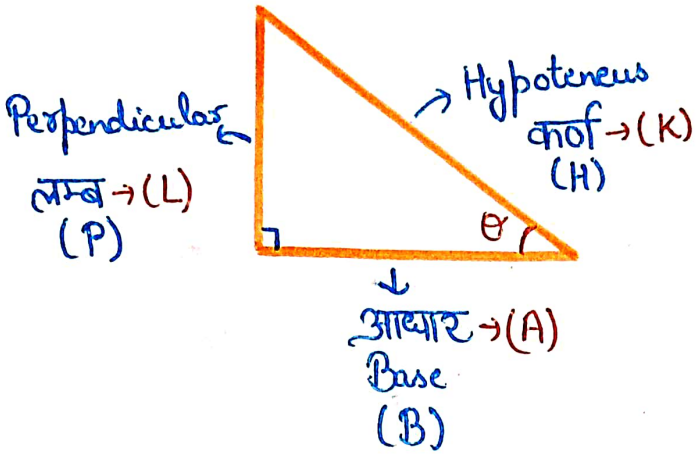
TRIGONOMETRY



- $\sin\theta = \frac{P}{H}$ $\frac{L}{K}$
- $\cos\theta = \frac{B}{H}$ $\frac{A}{K}$
- $\tan\theta = \frac{P}{B}$ $\frac{L}{A}$
- $\text{Cosec}\,\theta = \frac{1}{\sin\theta} = \frac{H}{P}$ $\frac{K}{L}$
- $\sec\theta = \frac{1}{\cos\theta} = \frac{H}{B}$ $\frac{K}{A}$
- $\cot\theta = \frac{1}{\tan\theta} = \frac{B}{P}$ $\frac{A}{L}$

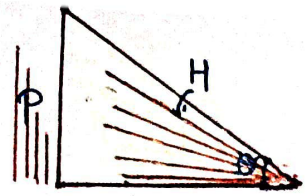
Trick

Sona	Chandi	Tole
↓	↓	↓
Pandit	Badri	Prasad
Har	Har	Bole

S	C	T
↓	↓	↓
L	A	L
K	K	A

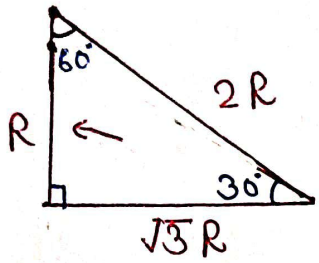
TRIGONOMETRIC TABLE OF ACUTE ANGLE

angle	0°	30°	45°	60°	90°
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
Cosec θ	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
Cot θ	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

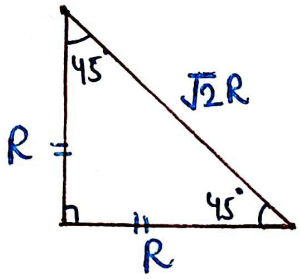


$\frac{P}{H} = 0$ $\quad \sin 0^\circ = 0$

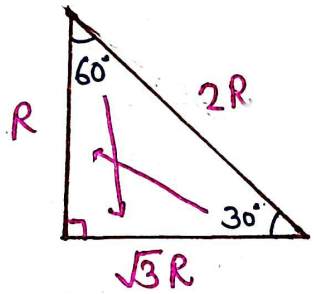
$P = 0$ $\quad \theta = 0$



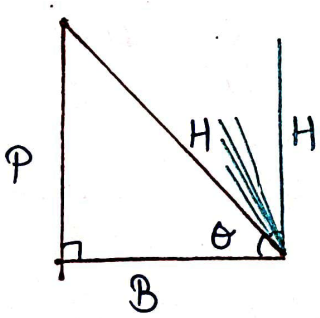
$\sin 30^\circ = \frac{\cancel{R}^{1}}{2\cancel{R}} = \frac{1}{2}$



$\sin 45^\circ = \frac{\cancel{R}^{1}}{\sqrt{2}\cancel{R}} = \frac{1}{\sqrt{2}}$



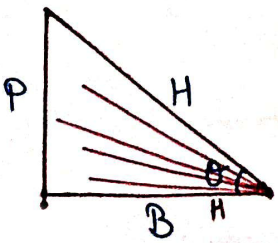
$\sin 60^\circ = \frac{\sqrt{3}\cancel{R}}{2\cancel{R}} = \frac{\sqrt{3}}{2}$



$P = H$

$\sin \theta = \frac{P}{H}$

$\sin 90^\circ = \frac{\cancel{P}}{\cancel{P}}$ 1

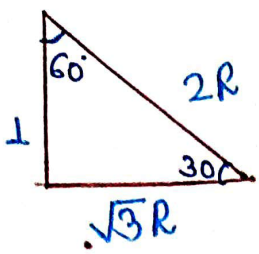


$B = H$

$\cos \theta = \frac{B}{H}$

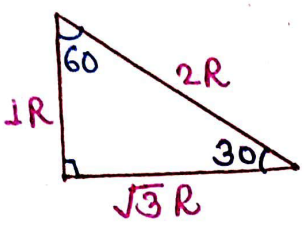
$\cos 0^\circ = 1$

*



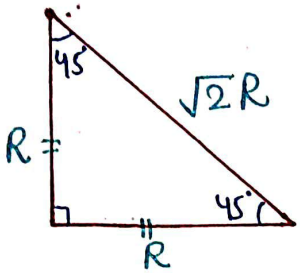
$$\cos 30^\circ = \frac{\sqrt{3}\cancel{R}}{2\cancel{R}} = \frac{\sqrt{3}}{2}$$

*



$$\cos 60^\circ = \frac{1\cancel{R}}{2\cancel{R}} = \frac{1}{2}$$

*



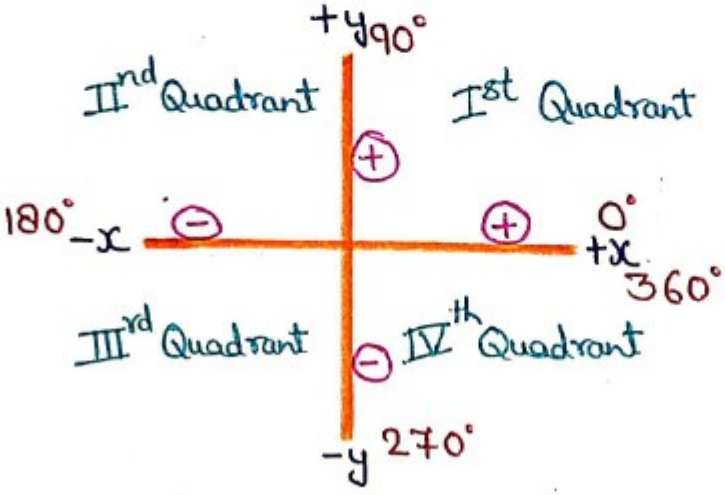
$$\cos 45^\circ = \frac{\cancel{R}\,1}{\sqrt{2}\cancel{R}} = \frac{1}{\sqrt{2}}$$

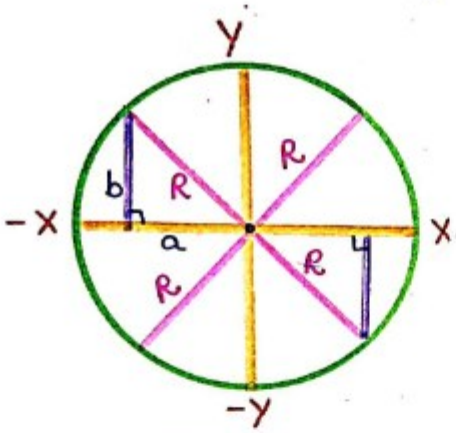
* $\left\{ \tan 90^\circ = \frac{1}{0} \right\} \Rightarrow$ Not defined.

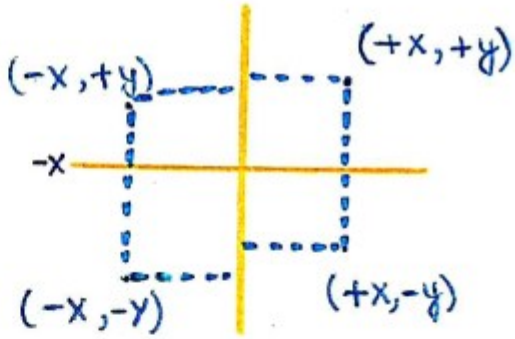
$$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}}$$

TRIGONOMETRY

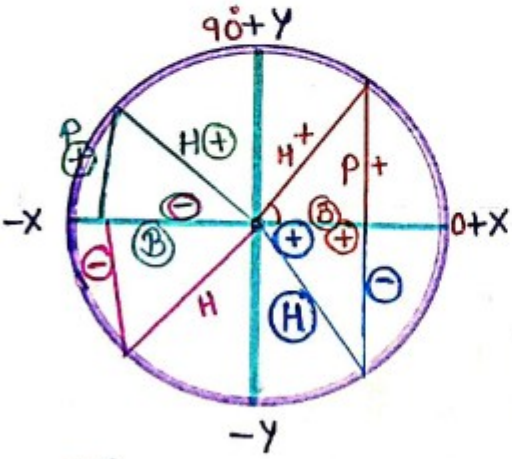
QUADRANT (चतुर्थांश)







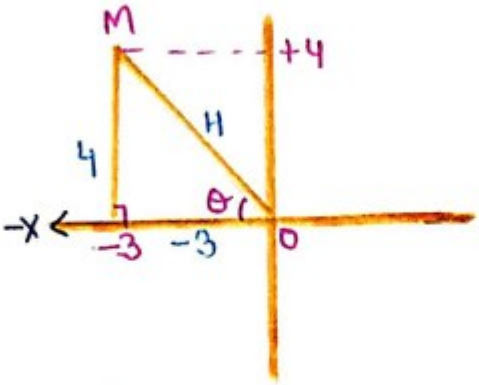
- $R = \sqrt{a^2+b^2}$
- Radius always positive



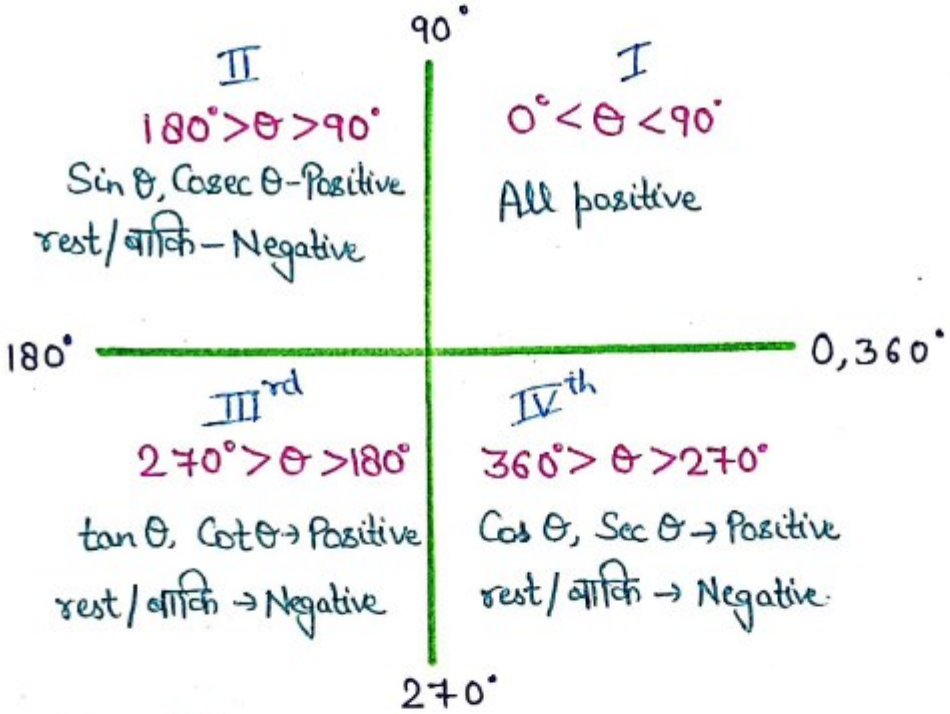
$\frac{\ominus}{\ominus} = \oplus$

	I	II	III	IV
$\frac{P}{H}$ Sin	+	+	−	−
$\frac{B}{H}$ Cos	+	−	−	+
$\frac{P}{H}$ tan	+	−	+	−
$\frac{H}{P}$ Cosec	+	+	−	−
$\frac{H}{B}$ Sec	+	−	−	+
$\frac{B}{P}$ Cot	+	−	+	−

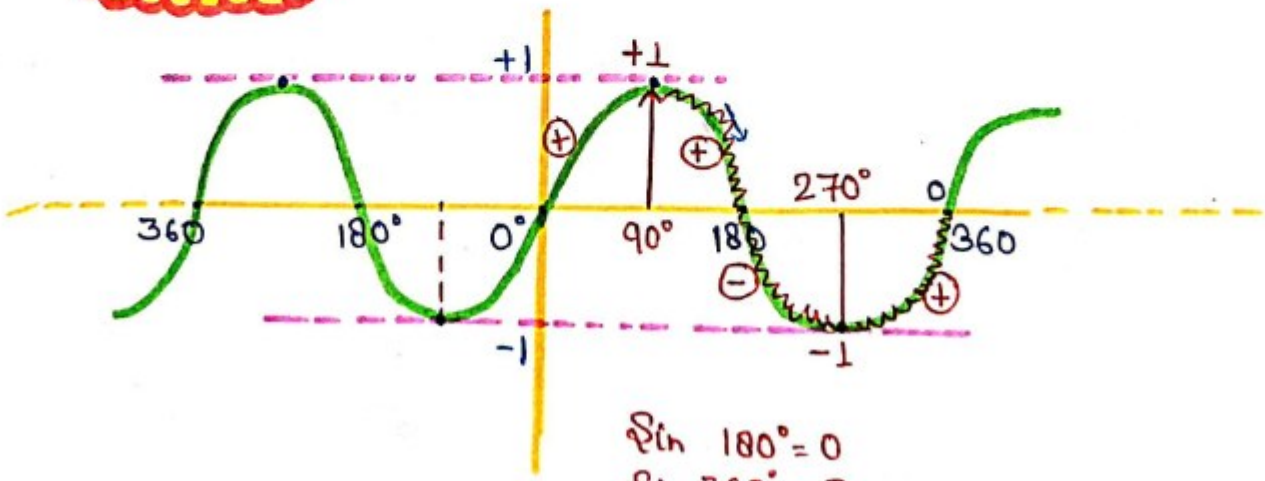
Ex:-



- $\sin\theta = \frac{4}{H}$
- $\cos\theta = \frac{-3}{H}$



Sine Graph



$\sin 180° = 0$

$\sin 360° = 0$

$\sin\theta = 0$

- if θ = even multiples of 90° (90° के सम गुणज)

Ex :- (A) (B)

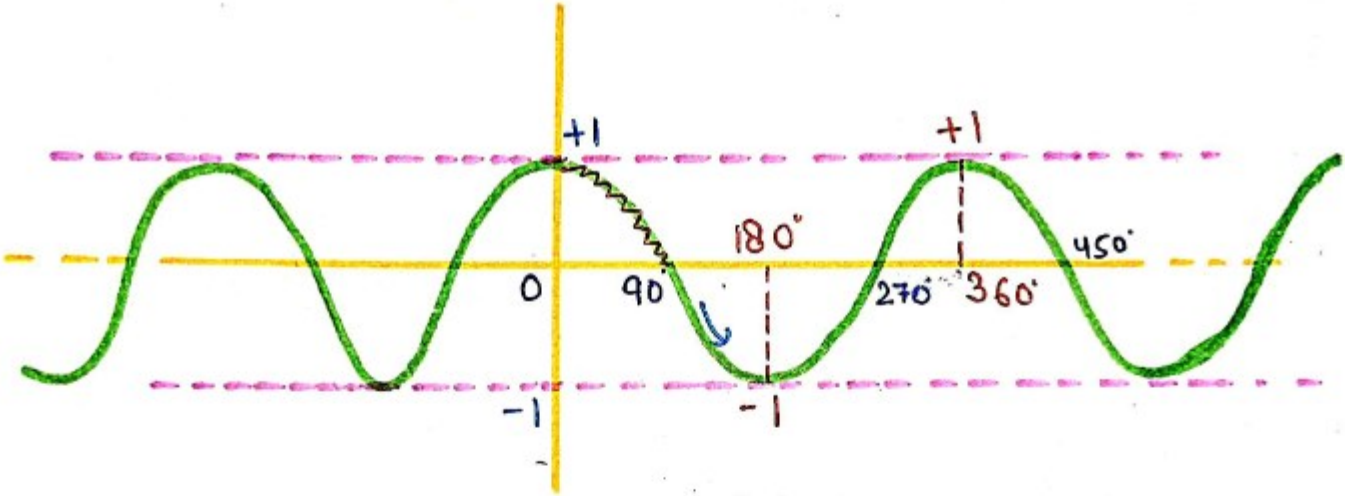
① $\sin 39 < \sin 72^\circ$

② $\sin 102^\circ > \sin 155^\circ$

③ $\sin 55^\circ > \sin 179^\circ$

	Min. Value	Max. Value
$\sin\theta$	-1	$+1$

Cosine Graph



	Min. Value	Max. Value
$\cos\theta$	-1	$+1$

$\cos 90^\circ = 0$

$\cos 270^\circ = 0$

$\cos\theta = 0$

- θ = odd multiples of 90° (90° के विषम गुणज)

