^2} =$

$3x$

$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= \sqrt{13} \times 3$

$x^3 - \frac{1}{x^3} = 27 + 9$

$x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right) \times \left(x - \frac{1}{x}\right)$

$x^5 - \frac{1}{x^5} =$

$\left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

$x^6 - \frac{1}{x^6} =$

$x^7 - \frac{1}{x^7} =$

$x^8 - \frac{1}{x^8} =$

Practice:

$$x + \frac{1}{x} = 3, 4, 5$$ ✓ ✓ ✓

$$x^2 + \frac{1}{x^2} = \qquad x^2 - \frac{1}{x^2} =$$

$$x^3 + \frac{1}{x^3} = \qquad x^3 - \frac{1}{x^3} =$$

$$x^4 + \frac{1}{x^4} = \qquad x^4 - \frac{1}{x^4} =$$

$$x^6 + \frac{1}{x^6} = \qquad x^6 - \frac{1}{x^6} =$$

$$x^8 + \frac{1}{x^8} = \qquad x^8 - \frac{1}{x^8} =$$

Practice:

Given	Find	
$x - \frac{1}{x} = 4$	$x^2 + \frac{1}{x^2} =$	$x^3 - \frac{1}{x^3} =$
$x + \frac{1}{x} = 5$	$x - \frac{1}{x} =$	$x^4 + \frac{1}{x^4} =$
$x - \frac{1}{x} = 6$	$x + \frac{1}{x} =$	$x^4 + \frac{1}{x^4} =$
$\frac{a}{b} + \frac{b}{a} = 3$	$\frac{a^6}{b^6} + \frac{b^6}{a^6} =$	$\frac{a^8}{b^8} + \frac{b^8}{a^8} =$
$\frac{x}{2} + \frac{2}{x} = 4$	$\frac{x}{2} - \frac{2}{x} =$	$\frac{x^2}{4} - \frac{4}{x^2} =$

$$\frac{x^2}{y} - \frac{y}{x^2} = \left(\frac{x}{2}\right)^2 - \left(\frac{2}{x}\right)^2$$

$$= \left(\frac{x}{2} + \frac{2}{x}\right)\left(\frac{x}{2} - \frac{2}{x}\right)$$

$$= 4 \times 2\sqrt{3} = 8\sqrt{3}$$

172. If $x - \frac{1}{x} = 5$, then value of $x^2 + \frac{1}{x^2}$ is

अगर $x - \frac{1}{x} = 5$ है तो $x^2 + \frac{1}{x^2}$ पता करें|

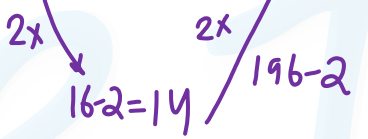
a) 5 b) 25 c) 27 d) 23

5^2+2

173. If $x + \frac{1}{x} = 4$, then the value of $x^4 + \frac{1}{x^4} = ?$

अगर $x + \frac{1}{x} = 4$ है तो $x^4 + \frac{1}{x^4} = ?$

a) 64 b) 194 c) 81 d) 124



174. If $x - \frac{1}{x} = 4$, then $x + \frac{1}{x}$ is equal to

अगर $x - \frac{1}{x} = 4$ है तो $x + \frac{1}{x}$ पता करें|

a) $5\sqrt{2}$ b) $2\sqrt{5}$ c) $4\sqrt{2}$ d) $4\sqrt{5}$

$\sqrt{()^2 + 4}$

$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

175. If $x + \frac{1}{x} = 7$, then $x^3 + \frac{1}{x^3}$ is equal to:

अगर $x + \frac{1}{x} = 7$, तो $x^3 + \frac{1}{x^3}$ बराबर है:

a) 300 b) 322

c) 364 d) 343

343
-21
322

176. If $x - \frac{1}{x} = 10$, then $x^3 - \frac{1}{x^3}$ is equal to :

अगर $x - \frac{1}{x} = 10$, तो $x^3 - \frac{1}{x^3}$ बराबर है:

a) 970
b) 1000
c) 1030
d) 1100

1000
+30
1030

177. If $x - \frac{1}{x} = 11$, What is the value of $(x^4 + \frac{1}{x^4})$?

यदि $x - \frac{1}{x} = 11$ है, तो $(x^4 + \frac{1}{x^4})$ का मान क्या होगा?