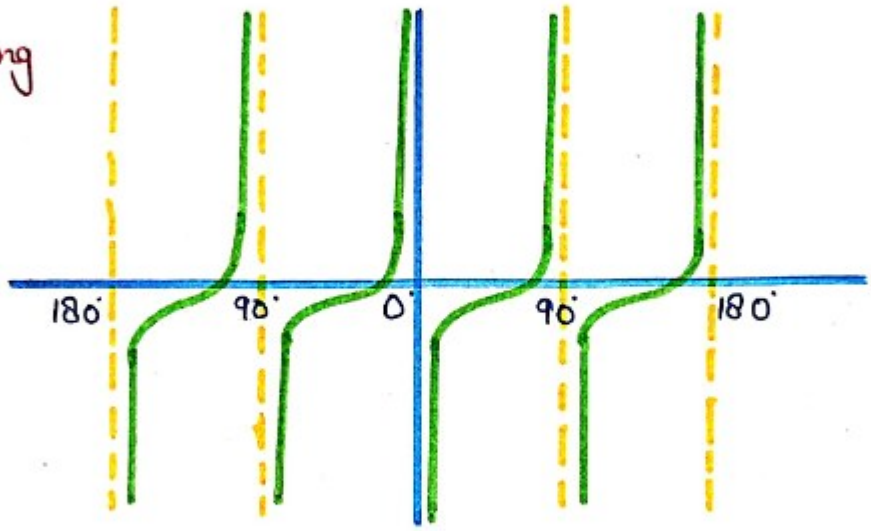
Ex:- (A) (B)

$\cos\theta$ $\cos 29^\circ > \cos 70^\circ$

$\cos 120^\circ > \cos 170^\circ$

Tan Graph

* always increasing



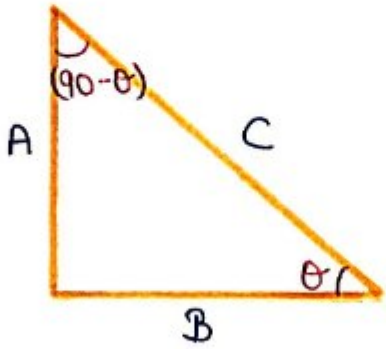
Ex:-

① $\tan 30^\circ < \tan 60^\circ$

② $\tan 127^\circ < \tan 145^\circ$

TRIGONOMETRY

CONVERSION OF TRIGONOMETRIC RATIO



$\sin\theta = \frac{A}{C}$ $\qquad$ $\cos(90-\theta) = \frac{A}{C}$

$$\boxed{\cos(90-\theta) = \sin\theta}$$

$\tan\theta = \frac{A}{B}$ $\qquad$ $\cot(90-\theta) = \frac{A}{B}$

$$\boxed{\cot(90-\theta) = \tan\theta}$$

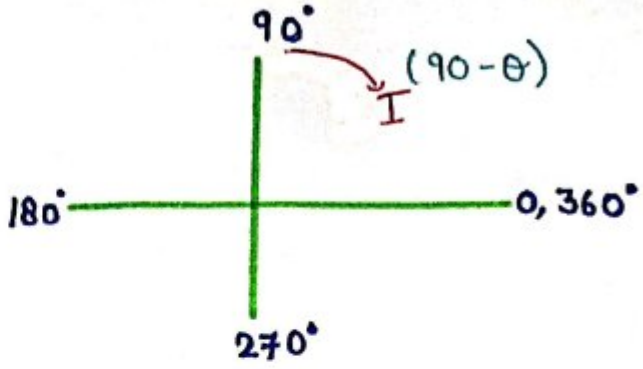
$\sec\theta = \frac{C}{B}$ $\qquad$ $\operatorname{cosec}(90-\theta) = \frac{C}{B}$

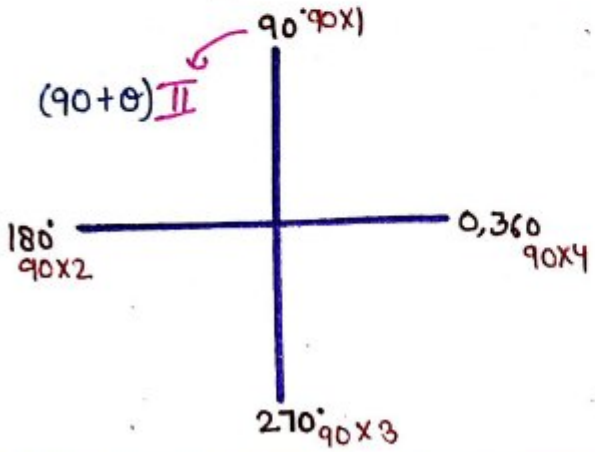
$$\boxed{\operatorname{cosec}(90-\theta) = \sec\theta}$$

Trick

S Sin, Cosec (+)	A All (+)
T tan, Cot (+)	C Cos, Sec (+)

$\sin(90-\theta) = +\cos\theta$
$\cos(90-\theta) = +\sin\theta$
$\tan(90-\theta) = +\cot\theta$
$\cot(90-\theta) = +\tan\theta$
$\sec(90-\theta) = +\operatorname{cosec}\theta$
$\operatorname{cosec}(90-\theta) = +\sec\theta$



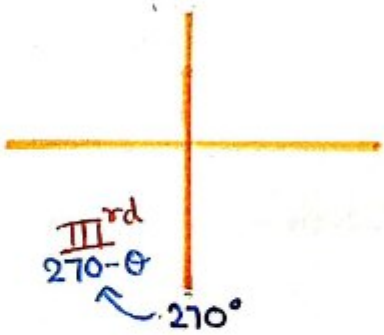


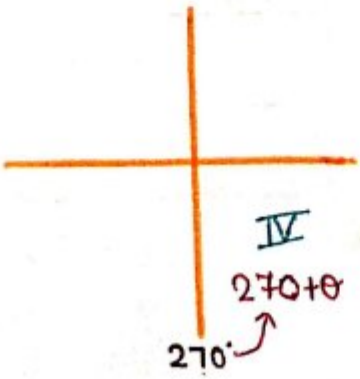
$\sin(90+\theta) = +\cos\theta$
$\cos(90+\theta) = -\sin\theta$
$\tan(90+\theta) = -\cot\theta$
$\cot(90+\theta) = -\tan\theta$
$\sec(90+\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90+\theta) = +\sec\theta$

$(90R \pm \theta)$
$\boxed{0° < \theta < 90}$
R → odd Number
विषम संख्या

Trigonometric Ratio
↓
CHANGES

$\sin(270-\theta) = -\cos\theta$
$\cos(270-\theta) = -\sin\theta$
$\tan(270-\theta) = +\cot\theta$
$\cot(270-\theta) = +\tan\theta$
$\sec(270-\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(270-\theta) = -\sec\theta$





$\sin(270+\theta) = -\cos\theta$
$\cos(270+\theta) = +\sin\theta$
$\tan(270+\theta) = -\cot\theta$
$\cot(270+\theta) = -\tan\theta$
$\sec(270+\theta) = +\operatorname{cosec}\theta$
$\operatorname{cosec}(270+\theta) = -\sec\theta$

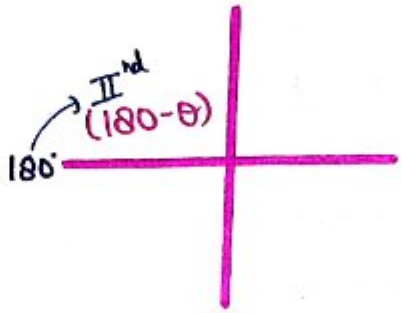
angle = $(90R \pm \theta)$

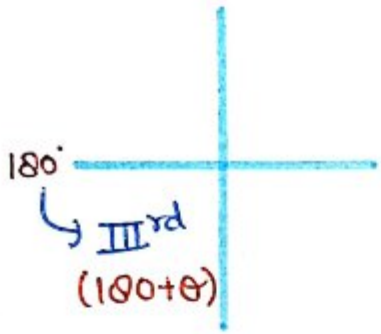
$\boxed{0^\circ < \theta < 90^\circ}$

R → even Number
सम संख्या

Trigonometric Ratio
↓
CHANGE नहीं होगे

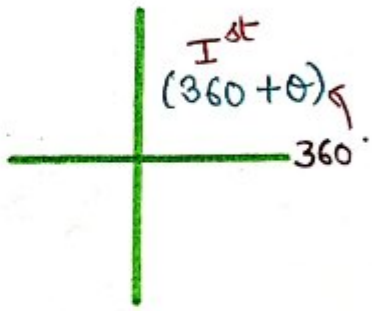
$\sin(180-\theta) = +\sin\theta$
$\cos(180-\theta) = -\cos\theta$
$\tan(180-\theta) = -\tan\theta$
$\cot(180-\theta) = -\cot\theta$
$\sec(180-\theta) = -\sec\theta$
$\operatorname{cosec}(180-\theta) = +\operatorname{cosec}\theta$

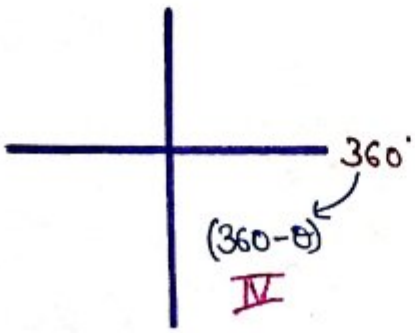




$\sin(180+\theta) = -\sin\theta$
$\cos(180+\theta) = -\cos\theta$
$\tan(180+\theta) = +\tan\theta$
$\cot(180+\theta) = +\cot\theta$
$\sec(180+\theta) = -\sec\theta$
$\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$

$\sin(360+\theta) = +\sin\theta$
$\cos(360+\theta) = +\cos\theta$
$\tan(360+\theta) = +\tan\theta$
$\cot(360+\theta) = +\cot\theta$
$\sec(360+\theta) = +\sec\theta$
$\operatorname{cosec}(360+\theta) = +\operatorname{cosec}\theta$





$\sin(360-\theta) = -\sin\theta$
$\cos(360-\theta) = +\cos\theta$
$\tan(360-\theta) = -\tan\theta$
$\cot(360-\theta) = -\cot\theta$
$\sec(360-\theta) = +\sec\theta$
$\operatorname{cosec}(360-\theta) = -\operatorname{cosec}\theta$

$\sin(-\theta) = \sin(0^\circ - \theta)$

$\sin(-\theta) = -\sin\theta$

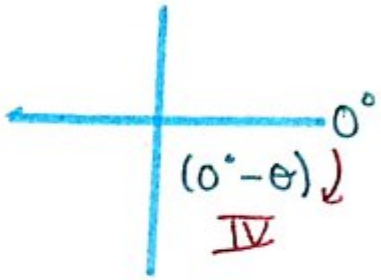
$\cos(-\theta) = \cos(0^\circ - \theta) = +\cos\theta$

$\sec(-\theta) = +\sec\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$



Q) $\cos(120^\circ)$

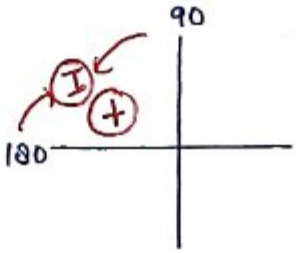
$\cos(90^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

$\cos(90 + \theta) = -\sin\theta$

Q) $\sin(150^\circ)$

$\sin(90+60)$
$= +\cos 60^\circ$
$= +\frac{1}{2}$

$\sin(180-30)$
$= +\sin 30^\circ$
$= +\frac{1}{2}$



Q) $\cos(225^\circ)$

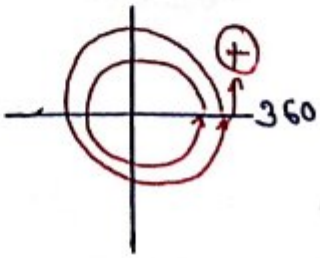
$\cos(180^\circ + 45^\circ)$
$= -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$

Q) $\tan(405^\circ)$

$\tan(360 + 45^\circ)$
$= +\tan 45^\circ$
$= +1$

Q) $\sin(750^\circ)$

$\sin(360 \times 2 + 30^\circ)$
$+\sin 30^\circ$
$= \frac{1}{2}$



Q) Find the value of $\cos 120^\circ$.

$\cos 120^\circ$ का सटीक मान ज्ञात कीजिए।

$\cos(90+30^\circ)$

$-\sin 30^\circ$

$= -\frac{1}{2} = -0.5$

Q) $\sin(1125^\circ)$

$\sin(360\times3+45^\circ)$

$+\sin 45^\circ$

$= \frac{1}{\sqrt{2}}$

Q) $\sin(-1125^\circ)$

$\sin(-\theta) = -\sin\theta$

$-\sin(1125^\circ)$

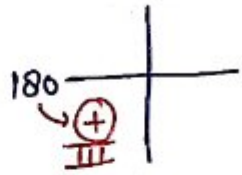
$-\frac{1}{\sqrt{2}}$

Q) Find the value of $\cos 225^\circ$

$\cos 225^\circ$ का मान ज्ञात कीजिए।

$\cos(180+45^\circ)$

$-\cos 45^\circ$



$= -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2} = \frac{-1.414}{2}$

$= -0.7071$

Q) What will be the value of $\sin(-405^\circ)$

$\sin(-405^\circ)$ का मान क्या होगा?

$-\sin(405^\circ)$

$-[\sin(360+45^\circ]$

$-[+\sin 45^\circ]$

$-\frac{1}{\sqrt{2}}$

Q) Find the value of cosec (-1410°)

cosec (-1410°) का मान ज्ञात कीजिए-

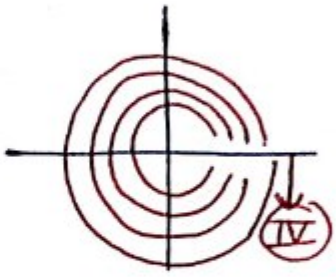
$-\operatorname{cosec}(1410)$

$-[\operatorname{Cosec}(1440-30^\circ)]$

$-[\operatorname{Cosec}(360\times4-30^\circ)]$

$-[-\operatorname{Cosec} 30^\circ]$

$+\operatorname{Cosec} 30^\circ = 2$



TRIGONOMETRY

Q) $\cos 570^\circ \sin 510^\circ + \sin(-330^\circ)(\cos 390^\circ) = ?$

$\cos(360+210)\ \sin(360+150)\ -\sin(360-30)\ \cos(360+30)$

$\cos 210^\circ \qquad \sin 150^\circ \qquad +\sin 30^\circ \qquad \cos 30$

$\cos(180+30^\circ) \qquad \sin(180-30) \qquad \frac{1}{2} \qquad = \frac{\sqrt{3}}{2}$

$-\cos 30^\circ \qquad \sin 30^\circ$

$= -\frac{\sqrt{3}}{2} \qquad \frac{1}{2}$

$-\frac{\sqrt{3}}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2}$

0

Q) $\operatorname{cosec} 2910^\circ + \sec 4260^\circ + \tan 2565^\circ + \cot 1755^\circ = ?$

$\operatorname{cosec}(360\times8+30) + \sec(360\times12-60) + \tan(360\times7+45) + \cot(360\times5-45)$

$\operatorname{cosec} 30^\circ + \sec 60^\circ + \tan 45^\circ - \cot 45^\circ$

$2 + 2 + 1 - 1$

4

Q) Find the value of $\cos 0^\circ + \cos 30^\circ - \tan 45^\circ + \operatorname{cosec} 60^\circ + \cot 90^\circ$?

$\cos 0^\circ + \cos 30^\circ - \tan 45^\circ + \operatorname{cosec} 60^\circ + \cot 90^\circ$ का मान ज्ञात कीजिए।

$\frac{\cos 90^\circ}{\sin 90^\circ} = \frac{0}{1} = 0$

$\cancel{1} + \frac{\sqrt{3}}{2} - \cancel{1} + \frac{2}{\sqrt{3}} + 0$

$\frac{3+4}{2\sqrt{3}} = \frac{7}{2\sqrt{3}}$

Q) The value of $\frac{\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$\frac{\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$ का मान है?

$$\frac{\left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1}{1}$$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$$\frac{1}{4} + 4 \times \frac{4}{3} - 1$$

$$\frac{1}{4} + \frac{16}{3} - 1$$

$$\frac{67}{12} - 1$$

$$\frac{55}{12}$$

Q) If $A = \frac{\sin 45^\circ - \sin 30^\circ}{\cos 45^\circ + \cos 60^\circ}$ and $B = \frac{\sec 45^\circ - \tan 45^\circ}{\operatorname{cosec} 45^\circ + \cot 45^\circ}$, then which one of the following is correct?

यदि $A = \frac{\sin 45^\circ - \sin 30^\circ}{\cos 45^\circ + \cos 60^\circ}$ और $B = \frac{\sec 45^\circ - \tan 45^\circ}{\operatorname{cosec} 45^\circ + \cot 45^\circ}$ तो निम्नलिखित में से कौन सा सही है?

$$A = \frac{\sin 45^\circ - \sin 30^\circ}{\cos 45^\circ + \cos 60^\circ}$$

$$\frac{\frac{1}{\sqrt{2}} - \frac{1}{2}}{\frac{1}{\sqrt{2}} + \frac{1}{2}}$$

$$\frac{2 - \sqrt{2}}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$$

$$\frac{(2 - \sqrt{2})^2}{4 - 2} = \frac{4 + 2 - 4\sqrt{2}}{2}$$

$$\frac{6 - 4\sqrt{2}}{2}$$

$$3 - 2\sqrt{2} \rightarrow (A)$$

$$B = \frac{\sec 45^\circ - \tan 45^\circ}{\operatorname{cosec} 45^\circ + \cot 45^\circ}$$

$$\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$\frac{(\sqrt{2} - 1)^2}{2 - 1} = \frac{2 + 1 - 2\sqrt{2}}{1}$$

$$3 - 2\sqrt{2} \rightarrow (B)$$

$$A = B$$

Q) Value of the expression

$$\frac{1+2\sin 60^\circ \cos 60^\circ}{\sin 60^\circ + \cos 60^\circ} + \frac{1-2\sin 60^\circ \cos 60^\circ}{\sin 60^\circ - \cos 60^\circ} \text{ is}$$

$$\frac{1+2\sin 60^\circ \cos 60^\circ}{\sin 60^\circ + \cos 60^\circ} + \frac{1-2\sin 60^\circ \cos 60^\circ}{\sin 60^\circ - \cos 60^\circ}$$ व्यंजक का मान है।

$$\frac{\sin^2 60 + \cos^2 60 + 2\sin 60 \cos 60}{\sin 60^\circ + \cos 60^\circ} + \frac{\sin^2 60 + \cos^2 60 - 2\sin 60 \cos 60}{\sin 60 - \cos 60}$$

$$\frac{(\sin 60 + \cos 60)^{\cancel{2}}}{\cancel{(\sin 60 + \cos 60)}} + \frac{(\sin 60 - \cos 60)^{\cancel{2}}}{\cancel{\sin 60 - \cos 60}}$$

$$\sin 60^\circ + \cancel{\cos 60} + \sin 60 - \cancel{\cos 60}$$

$$2\sin 60^\circ$$

$$\cancel{2} \times \frac{\sqrt{3}}{\cancel{2}}$$

$$\sqrt{3}$$

Q) The value of $\dfrac{4\tan^2 30^\circ + \frac{1}{4}\sin^2 90^\circ + \frac{1}{8}\cot^2 60^\circ + \sin^2 30^\circ \cos^2 45^\circ}{\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ}$ is

$$\frac{4\tan^2 30^\circ + \frac{1}{4}\sin^2 90^\circ + \frac{1}{8}\cot^2 60^\circ + \sin^2 30^\circ \cos^2 45^\circ}{\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ}$$ का मान है :

$$\frac{4\times\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{4}\times 1 + \frac{1}{8}\times\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{4}\times\frac{1}{2}}{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{1}{2}}$$

$$\frac{4\times\frac{1}{3} + \frac{1}{4} + \frac{1}{8}\times\frac{1}{3} + \frac{1}{8}}{\frac{3}{4} - \frac{1}{4}}$$

$$\frac{\frac{4}{3} + \frac{1}{4} + \frac{1}{24} + \frac{1}{8}}{\left(\frac{1}{2}\right)}$$

$$\frac{(32+6+1+3)\times 2}{24_{12}}$$

$$\frac{42}{12}\ \frac{7}{2} = 3\frac{1}{2}$$

Q) The value of $\frac{\operatorname{cosec}^2 30^\circ \sin^2 45^\circ + \sec^2 60^\circ}{\tan 60^\circ \operatorname{cosec}^2 45^\circ - \sec^2 60^\circ \tan 45^\circ}$ is

$\frac{\operatorname{cosec}^2 30^\circ \sin^2 45^\circ + \sec^2 60^\circ}{\tan 60^\circ \operatorname{cosec}^2 45^\circ - \sec^2 60^\circ \tan 45^\circ}$ का मान ज्ञात कीजिए।

$$\frac{(2)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + (2)^2}{\sqrt{3} \times (\sqrt{2})^2 - (2)^2 \times 1}$$

$$\frac{{}^{2}\cancel{4} \times \frac{1}{\cancel{2}} + 4}{2\sqrt{3} - 4}$$

$$\frac{6}{2\sqrt{3}-4} \Rightarrow \frac{\cancel{6}\ 3}{\cancel{2}(\sqrt{3}-2)}$$

$$\frac{3}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$$

$$\frac{3(\sqrt{3}+2)}{3-4}$$

$$-3(\sqrt{3}+2)$$

Q) If $\tan(A+B) = \sqrt{3}$ & $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0^\circ < (A+B) < 90^\circ$, $A > B$, then the values of A and B are.... respectively.

यदि $\tan(A+B) = \sqrt{3}$ & $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0^\circ < (A+B) < 90^\circ$, $A > B$, तो A और B के मान क्रमशः _____ है।

$A+B = 60^\circ$, $A-B = 30^\circ$

$A = \frac{90}{2}$ $B = \frac{30}{2}$

45° & 15°

Q) If $\tan\theta_1 = 1$, $\sin\theta_2 = \frac{1}{\sqrt{2}}$ then the value of $\sin(\theta_1 + \theta_2)$ equal to:

यदि $\tan\theta_1 = 1$, $\sin\theta_2 = \frac{1}{\sqrt{2}}$ तो $\sin(\theta_1 + \theta_2)$ का मान किसके बराबर है?

$\theta_1 = 45^\circ$, $\theta_2 = 45^\circ$

$\sin(45^\circ + 45^\circ)$

$\sin 90^\circ = 1$

RADIAN रेडियन

a° = a Degree

a^c = a radian

$$\boxed{1^c = 57.296^\circ}$$

$\pi = 3.14^c = 3.14 \times 57.296^\circ$

$= 179.9 \approx 180^\circ$

$$\boxed{\pi = 180^\circ}$$

$$\boxed{360^\circ = 2\pi}$$

$180^\circ = \pi$

$1^\circ = \frac{\pi^c}{180}$

$$\boxed{n^\circ = \left(\frac{\pi}{180} \times n\right)^c}$$

$90^\circ \rightarrow \frac{\pi}{\cancel{180}_{2}} \times \cancel{90} = \frac{\pi}{2}$

$45^\circ \rightarrow \frac{\pi}{\cancel{180}_{4}} \times \cancel{45} = \frac{\pi}{4}$

$30^\circ = \frac{\pi}{\cancel{180}_{6}} \times \cancel{30} = \frac{\pi}{6}$

$60^\circ = \frac{\pi}{\cancel{180}_{3}} \times \cancel{60} = \frac{\pi}{3}$

Q) $\frac{4}{3}\cot^2\frac{\pi}{6} + 3\cos^2 150^\circ - 4\,\text{cosec}^2\,45^\circ + 8\sin\frac{\pi}{2}$?

$\frac{4}{3}\times(\sqrt{3})^2 + 3\cos^2(180-30) - 4\times(\sqrt{2})^2 + 8\times 1$

$\frac{4}{\cancel{3}}\times\cancel{3} + 3\times\cos^2 30^\circ - 4\times 2 + 8$

$4 + 3\times\frac{3}{4} - \cancel{8} + \cancel{8}$

$4 + \frac{9}{4} = \frac{25}{4}$

Q) The value of $32\cot^2\frac{\pi}{4} - 8\sec^2\frac{\pi}{3} + 8\cos^3\frac{\pi}{6}$ is equal to:

$32\cot^2\frac{\pi}{4} - 8\sec^2\frac{\pi}{3} + 8\cos^3\frac{\pi}{6}$ का मान बराबर है।

$32\times 1 - 8\times(2)^2 + 8\times\left(\frac{\sqrt{3}}{2}\right)^3$

$\cancel{32} - \cancel{32} + \cancel{8}\times\frac{3\sqrt{3}}{\cancel{8}}$

$3\sqrt{3}$

Q) If $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$ then which of the following is / are correct?

यदि $A = \frac{\pi}{6}$ और $B = \frac{\pi}{3}$, तो निम्नलिखित में से कौनसा / से सही है / हैं?

I. $\sin A + \sin B = \cos A + \cos B \Rightarrow \sin 30 + \sin 60 = \cos 30 + \cos 60^\circ$

II. $\tan A + \tan B = \cot A + \cot B \Rightarrow \tan 30^\circ + \tan 60 = \cot 30 + \cot 60^\circ$

Both 1 and 2

Q) If $\cos\theta \geq \frac{1}{2}$ in the first quadrant then which one of the following is correct?

यदि पहले चतुर्थांश में $\cos\theta \geq \frac{1}{2}$ हो, तो निम्नलिखित में से कौन सा सही है?

$\boxed{90 < \theta < 0^\circ}$

↳ Ist Quadrant

$\cos\theta \geq \frac{1}{2}$

$\theta \geq 60^\circ$

$\theta \geq \frac{\pi}{3}$

Q) Consider the following :

(I) $\sin 1^\circ > \sin 1^c$ (II) $\cos 1^\circ < \cos 1^c$ (III) $\tan 1^\circ > \tan 1^c$

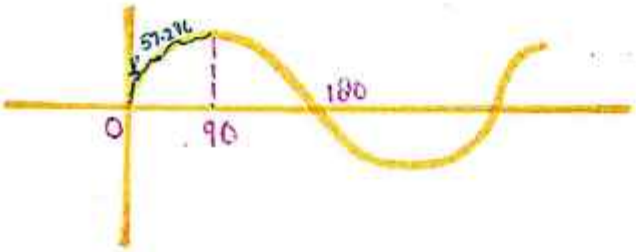
Which of the above are not correct?

निम्नलिखित पर विचार कीजिये:

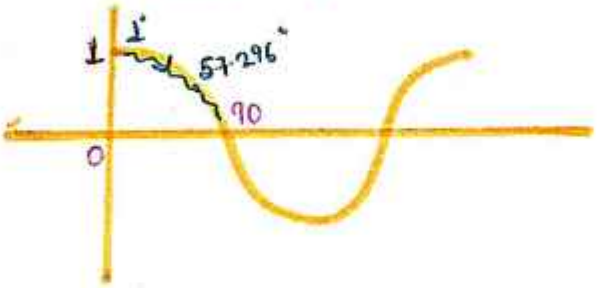
(I) $\sin 1^\circ > \sin 1^c$ (II) $\cos 1^\circ < \cos 1^c$ (iii) $\tan 1^\circ > \tan 1^c$

उपरोक्त में से कौन सा सही नहीं है।

(i) $\sin 1^\circ > \sin 1^c$ $\hookrightarrow 57.296^\circ$ $1^c = 57.296^\circ$



(II) $\cos 1^\circ < \cos 1^c$ $\hookrightarrow 57.296^\circ$



(III) $\tan 1^\circ > \tan 1^c$ $\hookrightarrow 57.296^\circ$

always increase

I, II and III

Q) Consider the following statements:
निम्नलिखित कथनों पर विचार करें:

1) The first quadrant contains only one value of x which satisfies $\sin x + \cos x = 2$.

प्रथम चतुर्थांश में x का केवल वही मान है जो $\sin x + \cos x = 2$ को संतुष्ट करता है।

$$\boxed{0^\circ < \theta < 90^\circ}$$

$$\sin x + \cos x = 2$$

2) The first quadrant contains only one value of x which satisfies $\sin x - \cos x = 0$

प्रथम चतुर्थांश में x का केवल एक मान है जो $\sin x - \cos x = 0$ को संतुष्ट करता है।

$$\sin x - \cos x = 0$$

$$\sin x = \cos x$$

$$x = 45^\circ \Rightarrow \sin x = \cos x$$

उपरोक्त में से कौन-सा/से सही है/हैं?

Which of the above is/are correct?

2 only

1. If $x \sin 45^\circ = y \operatorname{cosec} 30^\circ$, then $\frac{x^2}{y^2}$ is equal to

यदि $x \sin 45^\circ = y \operatorname{cosec} 30^\circ$, तो $\frac{x^2}{y^2}$ बराबर है

(a) 4^3
(b) 6^3
(c) 2^3
(d) 8^3

2. If $x\cos^2 30^\circ \cdot \sin 60^\circ = \frac{\tan^2 45^\circ \cdot \sec 60^\circ}{\operatorname{cosec} 60^\circ}$ then the value of x.

यदि $x\cos^2 30^\circ \cdot \sin 60^\circ = \frac{\tan^2 45^\circ \cdot \sec 60^\circ}{\operatorname{cosec} 60^\circ}$ तो x का मान।

(a) $\frac{1}{\sqrt{3}}$
(b) $2\frac{2}{3}$
(c) $\frac{1}{\sqrt{2}}$
(d) $\frac{1}{2}$

3. Find the value of $\operatorname{cosec}(-1410^\circ)$

$\operatorname{cosec}(-1410^\circ)$ का मान ज्ञात कीजि

(a) 3
(b) 4
(c) 7
(d) 2

4. $2\cos^2 135^\circ + \sin 150^\circ + \frac{1}{2}\cos 180^\circ + \tan 135^\circ$

(a) 0
(b) 2
(c) 1
(d) 3

5. Find the value of $\sin^2 200^\circ + \sin^2 250^\circ + \sin^2 280^\circ + \sin^2 370^\circ$

$\sin^2 200^\circ + \sin^2 250^\circ + \sin^2 280^\circ + \sin^2 370^\circ$ का मान ज्ञात कीजिए

(a) 2
(b) 3
(c) 4
(d) 6

6. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$

(a) $\frac{-1}{2}$ (b) 2
(c) -2 (d) $\frac{1}{2}$

7. $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

(a) 8

(b) 10

(c) 6

(d) 0

8. $2\sin^2\frac{\pi}{6}-\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

(a) $\frac{-1}{2}$

(b) $\frac{7}{2}$

(c) $\frac{5}{2}$

(d) $\frac{3}{2}$

9. $\frac{5\sin^2 30^\circ+\cos^2 45^\circ-4\tan^2 30^\circ}{2\sin 30^\circ\cdot\cos 30^\circ+\tan 45^\circ}=?$

(a) $\frac{5}{6(\sqrt{3}+2)}$

(b) $5-\sqrt{3}$

(c) $\sqrt{3}-2$

(d) $\frac{\sqrt{3}-2}{6}$

10. If $\cot\theta=\frac{1}{2}$ & θ is acute angle, the value of $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta+2\cos^3\theta+3\cos\theta}$ is यदि $\cot\theta=\frac{1}{2}$ और θ न्यूनकोण है, तो $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta+2\cos^3\theta+3\cos\theta}$ का मान है

(a) $\frac{23}{5}$

(b) $\frac{21}{5}$

(c) $\frac{21}{5\sqrt{5}}$

(d) $\frac{23\sqrt{5}}{5}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	B	D	A	A	A	B	A	A	B

Worksheet solution

Sol 1

$x \sin 45^\circ = y \operatorname{cosec} 30^\circ$

$x \times \frac{1}{\sqrt{2}} = y \times 2$

$x = 2\sqrt{2}y$

$\frac{x}{y} = \frac{2\sqrt{2}}{1}$

$\frac{x^2}{y^2} = \frac{8}{1} = 2^3$

Sol-2

$x \cos^2 30^\circ \cdot \sin 60^\circ = \frac{\tan^2 45^\circ \sec 60^\circ}{\operatorname{cosec} 60^\circ}$

$x \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{\sqrt{3}}{2} = \frac{(1)^2 \times 2}{\frac{2}{\sqrt{3}}}$

$x \times \frac{3}{4} \times \frac{\sqrt{3}}{2} = 1 \times \sqrt{3}$

$x = \frac{8}{3} = 2\frac{2}{3}$

Sol-3

$\operatorname{cosec}(-1410^\circ)$

$-\operatorname{cosec}[4 \times 360^\circ - 30^\circ]$

$-\operatorname{cosec}[-30^\circ]$

$\operatorname{cosec} 30^\circ = 2$

Sol 4

$2\cos^2 135^\circ + \sin 150^\circ + \frac{1}{2}\cos 180^\circ + \tan 135^\circ$

$2[\cos^2(90^\circ+45^\circ) + \sin(90^\circ+60^\circ) + \frac{1}{2}\cos 180^\circ + \tan(90^\circ+45^\circ)]$

$2\left(-\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2} + \frac{1}{2}(-1) + (-1)$

$= 0$

Sol 5

$\sin^2 200^\circ + \sin^2 250^\circ + \sin^2 280^\circ + \sin 370^\circ$

$\sin^2(180^\circ+20^\circ) + \sin^2(270-20^\circ) + \sin^2(270+10^\circ) + \sin^2(360+10)$

$\sin^2 20^\circ + \cos^2 20^\circ + \cos^2 10^\circ + \sin^2 10^\circ$

$1 + 1 = 2$

$(\because \sin^2 x + \cos^2 x = 1)$

Sol-6

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$$

$$\sin^2 30^\circ + \cos^2 60^\circ - \tan^2 45^\circ$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$\frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

Sol-7

$$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$$

$$2\sin^2 135^\circ + 2\cos^2 45^\circ + 2\sec^2 60^\circ$$

$(90^\circ + 45^\circ)$

$$2\sin^2 45^\circ + 2\cos^2 45^\circ + 2\sec^2 60^\circ$$

$$2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times(2)^2$$

$$= 10$$

Sol 8

$$2\sin^2\frac{\pi}{6} - \operatorname{cosec}^2\frac{7\pi}{6}\cdot\cos^2\frac{\pi}{3}$$

$$2\sin^2 30 - \operatorname{cosec}^2 210^\circ \times \cos^2 60$$

$$2\sin^2 30 - (-\operatorname{cosec}^2 30^\circ)\times\cos^2 60^\circ$$

$$2\left(\frac{1}{2}\right)^2 - (-2)^2\times\left(\frac{1}{2}\right)^2$$

$$2\times\frac{1}{4} - 4\times\frac{1}{4} = -\frac{1}{2}$$

Sol-9

$$\frac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ\cdot\cos 30^\circ + 4\tan 45^\circ}$$

$$\frac{5\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 4\left(\frac{1}{\sqrt{3}}\right)^2}{2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} + 1}$$

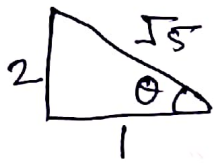
$$= \frac{\frac{5}{4} + \frac{1}{2} - \frac{4}{3}}{\frac{\sqrt{3}+2}{2}}$$

$$= \frac{\frac{15+6-16}{12}}{\frac{\sqrt{3}+2}{2}}$$

$$= \frac{5}{6(\sqrt{3}+2)}$$

Sol 10

$$\cot\theta = \frac{1}{2}$$



$$= \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$$

$$= \frac{8\times\frac{2}{\sqrt{5}} + 5\times\frac{1}{\sqrt{5}}}{\frac{8}{5\sqrt{5}} + \frac{2\times1}{5\sqrt{5}} + \frac{3}{\sqrt{5}}} = \frac{\frac{21}{\sqrt{5}}}{\frac{8+2+15}{5\sqrt{5}}}$$

$$= 21/5$$

TRIGONOMETRY

Q) If $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$, where $0 < \theta < \frac{\pi}{2}$, then which of the following statements is / are correct?