a) 14163 b) 14159 c) 15127 d) 15131

15129

2x

$x^2 + \frac{1}{x^2} = 121 + 2$

$= 123$

2x

$()^2 - 2$

Unit digit $3^2 - 2 = 7$

178. If $x + \frac{1}{x} = -13$, What is the value of $x^4 + \frac{1}{x^4}$?

यदि $x + \frac{1}{x} = -13$ है, तो $x^4 + \frac{1}{x^4}$ का मान क्या होगा?

a) 29243 b) 28561 c) 27887 d) 27891

$\downarrow 2x$

$169 - 2 = 167$

$\downarrow 2x$

$7^2 = 9$ ← (Unit digit)

179. If $\left(x-\frac{1}{x}\right)=\frac{7}{3}$, what is the value of $(x^3-\frac{1}{x^3})$?

यदि $\left(x-\frac{1}{x}\right)=\frac{7}{3}$ है, तो $(x^3-\frac{1}{x^3})$ का मान ज्ञात कीजिए?

a) $19\frac{20}{27}$ b) $19\frac{2}{3}$ c) $19\frac{19}{27}$ d) $19\frac{7}{9}$

$$\begin{array}{r} 73 \\ -54 \\ \hline 19 \end{array}$$

$$\frac{343}{27}+\frac{3\times 7}{3}$$

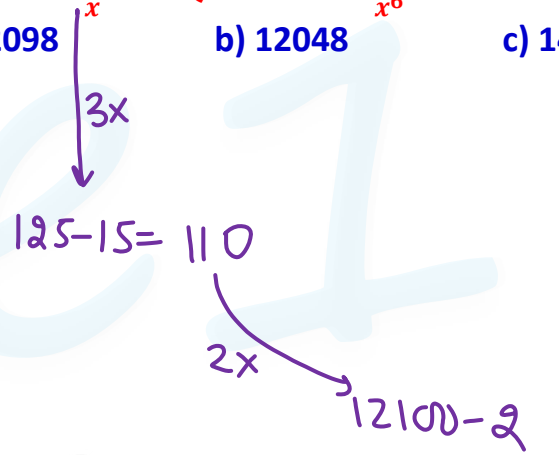
$$= 12\frac{19}{27}+7$$

$$= 19\frac{19}{27}$$

180. If $x + \frac{1}{x} = 5$, then the value of $x^6 + \frac{1}{x^6} = ?$