यदि $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$, जहां $0 < \theta < \frac{\pi}{2}$, तो निम्नलिखित में से कौन सा / से कथन सही है / हैं ?

1. There are two values of θ satisfying the above equations.
उपरोक्त समीकरणों को संतुष्ट करने वाले θ के दो मान हैं।

2. $\theta = 60°$ is satisfied the above equation. Select the correct answer using the code given below.
$\theta = 60°$ उपरोक्त समीकरण से संतुष्ट है।

$$\cos^2\theta - 3\cos\theta + 2 = \sin^2\theta$$

$$\cos^2\theta - 3\cos\theta + 2 = 1 - \cos^2\theta$$

$$\boxed{\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \sin^2\theta &= 1 - \cos^2\theta\end{aligned}}$$

$$2\cos^2\theta - 3\cos\theta + 1 = 0$$

$$\underbrace{2\cos^2\theta - 2\cos\theta}\ \underbrace{-\cos\theta + 1}_{2} = 0$$

$$2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$$

$$(\cos\theta - 1)(2\cos\theta - 1) = 0$$

$$\boxed{\cos\theta = 1} \qquad \boxed{\cos\theta = 1/2}$$

$$\theta = 0° \qquad \theta = 60°$$

Select the correct answer using the code given below.
नीचे दिए गए कूट का प्रयोग कर सही उत्तर चुनिए।

a 1 only / केवल 1

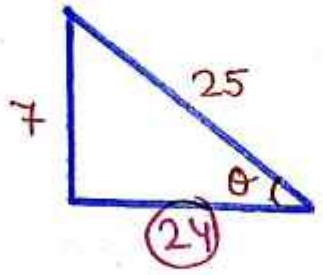
b) 2 only / केवल 2

c Both 1 and 2 / दोनो 1 और 2

d Neither 1 nor 2 / न तो 1 और न ही 2

Q) If $\sin\theta = \frac{7}{25}$, then the value of $\cos\theta$ and $\tan\theta$ respectively are .... (Where θ - in - first - Quadrant)

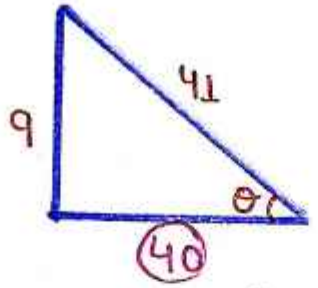
यदि $\sin\theta = \frac{7}{25}$ है, तो $\cos\theta$ और $\tan\theta$ के मान क्रमश: .... हैं। (जहां θ प्रथम चतुर्थांश में है)



$\cos\theta = \frac{24}{25}$, $\tan\theta = \frac{7}{24}$

Q) If $\sin\theta = \left(\frac{9}{41}\right)$, $0° < \theta < 90°$ then what is the value of $\cot\theta$?

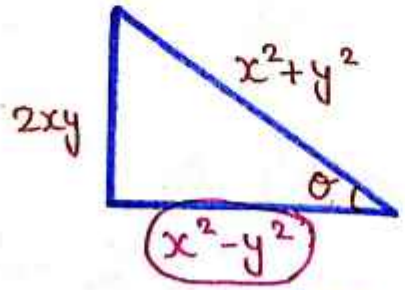
यदि $\sin\theta = \left(\frac{9}{41}\right)$, $0° < \theta < 90°$ तो $\cot\theta$ का मान क्या होगा?



$\cot\theta = \frac{40}{9}$

Q) If $\sin\theta = \frac{2xy}{x^2+y^2}$, then find the value of $\tan\theta$

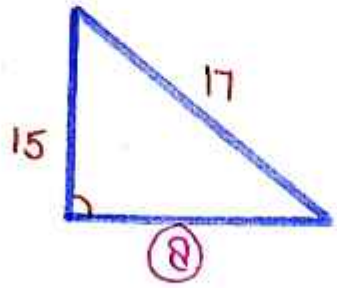
यदि $\sin\theta = \frac{2xy}{x^2+y^2}$ तो $\tan\theta$ का मान है।



$\tan\theta = \frac{2xy}{x^2-y^2}$

Q) If given that $\sin\theta = \frac{15}{17}$, find the value of $\frac{15\cot\theta + 17\sin\theta}{8\tan\theta + 16\sec\theta}$, where $0° < \theta < 90°$ is.

यदि $\sin\theta = \frac{15}{17}$ दिया गया है, $\frac{15\cot\theta + 17\sin\theta}{8\tan\theta + 16\sec\theta}$ का मान ज्ञात कीजिए। जहाँ $0° < \theta < 90°$ है।



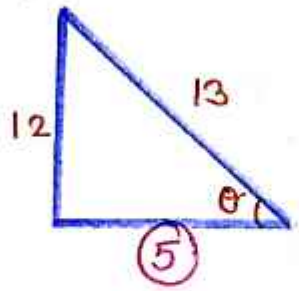
$$\frac{\cancel{15} \times \frac{8}{\cancel{15}} + \cancel{17} \times \frac{15}{\cancel{17}}}{\cancel{8} \times \frac{15}{\cancel{8}} + \overset{2}{\cancel{16}} \times \frac{17}{\cancel{8}}}$$

$$\frac{8+15}{15+34} = \frac{23}{49}$$

Q) If $\sin\theta = \frac{12}{13}$, $0 < \theta < 90°$, then $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \cdot \cos\theta} \times \frac{1}{\tan^2\theta} = \ldots\ldots$

यदि $\sin\theta = \frac{12}{13}$, $0 < \theta < 90°$ है तो $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \cdot \cos\theta} \times \frac{1}{\tan^2\theta} = \ldots$

$\cot^2\theta$



$$\frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{25}{144}$$

$$\frac{\frac{119}{\cancel{169}}}{\frac{120}{\cancel{169}}} \times \frac{25}{144}$$

$$\frac{119}{120} \times \frac{25}{144}$$

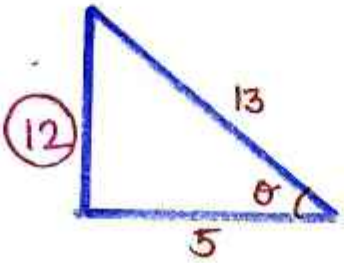
$$\frac{595}{3456}$$

Q) If $\sec\theta = \frac{13}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$ then find $\cot\theta$?

यदि $\sec\theta = \frac{13}{5}$ और $\frac{3\pi}{2} < \theta < 2\pi$ तो $\cot\theta$ का मान ज्ञात कीजिये?

$270 < \theta < 360$

IVth Quadrant.



$$\cot\theta = \frac{5}{12}$$

IVth Quadrant $\Rightarrow \frac{-5}{12}$

Q) If $\frac{\sin^2\theta}{\tan^2\theta - \sin^2\theta} = 5$, θ is an acute angle, then the value of $\frac{24\sin^2\theta - 15\sec^2\theta}{6\,\text{cosec}^2\theta - 7\cot^2\theta}$ is:

यदि $\frac{\sin^2\theta}{\tan^2\theta - \sin^2\theta} = 5$ है, θ न्यून कोण है, तो $\frac{24\sin^2\theta - 15\sec^2\theta}{6\,\text{cosec}^2\theta - 7\cot^2\theta}$ का मान ज्ञात करें।

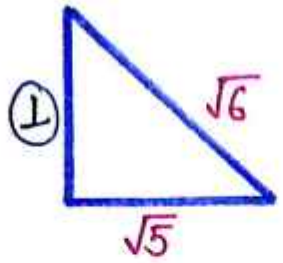
$$\sin^2\theta = 5\tan^2\theta - 5\sin^2\theta$$

$$6\sin^2\theta = 5\tan^2\theta$$

$$6\cancel{\sin^2\theta} = 5 \times \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$$

$$\boxed{\cos^2\theta = \frac{5}{6}}$$

$$\boxed{\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}}$$



$$\frac{{}^{4}\cancel{24}\times\frac{1}{\cancel{6}}-{}^{3}\cancel{15}\times\frac{6}{\cancel{5}}}{6\times\frac{6}{1}-7\times\frac{5}{1}}$$

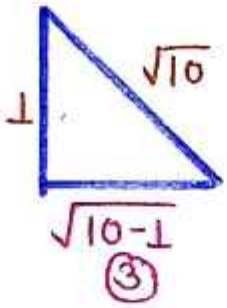
$$\frac{4-18}{36-35}=\frac{-14}{1}$$

Q) If $\sin\theta=\frac{1}{\sqrt{10}}$ where $0<\theta<\frac{\pi}{2}$ then $\frac{(6\tan\theta)(1+\tan\theta)^{-1}}{(1-\tan\theta)}$ is equal to?

यदि $\sin\theta=\frac{1}{\sqrt{10}}$ जहाँ $0<\theta<\frac{\pi}{2}$ तो $\frac{(6\tan\theta)(1+\tan\theta)^{-1}}{(1-\tan\theta)}$ किसके बराबर है?

$$\frac{6\tan\theta}{(1-\tan\theta)(1+\tan\theta)}$$

$$\frac{6\tan\theta}{1-\tan^2\theta}$$



$$\frac{{}^{2}\cancel{6}\times\frac{1}{\cancel{3}}}{1-\frac{1}{9}}$$

$$\frac{\cancel{2}}{\cancel{8}_{4}}\times 9=\frac{9}{4}=2.25$$

✱ PROPERTIES :-

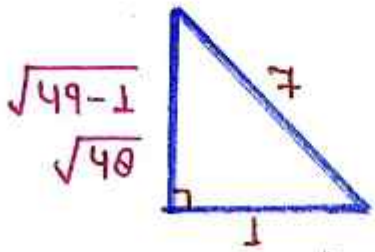
$$x=\sqrt{a\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\infty}}}}$$

$$\boxed{x=a}$$

Q) If $\cos\theta = \sqrt{\frac{1}{7}\sqrt{\frac{1}{7}\sqrt{\frac{1}{7}}}}$ ---- ∞ ($\theta \neq 0°$; θ is acute angle), then find $\tan\theta + \cot\theta = ?$

यदि $\cos\theta = \sqrt{\frac{1}{7}\sqrt{\frac{1}{7}\sqrt{\frac{1}{7}}}}$ ---- ∞ ($\theta \neq 0°$; θ न्यूनकोण है), तो $\tan\theta + \cot\theta$ ज्ञात कीजिए?

$$\cos\theta = \frac{1}{7}$$

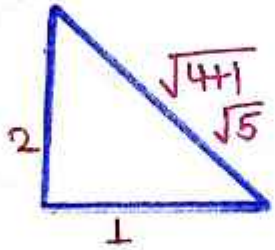


$$\frac{\sqrt{48}}{1} + \frac{1}{\sqrt{48}}$$

$$\frac{48+1}{\sqrt{48}} = \frac{49}{\sqrt{48}}$$

Q) If $\cot\theta = \frac{1}{2}$ & θ is acute angle, the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$ is

यदि $\cot\theta = \frac{1}{2}$ और θ न्यूनकोण है, तो $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$ का मान है?



$$\frac{8\times\frac{2}{\sqrt{5}} + 5\times\frac{1}{\sqrt{5}}}{\left(\frac{2}{\sqrt{5}}\right)^3 + 2\left(\frac{1}{\sqrt{5}}\right)^3 + 3\times\frac{1}{\sqrt{5}}}$$

$$\boxed{(\sqrt{a})^3 = a\sqrt{a}}$$

$$\frac{\frac{1}{\sqrt{5}}(16+5)}{\frac{8}{5\sqrt{5}} + 2\times\frac{1}{5\sqrt{5}} + \frac{3}{\sqrt{5}}}$$

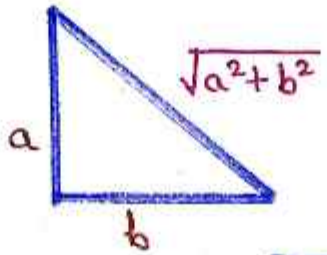
$$\frac{\frac{1}{\sqrt{5}}(21)}{\frac{1}{\sqrt{5}}\left(\frac{8}{5}+\frac{2}{5}+3\right)} = \frac{21}{\frac{10}{5}+3}$$

$$\frac{21}{5}$$

Q) If $\frac{\sin\theta}{a} = \frac{\cos\theta}{b}$, then find $\operatorname{cosec}\theta$?

यदि $\frac{\sin\theta}{a} = \frac{\cos\theta}{b}$, तो $\operatorname{cosec}\theta$ ज्ञात कीजिये ?

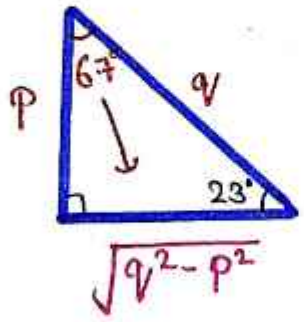
$$\frac{\sin\theta}{\cos\theta} = \frac{a}{b} \Rightarrow \boxed{\tan\theta = \frac{a}{b}}$$



$$\operatorname{cosec}\theta = \frac{\sqrt{a^2+b^2}}{a}$$

Q) If $\sin 23^\circ = \frac{p}{q}$ then $\sec 23^\circ - \sin 67^\circ = ?$

यदि $\sin 23^\circ = \frac{p}{q}$ तो $\sec 23^\circ - \sin 67^\circ = ?$

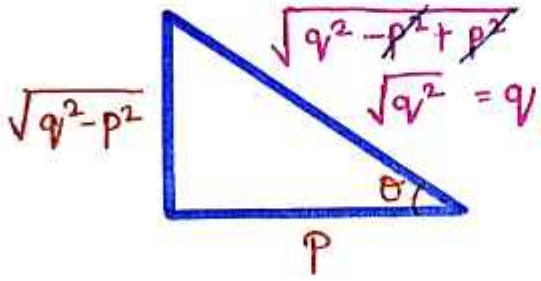


$$\frac{q}{\sqrt{q^2-p^2}} - \frac{\sqrt{q^2-p^2}}{q}$$

$$\frac{q^2-(q^2-p^2)}{q\sqrt{q^2-p^2}} = \frac{\cancel{q^2}-\cancel{q^2}+p^2}{q\sqrt{q^2-p^2}} = \frac{p^2}{q\sqrt{q^2-p^2}}$$

Q) If $\tan\theta = \frac{\sqrt{q^2-p^2}}{p}$, $0^\circ < \theta < 90^\circ$, then what is the value of $\sec\theta + \cos\theta + 2$?

यदि $\tan\theta = \frac{\sqrt{q^2-p^2}}{p}$, $0^\circ < \theta < 90^\circ$, तब $\sec\theta + \cos\theta + 2$ का मान क्या होगा ?



$$\frac{q}{p} + \frac{p}{q} + 2$$

$$\frac{q^2+p^2}{pq} + 2$$

$$\frac{q^2+p^2+2pq}{pq} = \frac{(q+p)^2}{pq}$$

TRIGONOMETRIC IDENTITIES

1) $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

2) $\boxed{\sec^2\theta - \tan^2\theta = 1}$

↳ $\sec^2\theta = 1 + \tan^2\theta$

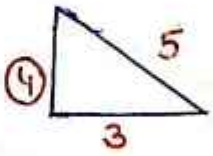
↳ $\tan^2\theta = \sec^2\theta - 1$

↳ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

↳ $\boxed{\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}}$

Ex:-

$$\sec\theta + \cancel{\tan\theta} = 3$$
$$+\ \sec\theta - \cancel{\tan\theta} = \frac{1}{3}$$
$$2\sec\theta = 3 + \frac{1}{3}$$
$$\cancel{2}\sec\theta = \frac{\cancel{10}\,5}{3}$$
$$\boxed{\sec\theta = \frac{5}{3}}$$



3) $\boxed{\text{Cosec}^2\theta - \cot^2\theta = 1}$

↳ $\text{Cosec}^2\theta = 1 + \cot^2\theta$

↳ $\cot^2\theta = \text{cosec}^2\theta - 1$

↳ $\boxed{(\text{cosec}\,\theta - \cot\theta) = \frac{1}{\text{cosec}\,\theta + \cot\theta}}$

Q) If $\sec\theta - \text{cosec}\,\theta = 0$, then what will be the value of $(\sec\theta + \text{cosec}\,\theta)$?
यदि $\sec\theta - \text{cosec}\,\theta = 0$ हो, तो $(\sec\theta + \text{cosec}\,\theta)$ का मान कितना होगा?

$$\boxed{\sec\theta = \text{Cosec}\,\theta}$$
$$\theta = 45^\circ$$
$$(\sec\theta + \text{cosec}\,\theta)$$
$$\sec 45^\circ + \text{cosec}\,45^\circ$$
$$\sqrt{2} + \sqrt{2}$$
$$2\sqrt{2}$$

Q) If $p\sin\theta = \sqrt{3}$ and $p\cos\theta = 1$, then what will be the value of p?
यदि $p\sin\theta = \sqrt{3}$ तथा $p\cos\theta = 1$ हो, तो p का मान कितना होगा?

$$p^2\sin^2\theta = 3 \qquad p^2\cos^2\theta = 1$$
$$+$$
$$p^2\sin^2\theta + p^2\cos^2\theta = 4$$
$$p^2(\sin^2\theta + \cos^2\theta) = 4$$
$$p^2 \times 1 = 4$$
$$p = \sqrt{4} = 2$$

Q) If $5\tan\theta = 4$, then what will be the value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right)$

यदि $5\tan\theta = 4$ हो, तो $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right)$ का मान क्या होगा ?