अगर $x + \frac{1}{x} = 5$ है तो $x^6 + \frac{1}{x^6} = ?$

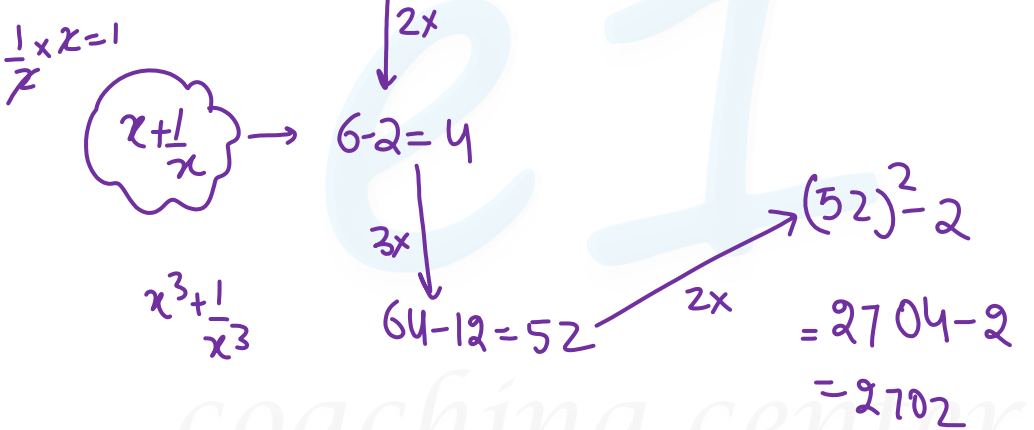
a) 12098 b) 12048 c) 14062 d) 12092



181. If $\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{6}$ then the value of will be: $x^6+\frac{1}{x^6}$

$x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}$ यदि $\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{6}$ है, तो $x^6+\frac{1}{x^6}$ का मान ज्ञात करें|

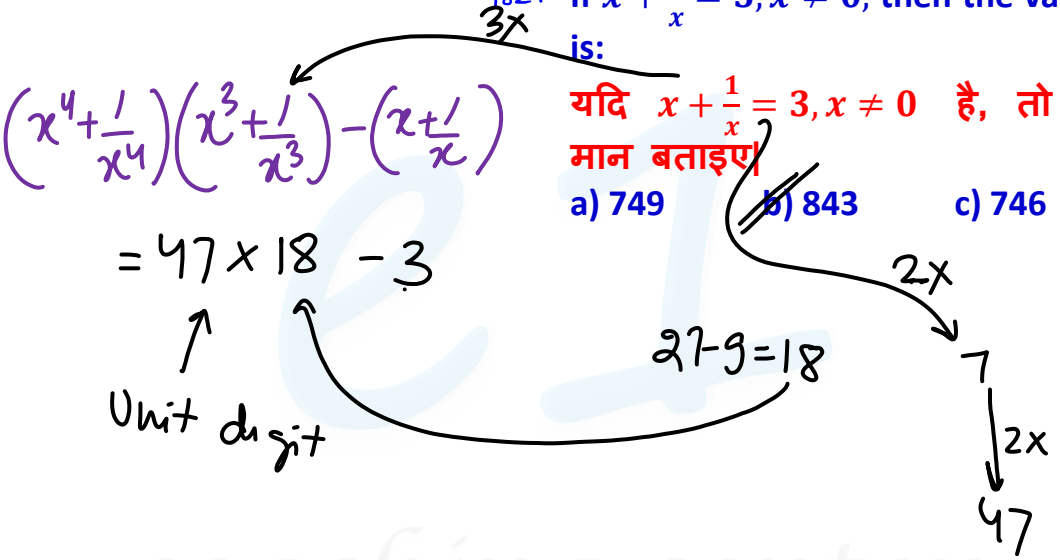
a) 2270 b) 2502 c) 2702 d) 2712



182. If $x + \frac{1}{x} = 3, x \neq 0$, then the value of $x^7 + \frac{1}{x^7}$ is:

यदि $x + \frac{1}{x} = 3, x \neq 0$ है, तो $x^7 + \frac{1}{x^7}$ का मान बताइए।

a) 749 b) 843 c) 746 d) 849



183. If $x+\frac{1}{x}=-3\sqrt{2}$, what is the value of $\left(x^5+\frac{1}{x^5}\right)$?

2x

18-2

यदि $x+\frac{1}{x}=-3\sqrt{2}$, तो $\left(x^5+\frac{1}{x^5}\right)$ का मान क्या होगा?

a) $-723\sqrt{2}$

b) $-720\sqrt{2}$

c) $-715\sqrt{2}$

d) $-717\sqrt{2}$

3x

$\left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)$

$= -45\sqrt{2} \times 16 + 3\sqrt{2}$

$=$

Unit digit $5\times6-3=7$ -ve

$-54\sqrt{2}-(-9\sqrt{2})$

$= -45\sqrt{2}$

184. If $x+\frac{1}{x}=-2\sqrt{3}$, What is the value of $x^5+\frac{1}{x^5}$?

यदि $x+\frac{1}{x}=-2\sqrt{3}$ है , तो $x^5+\frac{1}{x^5}$ का मान ज्ञात कीजिए|

a) $-178\sqrt{3}$ b) $182\sqrt{3}$

c) $-182\sqrt{3}$ d) $-180\sqrt{3}$

2x

12-2

3x

$\left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)$

$= -18\sqrt{3} \times 10 + 2\sqrt{3}$

$-24\sqrt{3}-(-6\sqrt{3})$

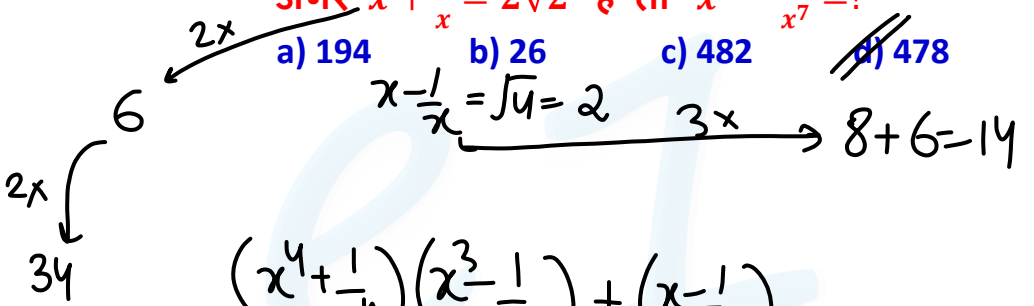
$= -18\sqrt{3}$

Unit digit

185. If $x + \frac{1}{x} = 2\sqrt{2}$, then the value of $x^7 - \frac{1}{x^7} = ?$

अगर $x + \frac{1}{x} = 2\sqrt{2}$ है तो $x^7 - \frac{1}{x^7} = ?$

a) 194 b) 26 c) 482 d) 478



186. If $x > 1$ and $x + \frac{1}{x} = 4$, then the value of $x^4 - \frac{1}{x^4} = ?$

अगर $x > 1$ और $x + \frac{1}{x} = 4$ है तो $x^4 - \frac{1}{x^4} = ?$

a) $\sqrt{3}$ b) $4\sqrt{3}$ c) $112\sqrt{3}$ d) $56\sqrt{3}$

$$\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= 14 \times 4 \times 2\sqrt{3}$$

187. If $x+\frac{1}{x}=\frac{\sqrt{3}+1}{2}$, then what is the value of $x^4+\frac{1}{x^4}$?

यदि $x+\frac{1}{x}=\frac{\sqrt{3}+1}{2}$ है, तो $x^4+\frac{1}{x^4}$ का मान क्या है?

a) $\frac{4\sqrt{3}-1}{4}$ b) $\frac{4\sqrt{3}+1}{2}$ c) $\frac{-4\sqrt{3}-1}{4}$ d) $\frac{-4\sqrt{3}-1}{2}$