$\boxed{\tan\theta = \frac{4}{5}}$

$$\frac{\cancel{\cos\theta}\left[5 \times \frac{\sin\theta}{\cos\theta} - 3\right]}{\cancel{\cos\theta}\left[5 \times \frac{\sin\theta}{\cos\theta} + 3\right]}$$

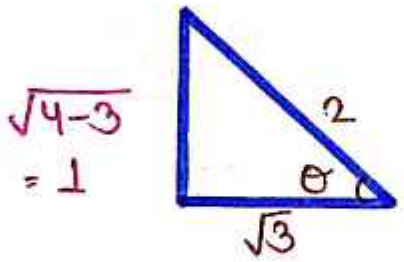
$$\frac{5\tan\theta - 3}{5\tan\theta + 3}$$

$$\frac{\cancel{5} \times \frac{4}{\cancel{5}} - 3}{\cancel{5} \times \frac{4}{\cancel{5}} + 3} = \frac{1}{7}$$

Q) If $\sec\theta + \tan\theta = \sqrt{3}$ then the positive value of $\sin\theta$ is -

यदि $\sec\theta + \tan\theta = \sqrt{3}$ तो $\sin\theta$ का धनात्मक मान है-

$$\sec\theta + \tan\theta = \sqrt{3}$$
$$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$$
$$+$$
$$\cancel{2}\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}} \Rightarrow \frac{\cancel{4}^{2}}{\sqrt{3}}$$



$$\sin\theta = \frac{1}{2}$$

Q) If $\sec\theta + \tan\theta = 2$, then what will be the value of $\sec\theta$?

यदि $\sec\theta + \tan\theta = 2$ हो, तो $\sec\theta$ का मान क्या होगा ?

$$\sec\theta + \tan\theta = 2$$
$$\sec\theta - \tan\theta = \frac{1}{2}$$
$$+$$
$$2\sec\theta = 2 + \frac{1}{2} = \frac{5}{2}$$
$$\sec\theta = \frac{5}{4}$$

Q) If $7\sin^2\theta + 3\cos^2\theta = 4$ $(0^\circ \le \theta \le 90^\circ)$ then what will be the value of θ?

यदि $7\sin^2\theta + 3\cos^2\theta = 4$ $(0^\circ \le \theta \le 90^\circ)$ हो, तो θ का मान कितना होगा?

By option $\frac{\pi}{6} = 30^\circ = 7 \times \frac{1}{4} + 3 \times \frac{3}{4}$

$$\frac{7}{4} + \frac{9}{4} = \frac{16}{4} \quad 4$$

Method

$$7\sin^2\theta + 3(1 - \sin^2\theta) = 4$$
$$7\sin^2\theta + 3 - 3\sin^2\theta = 4$$
$$4\sin^2\theta = 1$$
$$\sin^2\theta = \frac{1}{4}$$
$$\sin\theta = \sqrt{\frac{1}{4}} = \frac{1}{2}$$
$$\theta = 30^\circ \rightarrow \frac{\pi}{6}$$

1. $\sin\theta = \frac{4}{5}$, find $\frac{\sin\theta+\cos\theta}{\tan\theta+\cot\theta}$

(a) $\frac{84}{125}$
(b) $\frac{25}{84}$
(c) $\frac{33}{125}$
(d) $\frac{64}{125}$

2. $\cos\theta = \frac{3}{5}$, find $\frac{2\sin\theta+3\cos\theta}{\tan\theta-\cot\theta}$

(a) $\frac{105}{35}$
(b) $\frac{109}{22}$
(c) $\frac{204}{35}$
(d) $\frac{203}{35}$

3. $\tan\theta = \frac{4}{3}$, find $\frac{1+2\sin\theta+3\cos\theta}{1+3\sin\theta-2\cos\theta}$

(a) $\frac{265}{34}$
(b) $\frac{34}{265}$
(c) $\frac{39}{24}$
(d) 2

4. If $\cot\theta = \frac{8}{15}$, then what is the value of $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ where, θ is a positive acute angle?

यदि $\cot\theta = \frac{8}{15}$, तो $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ where, θ का मान क्या है, जहाँ θ एक धनात्मक न्यून कोण है?

(a) $\frac{1}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$
(d) $\frac{4}{5}$

5. If $\cos A = \frac{5}{13}$, then what is the value of $\frac{\sin A-\cot A}{2\tan A}$?

यदि $\cos A = \frac{5}{13}$, तो $\frac{\sin A-\cot A}{2\tan A}$ का मान क्या है?

(a) $\frac{395}{3644}$ (b) $\frac{395}{3844}$
(c) $\frac{395}{3744}$ (d) $\frac{385}{3744}$

6. If θ lies in the first quadrant and $\cot\theta = \frac{63}{16}$, then what is the value of $(\sin\theta + \cos\theta)$?

यदि θ प्रथम चतुर्थांश में स्थित है तथा $\cot\theta = \frac{63}{16}$ है, तो $(\sin\theta + \cos\theta)$का मान क्या है?

(a) 1
(b) $\frac{69}{65}$
(c) $\frac{79}{65}$
(d) 2

7. If $0 < \theta < 90^\circ$, $\sin\theta = \frac{3}{5}$ and $x = \cot\theta$, then what is the value of $1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5$?

यदि $0 < \theta < 90^\circ$, $\sin\theta = \frac{3}{5}$ तथा $x = \cot\theta$ है, तो $1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5$ का मान क्या है?

(a) 941
(b) 1000
(c) 1220
(d) 1365

8. If $\operatorname{cosec}\theta = \frac{p}{q}$ and θ is acute, then what is the value of $\left(\sqrt{p^2 - q^2}\right)\tan\theta$?

यदि $\operatorname{cosec}\theta = \frac{p}{q}$ तथा θ न्यूनकोण है, तो $\left(\sqrt{p^2 - q^2}\right)\tan\theta$ का मान क्या है?

(a) p
(b) q
(c) pq
(d) $\sqrt{p^2} + q^2$

9. If $\tan\theta = \frac{p}{q}$, then what is $\frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$ equal to?

यदि $\tan\theta = \frac{p}{q}$, तो $\frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$ किसके बराबर है?

(a) $\frac{p-q}{p+q}$
(b) $\frac{q^2-p^2}{q^2+p^2}$
(c) $\frac{p^2-q^2}{q^2+p^2}$
(d) 1

10. If $\sin 17^\circ = \frac{x}{y}$ then the value of $(\sec 17^\circ - \sin 73^\circ)$ is

यदि $\sin 17^\circ = \frac{x}{y}$ तो $(\sec 17^\circ - \sin 73^\circ)$ का मान है

(a) $\frac{y^2}{x\sqrt{y^2-x^2}}$

(b) $\frac{x^2}{y\sqrt{y^2-x^2}}$

(c) $\frac{x^2}{y\sqrt{x^2-y^2}}$

(d) $\frac{y^2}{x\sqrt{x^2-y^2}}$

11. If $\sin\theta = \frac{p^2-1}{p^2+1}$, then $\cos\theta$ is equal to:

यदि $\sin\theta = \frac{p^2-1}{p^2+1}$ है, तो $\cos\theta$ बराबर है :

(a) $\frac{2p}{1+p^2}$

(b) $\frac{p}{p^2-1}$

(c) $\frac{p}{1+p^2}$

(d) $\frac{2p}{p^2-1}$

12. If यदि $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, $0^\circ < \theta < 90^\circ$, then the value of $\sec\theta + \tan\theta$ is:

यदि $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, $0^\circ < \theta < 90^\circ$, तो $\sec\theta + \tan\theta$ का मान है:

(a) $\frac{\sqrt{a^2+b^2}+a}{b}$

(b) $\frac{\sqrt{a^2+b^2}+b}{2a}$

(c) $\frac{\sqrt{a^2+b^2}+a}{2b}$

(d) $\frac{\sqrt{a^2+b^2}+b}{a}$

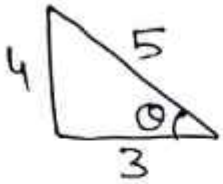
ANSWER SHEET

1	2	3	4	5	6	7	8	9	10	11	12
A	C	D	C	C	C	D	B	C	B	A	A

Worksheet solution

sol 1

$$\sin\theta = \frac{4}{5}$$

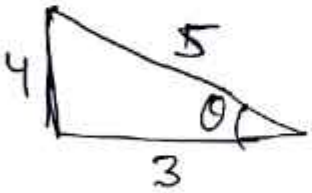


$$= \frac{\sin\theta + \cos\theta}{\tan\theta + \cot\theta}$$

$$= \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{3} + \frac{3}{4}} = \frac{84}{125}$$

sol 2

$$\cos\theta = \frac{3}{5}$$



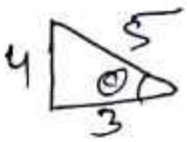
$$= \frac{2\sin\theta + 3\cos\theta}{\tan\theta - \cot\theta}$$

$$= \frac{2 \times \frac{4}{5} + 3 \times \frac{3}{5}}{\frac{4}{3} - \frac{3}{4}}$$

$$= \frac{204}{35}$$

sol 3

$$\tan\theta = \frac{4}{3}$$



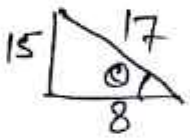
$$= \frac{1 + 2\sin\theta + 3\cos\theta}{1 + 3\sin\theta - 2\cos\theta}$$

$$= \frac{1 + 2 \times \frac{4}{5} + 3 \times \frac{3}{5}}{1 + 3 \times \frac{4}{5} - 2 \times \frac{3}{5}}$$

$$= \frac{22}{11} = 2$$

sol. 4

$$\cot\theta = \frac{8}{15}$$

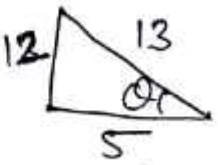


$$= \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\frac{1 - \frac{8}{17}}{1 + \frac{8}{17}}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

sol 5

$$\cos A = \frac{5}{13}$$

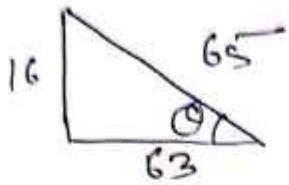


$$= \frac{\sin A - \cot A}{2\tan A} = \frac{\frac{12}{13} - \frac{5}{12}}{2 \times \frac{12}{5}}$$

$$= \frac{144 - 65}{13 \times 12 \times 2 \times \frac{12}{5}} = \frac{395}{3744}$$

Sol-6 $\cot\theta = \frac{63}{16}$

Pythagorean triplet $(65, 63, 16)$

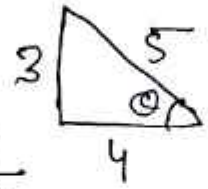


$= \sin\theta + \cos\theta$

$= \frac{16}{65} + \frac{63}{65} = \frac{79}{65}$

Sol 7

$\sin\theta = \frac{3}{5}$



$\therefore \cot\theta = x = \frac{4}{3}$

$= 1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5$

$= 1 + 3\times\frac{4}{3} + 9\times\frac{16}{9} + 27\times\frac{64}{27} + 81\times\frac{256}{81} + 243\times\frac{1024}{243}$

$= 1 + 4 + 16 + 64 + 256 + 1024$

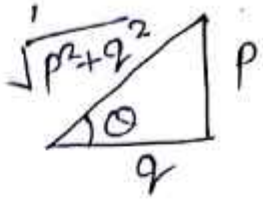
$= 1365$

Sol 8 $\operatorname{cosec}\theta = \frac{p}{q}$

$= (\sqrt{p^2-q^2}) \times \frac{q}{\sqrt{p^2-q^2}}$

$= q$

Sol-9 $\tan\theta = \frac{p}{q}$

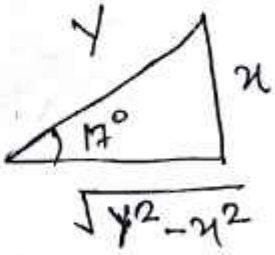


$= \frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$

$= \frac{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) - q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) + q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}$

$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{p}{q} + \frac{q}{p}} = \frac{p^2-q^2}{p^2+q^2}$

Sol 10 $\sin 17^\circ = \frac{x}{y}$



$= \sec 17^\circ - \sin 73^\circ$

$= \sec 17^\circ - \sin(90^\circ - 17)$

$= \sec 17^\circ - \cos 17^\circ$

$= \frac{y}{\sqrt{y^2-x^2}} - \frac{\sqrt{y^2-x^2}}{x}$

$= \frac{x^2}{y\sqrt{y^2-x^2}}$

$\boxed{\text{sol-11}}$ $\sin\theta = \dfrac{p^2-1}{p^2+1}$

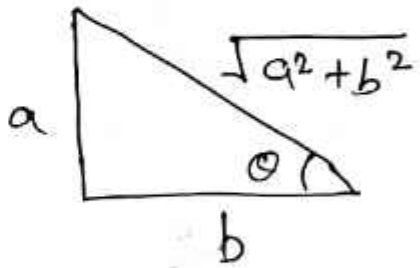
$$\therefore \cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\frac{(p^2-1)^2}{(p^2-1)^2}} = \sqrt{\frac{(p^2+1)^2-(p^2-1)^2}{(p^2+1)^2}}$$

$$= \sqrt{\frac{4p^2}{(p^2+1)^2}} \quad \left[\because (a+b)^2-(a-b)^2 = 4ab\right]$$

$$= \frac{2p}{p^2+1}$$

$\boxed{\text{sol-12}}$ $\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$



$$\therefore \sec\theta + \tan\theta$$

$$= \frac{\sqrt{a^2+b^2}}{b} + \frac{a}{b}$$

$$= \frac{\sqrt{a^2+b^2}+a}{b}.$$

TRIGONOMETRY

$\sec^2\theta - \tan^2\theta = 1$

$\sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$

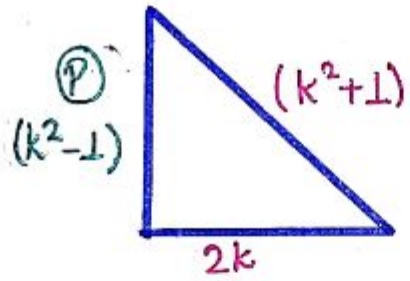
$\sec\theta + \tan\theta = k$

$\sec\theta - \tan\theta = \dfrac{1}{k}$

$+$

$2\sec\theta = k + \dfrac{1}{k} = \dfrac{k^2+1}{k}$

$$\boxed{\sec\theta = \frac{k^2+1}{2k}}$$



$P^2 = (k^2+1)^2 - (2k)^2$

$P^2 = k^4 + 1 + 2k^2 - 4k^2$

$k^4 + 1 - 2k^2$

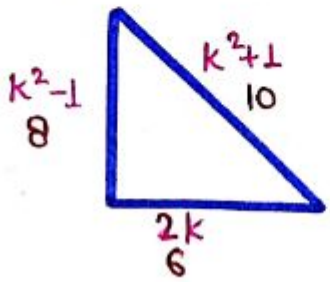
$P^2 = (k^2-1)^2$

$P = \boxed{k^2-1}$

Ex:-

$\sec\theta + \tan\theta = 3 \rightarrow k$

$\sin\theta = ?$

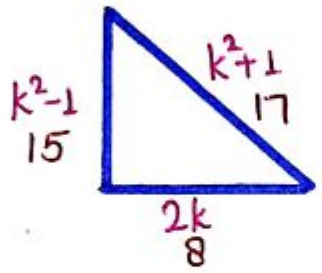


$\sin\theta = \cancel{\dfrac{8}{10}}\ \dfrac{4}{5}$

Ex:-

$\text{Cosec}\,\theta + \text{Cot}\,\theta = 4 \rightarrow k$

$\text{Cos}\,\theta = ?$

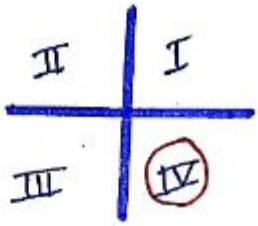


$\text{Cos}\,\theta = \frac{8}{17}$

SPECIAL CASE

$\text{Sec}\,\theta - \tan\theta = 3 \rightarrow k$

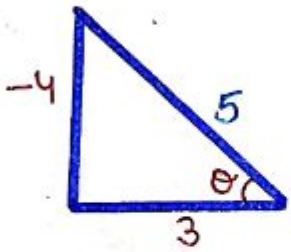
$\text{Sec}\,\theta + \tan\theta = \frac{1}{3}$



$2\tan\theta = \frac{1}{3} - 3$

$\cancel{2}\tan\theta = -\frac{\cancel{8}\,4}{3}$

$\boxed{\tan = -\frac{4}{3}}$



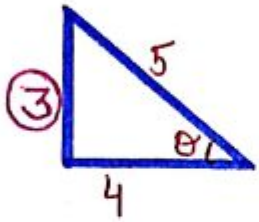
$\sin\theta = -\frac{4}{5}$

Q) If $\sec\theta = x + \frac{1}{4x}$ $(0^\circ < \theta < 90^\circ)$, then what will be $\sec\theta + \tan\theta$ equal to?

यदि $\sec\theta = x + \frac{1}{4x}$ $(0^\circ < \theta < 90^\circ)$ हों, तो $\sec\theta + \tan\theta$ किसके बराबर होगा?

Put, $x = 1$

$\sec\theta = 1 + \frac{1}{4} = \frac{5}{4}$



$\sec\theta + \tan\theta$

$\frac{5}{4} + \frac{3}{4} = \frac{8}{4}$ ②

$2x = 2$

Q) If $\sec\theta + \tan\theta = 2 + \sqrt{5}$, then what will be the value of $\sin\theta + \cos\theta$?

यदि $\sec\theta + \tan\theta = 2 + \sqrt{5}$ हो, तो $\sin\theta + \cos\theta$ का मान क्या होगा?

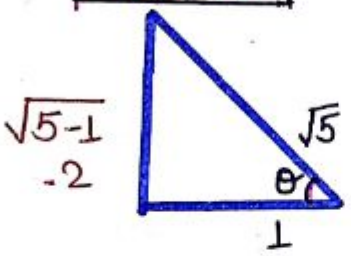
$\sec\theta + \tan\theta = \sqrt{5} + 2$

$\sec\theta - \tan\theta = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$

$\frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

$2\sec\theta = 2\sqrt{5}$

$\boxed{\sec\theta = \sqrt{5}}$



$\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}$

$\frac{3}{\sqrt{5}}$

Q) The numerical value of $\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$ is :

$\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$ का संख्यात्मक मान होगा।

$$\frac{1}{\operatorname{cosec}^2\theta} + \frac{3}{\sec^2\theta} + 2\sin^2\theta$$

$$\sin^2\theta + 3\cos^2\theta + 2\sin^2\theta$$

$$3\sin^2\theta + 3\cos^2\theta$$

$$3(\sin^2\theta + \cos^2\theta)$$

$$3 \times 1 = 3$$

IInd Method Value Putting

{ θ की कोई भी Value रख सकते है। }

$\theta = 45^\circ$

{ $\boxed{\frac{1}{0}, \frac{0}{1}}$ की form नही बननी चाहिए }

$$\frac{1}{1+1} + \frac{3}{1+1} + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{1}{2} + \frac{3}{2} + \cancel{2} \times \frac{1}{\cancel{2}}$$

$$\frac{\cancel{4}^{2}}{\cancel{2}} + 1 = 3$$

Q) The value of $\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$ is

$\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$ का मान है।

$\theta = 45^\circ$

$$\frac{4}{1+1} + \frac{1}{1+1} + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{4}{2} + \frac{1}{2} + 3 \times \frac{1}{2}$$

$$\frac{5}{2} + \frac{3}{2} = \frac{8}{2} \quad 4$$

IInd Method

$$\frac{4}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta} + 3\sin^2\alpha$$

$$4\cos^2\theta + \sin^2\theta + 3\sin^2\theta$$

$$4(\cos^2\theta + \sin^2\theta)$$

$$4 \times 1 = 4$$

Q) If $4x = \sec\theta$ and If $\frac{4}{x} = \tan\theta$, then what will be $8\left(x^2 - \frac{1}{x^2}\right)$?

यदि $4x = \sec\theta$ तथा $\frac{4}{x} = \tan\theta$ हो, तो $8\left(x^2 - \frac{1}{x^2}\right)$ क्या होगा?

$$x = \frac{\sec\theta}{4}, \quad \frac{1}{x} = \frac{\tan\theta}{4}$$

$$x^2 - \frac{1}{x^2} = \left(\frac{\sec^2\theta}{16} - \frac{\tan^2\theta}{16}\right)$$

$$\frac{1}{16}\left(\sec^2\theta - \tan^2\theta\right)$$

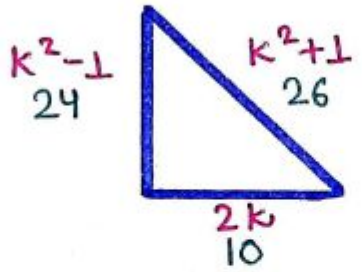
$$\frac{1}{16} \times 1 = \frac{1}{16}$$

$$8\left(x^2 - \frac{1}{x^2}\right)$$

$$\cancel{8} \times \frac{1}{\cancel{16}_2} = \frac{1}{2}$$

Q) If $\sec\theta + \tan\theta = 5$, then what will be the value of $\frac{\tan\theta + 1}{\tan\theta - 1}$?

यदि $\sec\theta + \tan\theta = 5$, तो $\frac{\tan\theta + 1}{\tan\theta - 1}$ का मान क्या होगा?



$$\frac{\frac{24}{10} + 1}{\frac{24}{10} - 1} \Rightarrow \frac{\frac{34}{\cancel{10}}}{\frac{14}{\cancel{10}}}$$

$$\frac{\cancel{34}}{\cancel{14}} \quad \frac{17}{7}$$

Q) What is the numerical value of $1+\frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$

$1+\frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$ का संख्यात्मक मान बताइए :

$$1+\tan^2 63 - \underset{\sec^2(90-63)}{\sec^2 27} + \operatorname{cosec}^2 63 - \underset{\operatorname{cosec}^2(90-63)}{\operatorname{cosec}^2 27}$$

$$\cancel{\sec^2 63^\circ} - \cancel{\operatorname{cosec}^2 63} + \cancel{\operatorname{cosec}^2 63} - \cancel{\sec^2 63^\circ}$$

$$= 0$$

Q) If θ be an acute angle and $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is

यदि θ एक न्यूनकोण हो और $7\sin^2\theta + 3\cos^2\theta = 4$ हो, तो $\tan\theta$ का मान है ?

$$7\sin^2\theta + 3(1-\sin^2\theta) = 4$$

$$7\sin^2\theta + 3 - 3\sin^2\theta = 4$$

$$4\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \sqrt{\frac{1}{4}} = +\frac{1}{2}$$

$$\boxed{\theta = 30^\circ}$$

$$\tan\theta$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Q) If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta = ?$

यदि $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, तो $\sec^4\theta - \tan^4\theta = ?$

$$\sec^4\theta - \tan^4\theta$$

$$(\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$$

$$\frac{7}{12} \times 1$$

$$\frac{7}{12}$$

Q) If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then $\sin\theta - \cos\theta = ?$

यदि $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, तो $(\sin\theta - \cos\theta = ?$

$$(\sin\theta + \cos\theta = \sqrt{2}\cos\theta)^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$1 + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$\boxed{2\sin\theta\cos\theta = 2\cos^2\theta - 1}$$

$$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$

$$1 - (2\cos^2\theta - 1)$$

$$1 - 2\cos^2\theta + 1 = 2 - 2\cos^2\theta$$

$$2(1 - \cos^2\theta)$$

$$2\sin^2\theta$$

$$\sin\theta - \cos\theta = \sqrt{2\sin^2\theta}$$

$$\pm 2\sin\theta$$

Q) $\sqrt{\dfrac{1}{\sin^4(90+A)} + \dfrac{1}{\cos^2(90+A) - 1}} = ?$

$$= \sqrt{\frac{1}{\cos^4 A} + \frac{1}{\sin^2 A - 1}}$$

$$= \sqrt{\frac{1}{\cos^4 A} + \frac{1}{-\cos^2 A}}$$

$$\left\{\begin{array}{l} 1 - \sin^2\theta = \cos^2\theta \\ \sin^2\theta - 1 = -\cos\theta \end{array}\right\}$$

$$= \sqrt{\frac{1}{\cos^2 A}\left[\frac{1}{\cos^2 A} - 1\right]}$$

$$= \frac{1}{\cos A}\sqrt{\frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{1}{\cos A}\sqrt{\tan^2 A}$$

$$\sec A \times \tan A$$

Q) $a^2\sin^2 x + b^2\cos^2 x = c^2$

$2\sin^2 x - \cos^2 x = ?$

$$a^2\sin^2x + b^2(1-\sin^2x) = c^2$$

$$a^2\sin^2x + b^2 - b^2\sin^2x = c^2$$

$$\sin^2x\,(a^2-b^2) = c^2-b^2$$

$$\boxed{\sin^2x = \frac{c^2-b^2}{a^2-b^2}}$$

$$\cos^2x = 1-\sin^2x \Rightarrow 1-\frac{c^2-b^2}{a^2-b^2}$$

$$\frac{a^2-\cancel{b^2}-c^2+\cancel{b^2}}{a^2-b^2} = \frac{a^2-c^2}{a^2-b^2}$$

$$2\sin^2x - \cos^2x$$

$$2\left(\frac{c^2-b^2}{a^2-b^2}\right) - \frac{a^2-c^2}{a^2-b^2}$$

$$\frac{2c^2-2b^2-a^2+c^2}{a^2-b^2}$$

$$\frac{3c^2-2b^2-a^2}{a^2-b^2}$$

1. What is the least value of $9\sin^2\theta + 16\cos^2\theta$?
$9\sin^2\theta + 16\cos^2\theta$ का न्यूनतम मान क्या है?
(a) 0
(b) 9
(c) 16
(d) 25

2. If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$, then what is $m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ equal to?
यदि $\operatorname{cosec}\theta - \sin\theta = m$ तथा $\sec\theta - \cos\theta = n$, तो $m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ किसके बराबर है?
(a) 0
(b) 1
(c) mn
(d) m^2n^2

3. $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} = ?$
(a) $1 - \sin\theta\cos\theta$
(b) $1 + \sin\theta\cos\theta$
(c) $1 + \sin\theta$
(d) None of these

4. What will be the value of $(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$?
$(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$ का मान क्या होगा?
(a) -2
(o) 2
(c) -1
(d) 1

5. Find the value of $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1} - \tan^2\theta, 0^\circ < \theta < 90^\circ$.
$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1} - \tan^2\theta, 0^\circ < \theta < 90^\circ$, का मान ज्ञात करें।
(a) $2\sec^2\theta$
(b) $\sec^2\theta + 1$
(c) $\sec^2\theta$
(d) $1 - \tan^2\theta$

6. If $x + y = 90^\circ$, then what is $\sqrt{\cos x\operatorname{cosec} y - \cos x\sin y}$ equal to?

यदि $x + y = 90^\circ$, तो $\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$ किसके बराबर है?

(a) $\cos x$ (b) $\sin x$

(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

7. $\frac{[1-\tan(90-\theta)+\sec(90-\theta)]}{[\tan(90-\theta)+\sec(90-\theta)+1]}$ What is the value of "of"?/का मान क्या है ?

(a) $\cot\frac{\theta}{2}$ (b) $\tan\frac{\theta}{2}$

(c) $\sin\theta$ (d) $\cos\theta$

8. If $p = \cot\theta + \tan\theta$ and $q = \sec\theta - \cos\theta$, then $(p^2p)^{\frac{2}{3}}(q^2p)^{\frac{2}{3}}$ is equal to

यदि $p = \cot\theta + \tan\theta$ तथा $q = \sec\theta - \cos\theta$, तो $((p^2p)^{\frac{2}{3}}(q^2p)^{\frac{2}{3}}$ बराबर है

(a) 0 (b) 1

(c) 2 (d) 3

9. Consider the following :

निम्न पर विचार करें :

I. $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \cos^2\theta(1+\tan\theta)(1-\tan\theta)$

II. $\frac{1+\sin\theta}{1-\sin\theta} = (\tan\theta + \sec\theta)^2$

Which of the statements given above is/are correct?

उपर्युक्त में से कौन सा/से कथन सही है/हैं?

(a) Only I

(b) Only II

(c) Both I and II

(d) Neither I nor II

10. If $\tan^2\theta - 3\sec\theta + 3 = 0, 0^\circ < \theta < 90^\circ$, then the value of $(\sin\theta + \cot\theta)$ is:

यदि $\tan^2\theta - 3\sec\theta + 3 = 0, 0^\circ < \theta < 90^\circ$, है, तो $(\sin\theta + \cot\theta)$ का मान है :

(a) $\frac{5\sqrt{3}}{6}$ (b) $2\sqrt{3}$

(c) $\frac{5\sqrt{3}}{3}$ (d) $3\sqrt{3}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
B	B	B	B	B	B	B	B	C	A

Worksheet solution

Sol 1

$$y = 9\sin^2\theta + 16\cos^2\theta$$
$$= 9(\sin^2\theta + \cos^2\theta) + 7\cos^2\theta$$
$$= 9 + 7(0) = 9$$

Sol 2

$$m = \operatorname{cosec}\theta - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

$$n = \sec\theta - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$$m^{\frac{4}{3}} \cdot n^{\frac{2}{3}} + m^{\frac{2}{3}} \cdot n^{\frac{4}{3}} = (m \cdot n)^{\frac{2}{3}} \cdot \left\{ m^{\frac{2}{3}} + n^{\frac{2}{3}} \right\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} \right)^{\frac{2}{3}} \cdot \left\{ \left(\frac{\cos^2\theta}{\sin\theta} \right)^{\frac{2}{3}} + \left(\frac{\sin^2\theta}{\cos\theta} \right)^{\frac{2}{3}} \right\}$$

$$= (\sin\theta \cdot \cos\theta)^{\frac{2}{3}} \cdot \left\{ \frac{(\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}}}{(\sin\theta \cdot \cos\theta)^{\frac{2}{3}}} \right\}$$

$$= (\sin\theta \cdot \cos\theta)^{\frac{2}{3}} \cdot \left\{ \frac{\cos^2\theta + \sin^2\theta}{(\sin\theta \cdot \cos\theta)^{\frac{2}{3}}} \right\}$$

$$= 1$$

Sol 3

$$= \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{[\cos\theta - \sin\theta][\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta]}{(\cos\theta - \sin\theta)}$$

$$= 1 + \cos\theta\sin\theta$$

$\boxed{\text{sol 4}}$

$$= (1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$$

$$= [1 + (\sin^2 A)^2 - (\cos^2 A)^2]\operatorname{cosec}^2 A$$

$$= [1 + (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)]\operatorname{cosec}^2 A$$

$$= [1 + \sin^2 A - \cos^2 A]\operatorname{cosec}^2 A$$

$$= 2\sin^2 A \times \operatorname{cosec}^2 A = 2$$

$\boxed{\text{sol 5}}$

$$= \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} - \tan^2\theta$$

$$= \frac{\operatorname{cosec}\theta\,(\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1)}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)} - \tan^2\theta$$

$$= \frac{\operatorname{cosec}\theta \cdot 2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} - \tan^2\theta$$

$$= \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta} - \tan^2\theta$$

$$= 2 \times \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} - \tan^2\theta$$

$$= 2\sec^2\theta - \tan^2\theta$$

$$= \sec^2\theta + (\sec^2\theta - \tan^2\theta)$$

$$= \sec^2\theta + 1$$

sol 6

$$= \sqrt{\cos x \operatorname{cosec} y - \cos x \sin y} \qquad (\because x+y=90^\circ)$$

$$= \sqrt{\cos x \cdot \operatorname{cosec}(90^\circ - x) - \cos x \cdot \sin(90^\circ - x)}$$

$$= \sqrt{\cos x \cdot \sec x - \cos^2 x}$$

$$= \sqrt{1 - \cos^2 x} = \sqrt{\sin^2 x} = \sin x$$

sol 7

$$= \frac{[1 - \tan(90-\theta) + \sec(90-\theta)]}{[\tan(90-\theta) + \sec(90-\theta) + 1]}$$

$$= \frac{1 - \cot\theta + \operatorname{cosec}\theta}{\cot\theta + \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\operatorname{cosec}\theta - \cot\theta)}{\cot\theta + \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) + (\operatorname{cosec}\theta - \cot\theta)}{\cot\theta + \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta + 1)}{\cot\theta + \operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1 - \cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$\boxed{\text{sol 8}}$ $p = \cot\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

$q = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{\sin^2\theta}{\cos\theta}$

$= (p^2 q)^{\frac{2}{3}} - (q^2 p)^{\frac{2}{3}}$

$= \left(\frac{1}{\sin^2\theta \cdot \cos^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^4\theta}{\cos^2\theta} \times \frac{1}{\sin\theta\cos\theta}\right)^{\frac{2}{3}}$

$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}}$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$

$\boxed{\text{sol 9}}$ (I) R.H.S $\cos^2\theta\,(1+\tan\theta)(1-\tan\theta)$

$= \cos^2\theta\,(1-\tan^2\theta)$

$= \cos^2\theta\left(\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right)$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \text{L.H.S}$

(II) L.H.S $\frac{1+\sin\theta}{1-\sin\theta} \times \frac{(1+\sin\theta)}{(1-\sin\theta)}$

$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1+\sin\theta}{\cos\theta}\right)$

$= (\sec\theta + \tan\theta)^2$

option (c) Both (i) and (ii)

Sol 10

$\Rightarrow \tan^2\theta - 3\sec\theta + 3 = 0$

$\Rightarrow \sec^2\theta - 1 - 3\sec\theta + 3 = 0$

$\Rightarrow \sec^2\theta - 3\sec\theta + 2 = 0$

$\Rightarrow \sec^2\theta - 2\sec\theta - \sec\theta + 2 = 0$

$\Rightarrow \sec\theta(\sec\theta - 2) - 1(\sec\theta - 2) = 0$

$\Rightarrow (\sec\theta - 1)(\sec\theta - 2) = 0$

$\sec\theta = 2 \qquad \sec\theta \neq 1 \; (0^\circ < \theta < 90^\circ)$

$\sec\theta = 2 \Rightarrow \sec\theta = \sec 60^\circ$

$\theta = 60^\circ$

$= (\sin\theta + \cot\theta)$

$= \sin 60^\circ + \cot 60^\circ$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} = \frac{3+2}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$

$= \frac{5\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3}}{6}$

TRIGONOMETRY

- $a^3 + b^3 = (a+b)(a^2+b^2-ab)$
- $a^3 - b^3 = (a-b)(a^2+b^2+ab)$

Q) What is $(\sin^2 x - \cos^2 x)(1-\sin^2 x \cos^2 x)$ equal to ?

$(\sin^2 x - \cos^2 x)(1-\sin^2 x \cos^2 x)$ किसके बराबर है?

$$(\sin^2 x - \cos^2 x)((\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x)$$

$$(\sin^4 x + \cos^4 x + 2\sin^2 x \cdot \cos^2 x - \sin^2 x \cos^2 x)$$

$$(\sin^2 x - \cos^2 x)(\sin^4 x + \cos^4 x + \sin^2 x \cdot \cos^2 x)$$

$$(\sin^2 x)^3 - (\cos^2 x)^3$$

$$\sin^6 x - \cos^6 x$$

Q) Consider the following equations:

निम्नलिखित समीकरणों पर विचार करें:

I. $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \sec^2 x$

II. $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$

III. $\operatorname{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$

Which of the above statements are correct?