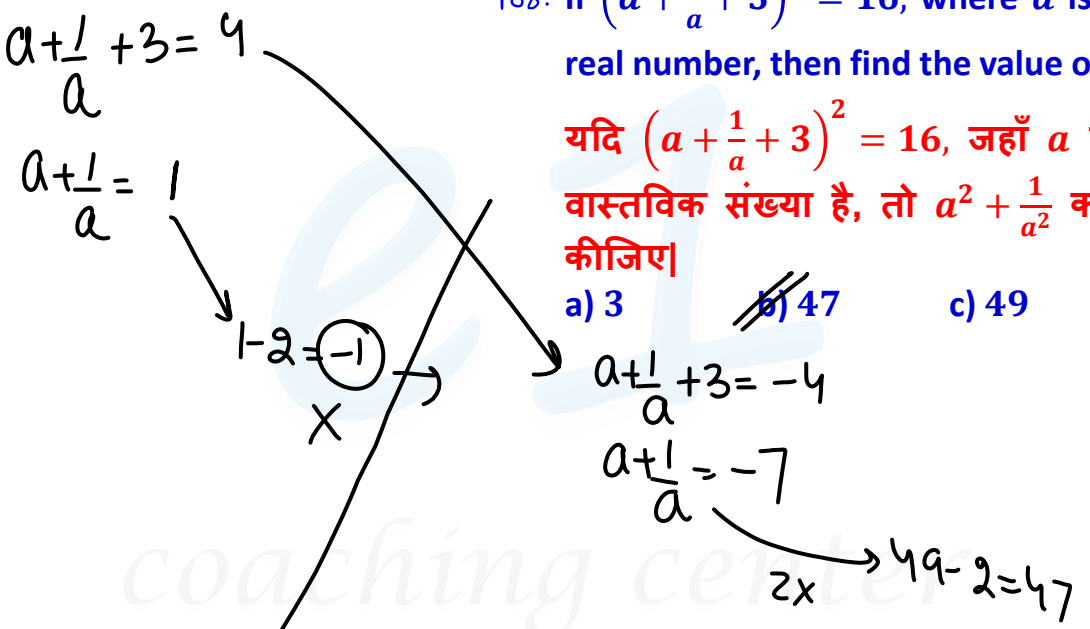
$x^2+\frac{1}{x^2}=\frac{4+2\sqrt{3}}{4}-2=\frac{4+2\sqrt{3}-8}{4}=\frac{\sqrt{3}-2}{2}$ $\quad ()^2-2 \quad -4$

$x^4+\frac{1}{x^4}=\frac{7-4\sqrt{3}}{4}-2=\frac{7-4\sqrt{3}-8}{4}=\frac{-4\sqrt{3}-1}{4}$ $\quad ()^2-2$

188. If $\left(a+\frac{1}{a}+3\right)^2 = 16$, where a is a non-zero real number, then find the value of $a^2+\frac{1}{a^2}$.

यदि $\left(a+\frac{1}{a}+3\right)^2 = 16$, जहाँ a एक शुन्येतर वास्तविक संख्या है, तो $a^2+\frac{1}{a^2}$ का मान ज्ञात कीजिए|

a) 3 b) 47 c) 49 d) 7



189. If A is the average of x and $\frac{1}{x}$ then find the average of x^3 and $\frac{1}{x^3}$ =?

यदि x और $\frac{1}{x}$ का औसत A है तो x^3 और $\frac{1}{x^3}$ का औसत ज्ञात कीजिये |

a) $4A^3 - 3A$ b) $8A^3 - 6A$ c) $3A^3 - 4A$ d) $4A^3 - 2A$

$x + \frac{1}{x} = 2A$

$\frac{x^3 + \frac{1}{x^3}}{2}$

$\frac{x^3 + \frac{1}{x^3}}{2} = \frac{8A^3 - 6A}{2} = 4A^3 - 3A$

190. If $x = 3 + \sqrt{8}$, then $x^2 + \frac{1}{x^2}$ is equal to

अगर $x = 3 + \sqrt{8}$ है तो $x^2 + \frac{1}{x^2}$ पता करें|

a) 38 b) 36 c) 34 d) 30

$\frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{3-\sqrt{8}}{9-8}$

$\frac{1}{x} = 3 - \sqrt{8}$

$x + \frac{1}{x}$

$x + \frac{1}{x} = 6 \xrightarrow{2x} 36 - 2 = 34$

191. If $x = \sqrt{3} + \sqrt{2}$, then the value of $x^3 + \frac{1}{x^3}$ is

अगर $x = \sqrt{3} + \sqrt{2}$ है तो $x^3 + \frac{1}{x^3}$ पता करें|

a) $10\sqrt{2}$ b) $30\sqrt{3}$ c) $18\sqrt{3}$ d) $24\sqrt{3}$

$\frac{1}{\sqrt{3}+\sqrt{2}}$

$\frac{1}{x} = \sqrt{3} - \sqrt{2}$

$x + \frac{1}{x} = 2\sqrt{3}$ $\xrightarrow{3x}$ $24\sqrt{3} - 6\sqrt{3}$