उपर्युक्त में से कौन से कथन सही हैं?

a) I and II b) II and III c) I and III d) All of these

(I) $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \sec^2 x$

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$\frac{1}{\sin^2 x \cdot \cos^2 x}$$

$$= \operatorname{cosec}^2 x \cdot \sec^2 x$$

(II) $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$

$$\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{1+\sin^2 x}{\cos^2 x}$$

(III) $\operatorname{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$

$\cancel{1} + \cot^2 x + \sec^2 x - \cancel{1}$

$\cot^2 x + \sec^2 x$

$\therefore \sec^2 x - \tan^2 x = 1$

$\tan^2 x = \sec^2 x - 1$

Q) If $\sin^2 \alpha = \cos^3 \alpha$, then ~~then~~ the value of $(\cot^6 \alpha - \cot^2 \alpha)$ is –

यदि $\sin^2 \alpha = \cos^3 \alpha$, तो $(\cot^6 \alpha - \cot^2 \alpha)$ का मान है –

$\sin^2 \alpha = \cos \alpha \cos^2 \alpha$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\cos \alpha}$$

$$\boxed{\cot^2 x = \sec \alpha}$$

$\downarrow$

$\cot^4 \alpha = \sec^2 \alpha$

$\cot^2 \alpha (\cot^4 \alpha - 1)$

$\cot^2 \alpha (\sec^2 \alpha - 1)$

$\cot^2 \alpha \cdot \tan^2 \alpha$

$= 1$

Q) If $a^2 = \dfrac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$, then what is the value of $\dfrac{a+1}{a-1}$?

यदि $a^2 = \dfrac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$ है, तो $\dfrac{a+1}{a-1}$ का मान क्या है?

$$\frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$$

$$a^2 = \frac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2}$$

$$\boxed{\frac{a}{1} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}}$$

$$\frac{a+1}{a-1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

- $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
- $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

Q) If $\sin\theta + \sin^2\theta = 1$, then what is the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$?

यदि $\sin\theta + \sin^2\theta = 1$ हो, तो $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ का मान क्या होगा ?

$\sin\theta = 1 - \sin^2\theta$

$\boxed{\sin\theta = \cos^2\theta}$

$\downarrow$

$\boxed{\sin^2\theta = \cos^4\theta}$

$[(\cos^4\theta)^3 + (\cos^2\theta)^3 + 3\cos^8\theta \cdot \cos^2\theta + 3 \times \cos^4\theta \times \cos^4\theta] - 1$

$[\cos^4\theta + \cos^2\theta]^3 - 1$

$\downarrow$

$[\sin^2\theta + \cos^2\theta]^3 - 1$

$1 - 1 = 0$

Q) If $\cos A = 1 - \sin^2(90^\circ - A)$, then $\sin^{12}A + 3\sin^{10}A + 3\sin^8 A + \sin^6 A + 2\sin^4 A + 2\sin^2 A - 2 = ?$

यदि $\cos A = 1 - \sin^2(90^\circ - A)$, तो $\sin^{12}A + 3\sin^{10}A + 3\sin^8 A + \sin^6 A + 2\sin^4 A + 2\sin^2 A - 2 = ?$

$\cos A = 1 - \cos^2 A$

$\boxed{\cos A = \sin^2 A}$

$\downarrow \quad \downarrow$

$\cos^2 A = \sin^4 A$

$[\sin^4 A + \sin^2 A]^3 + 2[\sin^4 A + \sin^2 A - 1]$

$\downarrow$

$[\cos^2 A + \sin^2 A]^3 + 2[\cos^2 A + \sin^2 A - 1]$

$1^3 + 2(1 - 1)$

$1 + 2 \times 0 = 1$

$\boxed{\sin\theta + \operatorname{cosec}\theta = 2}$

$\Downarrow \quad \Downarrow$

$1 \quad 1$

$\boxed{\theta = 90^\circ}$

$\boxed{\sin^n\theta + \operatorname{cosec}^n\theta = 2}$

$\boxed{\cos\theta + \sec\theta = 2}$

$\Downarrow \quad \Downarrow$

$1 \quad 1$

$\boxed{\theta = 0}$

$\boxed{\cos^n\theta + \sec^n\theta = 2}$

$\boxed{\tan\theta + \cot\theta = 2}$

$\Downarrow \quad \Downarrow$

$1 \quad 1$

$\boxed{\theta = 45^\circ}$

$\boxed{\tan^n\theta + \cot^n\theta = 2}$.

Q) If $\cos\theta + \sec\theta = 2$, the value of $\cos^6\theta + \sec^6\theta$ is :

यदि $\cos\theta + \sec\theta = 2$, तो $\cos^6\theta + \sec^6\theta$ का मान है :

$\boxed{\theta = 0^\circ}$

$\Downarrow$

2

Q) If $\sin\theta + \operatorname{cosec}\theta = 2$, then what is the value of $\sin^4\theta + \cos^4\theta$?

यदि $\sin\theta + \operatorname{cosec}\theta = 2$ है, तो $\sin^4\theta + \cos^4\theta$ का मान क्या है ?

$\boxed{\theta = 90^\circ}$

$\sin^4 90^\circ + \cos^4 90^\circ$

$1 + 0$

1

Q) If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then find the value of $\tan^3\theta + \cot^3\theta + 6\tan^3\theta \cdot \cot^2\theta$

यदि θ एक न्यूनकोण है और $\tan\theta + \cot\theta = 2$ है, तो $\tan^3\theta + \cot^3\theta + 6\tan^3\theta \cdot \cot^2\theta$ का मान ज्ञात कीजिए

$\boxed{\theta = 45^\circ}$

$\tan^3\theta + \cot^3\theta + 6\tan^3\theta \cdot \cot^2\theta$

$1 \quad 1 \quad 6 \times 1 \times 1$

$1 + 1 + 6$

8

Q) If $\sin\theta + \operatorname{cosec}\theta = 2$, then what will be the value of $\sin^n\theta + \operatorname{cosec}^n\theta$

यदि $\sin\theta + \operatorname{cosec}\theta = 2$ हो, तो $\sin^n\theta + \operatorname{cosec}^n\theta$ का मान क्या होगा ?

$\Downarrow$

2

Q) If $\cos\theta + \sec\theta = 2$, then $(\cos^{117}\theta + \sec^{117}\theta)$ is equal to :

यदि $\cos\theta + \sec\theta = 2$ है, तो $(\cos^{117}\theta + \sec^{117}\theta)$ बराबर है :

$\Downarrow$

2

Q) If θ is an acute angle and $\cot\theta + \tan\theta = 2$ then find the value of $\tan^{12}\theta + \cot^{12}\theta + 2\tan^5\theta \cot^7\theta$.

यदि θ एक न्यून कोण है और $\cot\theta + \tan\theta = 2$ है तो $\tan^{12}\theta + \cot^{12}\theta + 2\tan^5\theta \cot^7\theta$ का मान ज्ञात कीजिए।

$\boxed{\theta = 45^\circ}$

$1 + 1 + 2 \times 1 \times 1$

$1 + 1 + 2$

4

#

- $\sin(A+B) = \sin A \cos B + \cos A \cdot \sin B$

$\boxed{B \to (-B)}$ • $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\boxed{A \to (90+A)}$ • $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\boxed{B \to (-B)}$ • $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\boxed{\begin{array}{c}\sin(90+A+B) \\ \cos(A+B)\end{array}}$

- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

- $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

- $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Q) Find the value of $\sin 15°$.

$\sin 15°$ का मान ज्ञात कीजिए।

$\sin(15°) = \sin(45° - 30°)$

$\sin 45° - \cos 30° - \cos 45° \cos 30°$

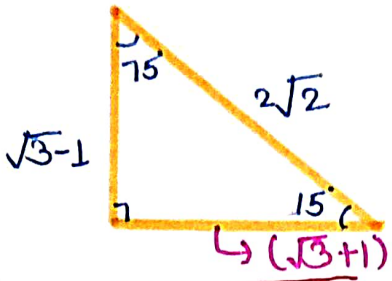
$\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$\dfrac{1}{\sqrt{2}}\left[\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right]$

$\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}-1}{2}\right)$

$\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

- $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$



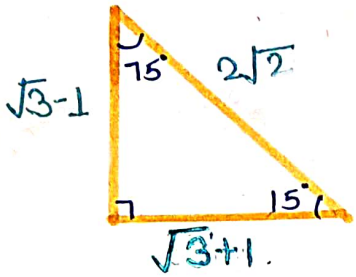
$$\sqrt{8-(\sqrt{3}-1)^2}$$

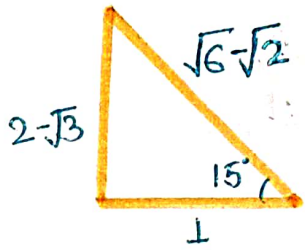
$$\sqrt{8-(3+1-2\sqrt{3})}$$

$$\sqrt{8-(4-2\sqrt{3})}$$

$$\sqrt{8-4+2\sqrt{3}}$$

$$\sqrt{4+2\sqrt{3}} = (\sqrt{3}+1)$$





- $\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

- $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

- $\cos 75^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

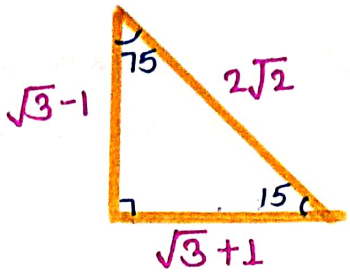
$$\frac{(\sqrt{3}-1)^2}{3-1} = \frac{4-2\sqrt{3}}{2}$$

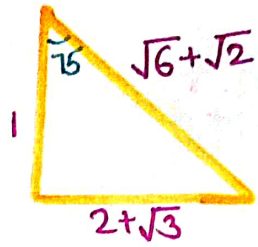
$$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$$

$$\tan 15^\circ = 2-\sqrt{3}$$

Q) Find the value of tan 75°

tan 75° का मान ज्ञात कीजिए।



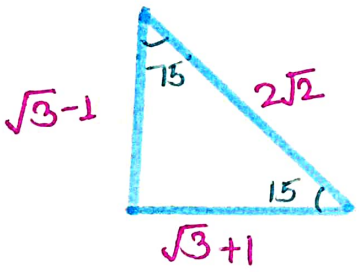
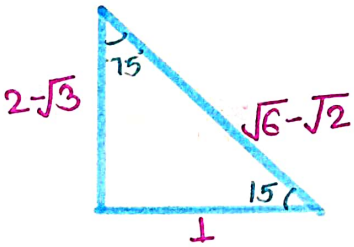
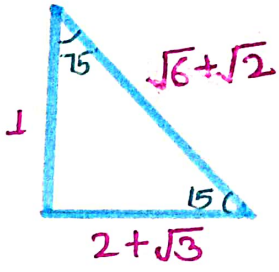


$$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$\frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}}$$

$$(2+\sqrt{3})$$

I	II	III
		

1. For $0^\circ < \theta < 90^\circ$, $\tan\theta + \cot\theta = 2$ then θ is equal to:

$0^\circ < \theta < 90^\circ$, के लिए $\tan\theta + \cot\theta = 2$ तो θ बराबर है :

(a) 30°
(b) 60°
(c) 45°
(d) 0°

2. If $\tan A + \cot A = 2$, then $\tan 3A + \cot 3A$ = ?

यदि $\tan A + \cot A = 2$ है, तो $\tan 3A + \cot 3A$ = ?

(A) -2
(B) 1
(C) 0
(D) 3

3. If $\cos\theta + \sec\theta = 2$, then find the value of $\sin^6\theta + \cos^6\theta$.

यदि $\cos\theta + \sec\theta = 2$ है, तो $\sin^6\theta + \cos^6\theta$ का मान ज्ञात कीजिए।

(a) $\frac{1}{3}$
(b) 1
(c) $\frac{1}{2}$
(d) 2

4. Find the value of $\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ$.

$\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ$ का मान ज्ञात कीजिए।

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 1 (d) $\frac{1}{2}$

5. If $0^0 \le \theta \le 90^\circ$, and $\sec^{107}\theta + \cos^{107}\theta = 2$, then, $(\sec\theta + \cos\theta)$ is equal to:

यदि $0^0 \le \theta \le 90^\circ$ और $\sec^{107}\theta + \cos^{107}\theta = 2$ है, तो $(\sec\theta + \cos\theta)$ का मान ज्ञात कीजिए।

(a) 2^{-107} (b) 2
(c) $1/2$ (d) 1

6. If $\tan\theta + \cot\theta = 2, \theta$ is an acute angle, then find the value of $2\tan^{25}\theta + 3\cot^{20}\theta + 5\tan^{30}\theta\cot^{15}\theta$.

यदि $\tan\theta + \cot\theta = 2$ है, θ एक न्यून कोण है, तो $2\tan^{25}\theta + 3\cot^{20}\theta + 5\tan^{30}\theta\cot^{15}\theta$ का मान ज्ञात कीजिए।

a. 12 b. 10
c. 8 d. 6

7. If $\sin\theta + \operatorname{cosec}\theta = 2$, then what is the value of $(\sin^{153}\theta + \operatorname{cosec}^{253}\theta)$?

यदि $\sin\theta + \operatorname{cosec}\theta = 2$ है, तो $(\sin^{153}\theta + \operatorname{cosec}^{253}\theta)$ का मान है?

(a) $\frac{1}{153\times253}$
(b) $\frac{253}{153}$
(c) 2
(d) $\frac{153}{253}$

8. If $\sin(A+B) = \frac{\sqrt{3}}{2}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, then $(2A+3B)$ is equal to:

यदि $\sin(A+B) = \frac{\sqrt{3}}{2}$ और $\tan(A-B) = \frac{1}{\sqrt{3}}$ तो $(2A+3B)$ बराबर है:

(a) 120°
(b) 135°
(c) 130°
(d) 125°

9. What is the value of $\frac{\sin(A+B)}{\sin A\cos B}$?

$\frac{\sin(A+B)}{\sin A\cos B}$ का मान क्या है?

(a) $1+\cot A\tan B$
(b) $1+\tan A\cot B$
(c) $1-\sin A\cos B$
(d) $1-\cot A\tan B$

10. Using trigonometric formulas, find the value of $\left(\frac{\sin(x-y)}{\sin(x+y)}\right)\left(\frac{\tan x+\tan y}{\tan x-\tan y}\right)$.

त्रिकोणमितीय सूत्रों का उपयोग करके $\left(\frac{\sin(x-y)}{\sin(x+y)}\right)\left(\frac{\tan x+\tan y}{\tan x-\tan y}\right)$ का मान ज्ञात करें।

(a) -2
(b) 2
(c) 0
(d) 1

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	A	B	A	B	B	C	B	A	D

Worksheet solution

sol 1

$\tan\theta + \cot\theta = 2$

$\therefore \theta = 45$ (Property)

sol 2

$\tan A + \cot A = 2$ $(\because A = 45^\circ)$

$\tan 3A + \cot 3A = ?$

$\tan 3\times 45 + \cot 3\times 45$

$\tan 135^\circ + \cot 135^\circ = -2$

sol 3

$\cos\theta + \sec\theta = 2$

$\theta = 0^\circ$

$= \sin^6\theta + \cos^6\theta$

$= \sin^6 0^\circ + \cos^6 0^\circ$

$= 0 + (1)^6 = 1$

sol 4

$\sin A \cdot \cos B + \cos A \cdot \sin B = \sin(A+B)$

$\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ = \sin(25^\circ + 35^\circ)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

sol 5

$\sec^{107}\theta + \cos^{107}\theta = 2$

$\theta = 0^\circ$ $(\because \sec^n\theta + \cos^n\theta = 2)$

$= \sec\theta + \cos\theta$

$= \sec 0^\circ + \cos 0^\circ$

$= 1 + 1 = 2$

$\boxed{\text{Sol-6}}$

$$\tan\theta + \cot\theta = 2$$

$$\theta = 45^\circ$$

$$= 2\tan^{25}\theta + 3\cot^{20}\theta + 5\tan^{30}\theta \cot^{15}\theta$$

$$= 2\tan^{25}45^\circ + 3\cot^{20}45^\circ + 5\tan^{30}45^\circ \cot^{15}45^\circ$$

$$= 2(1)^{25} + 3(1)^{20} + 5(1)^{30} \cdot (1)^{15}$$

$$= 2 + 3 + 5 = 10$$

$\boxed{\text{Sol7}}$

$$\sin\theta + \operatorname{cosec}\theta = 2$$

$$\theta = 90^\circ$$

$$= \sin^{153}\theta + \operatorname{cosec}^{253}\theta$$

$$= \sin^{153}90^\circ + \operatorname{cosec}^{253}90^\circ$$

$$= (1)^{153} + (1)^{253} = 2$$

$\boxed{\text{Sol8}}$

$$\sin(A+B) = \frac{\sqrt{3}}{2}, \quad (A+B) = 60^\circ \quad \text{--- (i)}$$

$$\tan(A-B) = \frac{1}{\sqrt{3}}, \quad (A-B) = 30^\circ \quad \text{--- (ii)}$$

$$A+B = 60^\circ$$
$$A-B = 30$$
$$\overline{2A = 90^\circ}$$
$$A = 45^\circ, \quad B = 15^\circ$$

$$\therefore (2A+3B) = 90^\circ + 45^\circ = 135^\circ$$

$\boxed{\text{sol-9}}$

$$\frac{\sin(A+B)}{\sin A \cos B} = \frac{\sin A \cdot \cos B + \cos A \cdot \sin B}{\sin A \cdot \cos B}$$

$$= 1 + \cot A \tan B$$

$\boxed{\text{sol 10}}$

$$= \left(\frac{\sin(x-y)}{\sin(x+y)}\right) \times \left[\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}\right]$$

$$= \frac{\sin(x-y)}{\sin(x+y)} \times \frac{\frac{\sin x \cdot \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}}$$

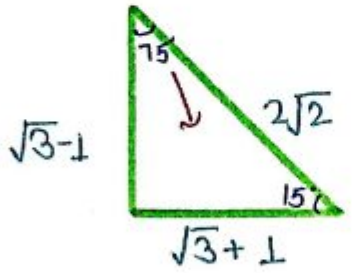
$$= \frac{\sin(x-y)}{\sin(x+y)} \times \frac{\sin(x+y)}{\sin(x-y)}$$

$$= 1$$

TRIGONOMETRY

Q) What is the value of $\sin 75^\circ + \sin 15^\circ$?

$\sin 75^\circ + \sin 15^\circ$ का मान क्या है?



$$\frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\frac{\sqrt{3}+\cancel{1}+\sqrt{3}-\cancel{1}}{2\sqrt{2}}$$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{2}\sqrt{2}} = \sqrt{\frac{3}{2}}$$

$$\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}}$$

Q) $\sin\left(\underbrace{\frac{\pi}{4}-A}_{A}\right)\sin\left(\underbrace{\frac{\pi}{4}-B}_{B}\right) - \cos\left(\underbrace{\frac{\pi}{4}-A}_{A}\right)\cos\left(\underbrace{\frac{\pi}{4}-B}_{B}\right) = ?$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$-\left[\cos\left(\frac{\pi}{4}-A\right)\cos\left(\frac{\pi}{4}-B\right) - \sin\left(\frac{\pi}{4}-A\right)\sin\left(\frac{\pi}{4}-B\right)\right]$$

$$= -\cos\left(\frac{\pi}{4}-A+\frac{\pi}{4}-B\right)$$

$$-\cos\left[\frac{\pi}{2}-(A+B)\right]$$

$$-\sin(A+B)$$

Q) What is the value of $[(\sin 59^\circ \cos 31^\circ + \cos 59^\circ \sin 31^\circ) \div (\cos 20^\circ \cos 25^\circ - \sin 20^\circ \sin 25^\circ)]$?

$[(\sin 59^\circ \cos 31^\circ + \cos 59^\circ \sin 31^\circ) \div (\cos 20^\circ \cos 25^\circ - \sin 20^\circ \sin 25^\circ)]$ का मान क्या है?

$$[\sin(59+31)] \div [\cos(20+25)]$$

$$\sin 90 \div \cos 45^\circ$$

$$1 \div \frac{1}{\sqrt{2}} \Rightarrow 1 \times \frac{\sqrt{2}}{1} = \sqrt{2}$$

Q) If $\tan A = \frac{15}{8}$ and $\tan B = \frac{7}{24}$, then $\tan(A+B)$?

यदि $\tan A = \frac{15}{8}$ तथा $\tan B = \frac{7}{24}$ है, तो $\tan(A+B)$?

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\frac{3}{3} \times \frac{15}{8} + \frac{7}{24}}{1 - \frac{\cancel{15}^{5}}{8} \times \frac{7}{\cancel{24}_{8}}} = \frac{\frac{52}{24}}{\frac{64-35}{64}}$$

$$\frac{52 \times \cancel{64}^{8}}{\cancel{24}_{3} \times 29} = \frac{416}{87}$$

Q) $\cos(90-A) \cdot \sin[\pi - (A-B)] + \sin(90-A) \cos[\pi - (B-A)] = ?$

$$\sin A \cdot \sin(A-B) + \cos A \cdot -\cos(B-A)$$

$$\sin A \cdot \sin(A-B) - \cos A \cdot \cos(A-B)$$

$$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$-[\cos A \cos(A-B) - \sin A \sin(A-B)]$$

$$-\cos(A + A - B)$$

$$-\cos(2A - B)$$

Q) $\cos(90-A) \cdot \sin[\pi + (A-B)] + \sin(90-A) \cos[\pi - (B-A)] = ?$

$$\sin A - \sin(A-B) + \cos A \cdot -\cos(B-A)$$

$$-\sin A \cdot \sin(A-B) - \cos A \cdot \cos(A-B)$$

$$\therefore \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$-[\cos A \cos(A-B) + \sin A \sin(A-B)]$$

$$-[\cos(\cancel{A} - \cancel{A} + B)]$$

$$-\cos B$$

$\boxed{\text{If } A+B=90^\circ}$

$\boxed{A=90-B}$

↳ $\sin A = \sin(90-B)$

$\boxed{\sin A = \cos B}$

↳ $\sin A = \dfrac{1}{\sec B}$

$\boxed{\sin A \cdot \sec B = 1}$

↳ $\dfrac{1}{\operatorname{cosec} A} = \cos B$

$\boxed{\operatorname{cosec} A \cdot \cos B = 1}$

↳ $\tan A = \tan(90-B)$

$\boxed{\tan A = \cot B}$

↳ $\tan A = \dfrac{1}{\tan B}$

$\boxed{\tan A \cdot \tan B = 1}$

↳ $\dfrac{1}{\cot A} = \cot B$

$\boxed{\cot A \cdot \cot B = 1}$

↳ $\sec A = \sec(90-B)$

$\boxed{\sec A = \operatorname{cosec} B}$

↳ $\boxed{\sin^2 A + \sin^2 B = 1}$

↳ $\boxed{\cos^2 A + \cos^2 B = 1}$

Ex:- $\sin 3x \cdot \sec 2x = 1$

$x = ?$

$5x = 90^\circ$

$x = 18^\circ$.

Q) If $\tan(x+y)\tan(x-y)=1$, then what will be the value of $\tan\left(\frac{2x}{3}\right)$?

यदि $\tan(x+y)\tan(x-y)=1$, हो, तो $\tan\left(\frac{2x}{3}\right)$ का मान क्या होगा ?

$\therefore \tan A \cdot \tan B = 1$

$A+B=90^\circ$

$x+\cancel{y}+x-\cancel{y}=90$

$\boxed{2x=90^\circ}$

$\tan\left(\frac{2x}{3}\right)=\tan\left(\frac{\cancel{90}^{30}}{\cancel{3}}\right)$

$\tan 30^\circ=\frac{1}{\sqrt{3}}$

Q) What will be the value of $\frac{\sin 39^\circ}{\cos 51^\circ}+2\tan 11^\circ \tan 31^\circ \tan 45^\circ \tan 59^\circ \tan 79^\circ - 3(\sin^2 21+\sin^2 69^\circ)$

$\frac{\sin 39^\circ}{\cos 51^\circ}+2\tan \underbrace{11^\circ \tan 31^\circ \tan 45^\circ \tan 59^\circ \tan 79}_{1}-3(\sin^2 21^\circ+\sin^2 69^\circ)$ का मान कितना होगा ?

$\sin 39 \sec 51^\circ + 2\times1\times1\times1-3\times1$

$1+2-3$

$3-3=0$

Q) Find the value of $\sin^2 15^\circ+\sin^2 25^\circ+\sin^2 35^\circ+\sin^2 45^\circ+\sin^2 55^\circ+\sin^2 65^\circ+\sin^2 75^\circ$?

$\sin^2 15^\circ+\sin^2 25^\circ+\sin^2 35^\circ+\sin^2 45^\circ+\sin^2 55^\circ+\sin^2 65^\circ+\sin^2 75^\circ$ का मान ज्ञात कीजिए।

$1+1+1+\left(\frac{1}{\sqrt{2}}\right)^2$

$3+\frac{1}{2}=\frac{7}{2}$

Q) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$ is equal-

$\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$ बराबर है -

$1\times1\times\sqrt{3}$

$=\sqrt{3}$

Q) If $\tan 2\theta \cdot \tan 4\theta = 1$, then what will be the value of $\tan 3\theta$?
यदि $\tan 2\theta \cdot \tan 4\theta = 1$, तो $\tan 3\theta$ का मान क्या होगा ?

$$2\theta + 4\theta = 90^\circ$$
$$6\theta = 90^\circ$$
$$\boxed{\theta = 15^\circ}$$
$$\tan 3\theta = \tan(15 \times 3)$$
$$\tan 45^\circ = 1$$

Q) The value of $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cot 70^\circ \cot 80^\circ$ is -
$\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cdot \cot 80^\circ$ का मान है -

$$1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Q) $\left[\frac{\operatorname{cosec} 31^\circ - \sec 59^\circ}{(\sin 18^\circ \div \cos 72^\circ)} \times \frac{\tan 26^\circ}{\cot 64^\circ}\right] + \left[\frac{\tan 30^\circ + \tan 15^\circ}{(1 - \tan 30^\circ \tan 15^\circ)}\right] = \_\_\_$

$\tan 26 \tan 64 \rightarrow 1$

$\tan(30 + 15^\circ) \rightarrow \tan 45^\circ = 1$

$$\frac{\operatorname{Cosec}(90-59) - \sec 59}{\sin 18 \div \cos 72^\circ}$$
$$\frac{\sec 59 - \sec 59}{\sin 18 \div \cos 72^\circ}$$
$$= 0$$
$$0 \times 1 + 1 = 1$$

Q) If $\sin\alpha \sec(30^\circ + \alpha) = 1 \; (0 < \alpha < 60^\circ)$, then the value of $\sin\alpha + \cos 2\alpha$ is -
यदि $\sin\alpha \sec(30^\circ + \alpha) = 1 \; (0 < \alpha < 60^\circ)$, तो $\sin\alpha + \cos 2\alpha$ का मान है -

$$\alpha + 30 + \alpha = 90^\circ$$
$$2\alpha = 60$$
$$\boxed{\alpha = 30^\circ}$$
$$\sin\alpha + \cos 2\alpha$$
$$\sin 30^\circ + \cos 60^\circ$$
$$\frac{1}{2} + \frac{1}{2} = 1$$

Q) $\sin^2 5° + \sin^2 6° + \ldots + \sin^2 84° + \sin^2 85° = ?$

$$\text{No. of terms} = \frac{\text{last term} - \text{first term}}{\text{common diff.}} + 1$$

$$\frac{85-5}{1} + 1$$

$$80 + 1 = 81$$

$$\frac{\text{No. of terms}}{2} = \frac{81}{2} = 40.5$$

IInd method

$1 + 1 + \ldots$ 40 बार $+ \sin^2 45°$

$$40 + \frac{1}{2}$$

$$40\frac{1}{2}$$

Q) What will be the value of $(\sin^2 1° + \sin^2 3° + \sin^2 5° + \ldots + \sin^2 85° \sin^2 87° + \sin^2 89°)$

$(\sin^2 1° + \sin^2 3° + \sin^2 5° + \ldots + \sin^2 85° + \sin^2 87° + \sin^2 89°)$ का मान कितना होगा?

$$\text{No. of terms} = \frac{89-1}{2} + 1$$

$$\frac{\cancel{88}^{44}}{\cancel{2}} + 1 = 44 + 1 = 45$$

$$= \frac{45}{2} = 22\frac{1}{2}$$

Q) If $\sec(7\theta + 28°) = \operatorname{cosec}(30° - 3\theta)$, then what will be the value of θ?

यदि $\sec(7\theta + 28°) = \operatorname{cosec}(30° - 3\theta)$ हो, तो θ का मान कितना होगा?

if $A + B = 90°$

$\sec A = \operatorname{cosec} B$

$7\theta + 28 + 30 - 3\theta = 90°$

$4\theta = 90 - 58$

$\cancel{4}\theta = \cancel{32}\,8$

$\theta = 8°$

Q) If $0<x<\frac{\pi}{2}$ and $\sec x = \operatorname{cosec} y$ then what will be $\sin(x+y)$?

यदि $0<x<\frac{\pi}{2}$ तथा $\sec x = \operatorname{cosec} y$ हो तो $\sin(x+y)$ कितना होगा?

$\boxed{x+y=90^\circ}$

$\sin(x+y)$

$\sin 90^\circ$

1

Q) The value of $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$ is

$\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$ का मान है –

$\boxed{A+B=90^\circ=\frac{\pi}{2}}$

$\frac{\pi}{20}+\frac{9\pi}{20}=\frac{\cancel{10}\pi}{\cancel{20}_{2}}\left(\frac{\pi}{2}\right)$ $\quad \frac{3\pi}{20}+\frac{7\pi}{20}=\frac{\cancel{10}\pi}{\cancel{20}_{2}}\left(\frac{\pi}{2}\right)$ $\quad \cot\frac{\cancel{5}\pi}{\cancel{20}_{4}}=\left(\frac{\pi}{4}\right)$

$1\times1\times\cot\frac{\pi}{4}$

$1\times1\times1=1$

Q) Consider the following : निम्न पर विचार करें:

1. $\frac{\cos 75^\circ}{\sin 15^\circ}+\frac{\sin 12^\circ}{\cos 78^\circ}-\frac{\cos 18^\circ}{\sin 72^\circ}=1$ $\quad 1+1-1=1$

2. $\frac{\cos 35^\circ}{\sin 55^\circ}-\frac{\sin 11^\circ}{\cos 79^\circ}+\underbrace{\cos 28^\circ \operatorname{cosec} 62^\circ}_{1}=1$ $\quad 1-1+1=1$

3. $\frac{\sin 80^\circ}{\cos 10^\circ}-\underbrace{\sin 59^\circ \sec 31^\circ}_{1}=0$ $\quad 1-1=0$

$\left\{\begin{array}{l} A+B=90^\circ \\ \frac{\sin A}{\cos B}=1 \end{array}\right\}$

Which of the above are correct?

उपरोक्त में से कौन सा सही है?

a) 1 and 2 only

b) 2 and 3 only

c) 1 and 3 only

d) 1, 2 and 3

Q) What is the value of $\sin 24^\circ \sin 66^\circ - \cos 24^\circ \cos 66^\circ + \tan 24^\circ \tan 66^\circ - \cot 24^\circ \cot 66^\circ$?

$\sin 24^\circ \sin 66^\circ - \cos 24^\circ \cos 66^\circ + \underbrace{\tan 24^\circ \tan 66^\circ}_{1} - \underbrace{\cot 24^\circ \cot 66^\circ}_{1}$ का मान क्या है?

$24+66=90^\circ$

$-[\cos 24 \cos 66 - \sin 24 \sin 66] + 1 - 1$

$-[\cos(24+66)] + 0$

$-\cos 90^\circ = 0$

Q) What is the value of $\frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sec^2 50^\circ - \cot^2 40^\circ} + 4\tan 13^\circ \tan 37^\circ \tan 53^\circ \tan 77^\circ$?

$\frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sec^2 50^\circ - \cot^2 40^\circ} + 4 \underbrace{\tan 13^\circ \tan 37^\circ}_{1} \underbrace{\tan 53^\circ \tan 77^\circ}_{1}$ का मान क्या है?

$4 \times 1 \times 1 = 4$

$\frac{1}{\sec^2 50^\circ - \tan^2 50^\circ} + 4$

$\left\{ \begin{array}{l} \cot 40^\circ = \tan 50^\circ \\ \cot(90-50^\circ) = \tan 50^\circ \end{array} \right\}$

$\frac{1}{1} + 4$

$1+4 = 5$

Q) If $\tan\theta \cdot \tan 2\theta = 1$, then the value of $\sin^2 2\theta + \tan^2 2\theta$ is :

यदि $\underbrace{\tan\theta \cdot \tan 2\theta}_{} = 1$, तो $\sin^2 2\theta + \tan^2 2\theta$ का मान है।

$3\theta = 90^\circ$

$\theta = 30^\circ$

$\sin^2 60 + \tan^2 60$

$\left(\frac{\sqrt{3}}{2}\right)^2 + (\sqrt{3})^2$

$\frac{3}{4} + 3$

$\frac{15}{4} = 3\frac{3}{4}$

Q) If θ is a positive acute angle, and $\tan 2\theta \tan 3\theta = 1$, then what is the value of $\left(2\cos^2 \frac{5\theta}{2} - 1\right)$?

यदि θ एक धनात्मक न्यूनकोण हो, और $\underbrace{\tan 2\theta \tan 3\theta}_{} = 1$ हो, तो $\left(2\cos^2 \frac{5\theta}{2} - 1\right)$ का मान कितना होगा?

$5\theta = 90^\circ$

$\theta = 18^\circ$

$$2\cos^2\left(\frac{90}{2}\right)-1$$

$$2\cos^2 45^\circ - 1$$

$$\cancel{2}\times\frac{1}{\cancel{2}}-1=0$$

Q) If $\sin(3x-20^\circ)=\cos(3y+20^\circ)$, then what will be the value of $(x+y)$?

यदि $\sin(3x-20^\circ)=\cos(3y+20^\circ)$ हो, तो $(x+y)$ का मान कितना होगा?

$$3x-\cancel{20}+3y+\cancel{20}=90^\circ$$

$$\cancel{3}(x+y)=\cancel{90^\circ}\ 30^\circ$$

$$x+y=30^\circ$$

Q) What is the value of $(\sin^2 25^\circ+\sin^2 65^\circ)$?

$(\sin^2 25^\circ+\sin^2 65^\circ)$ का मान कितना होगा?

$=1$

Q) What is the numerical value of $\cot 18^\circ\left(\cot 72^\circ\cos^2 22^\circ+\frac{1}{\tan 72^\circ\sec^2 68^\circ}\right)$

$\cot 18^\circ\left(\cot 72^\circ\cos^2 22^\circ+\frac{1}{\tan 72^\circ\sec^2 68^\circ}\right)$ का संख्यात्मक मान कितना है?

$$\cot 18^\circ\left[\cot 72^\circ\cos^2 22^\circ+\cot 72^\circ\cdot\cos^2 68^\circ\right]$$

$$\cot 18^\circ\cdot\cot 72^\circ\left[\cos^2 22+\cos^2 68^\circ\right]$$

$$1\times 1$$

$$1$$

Q) The value of $\left(\sin^2 7\frac{1}{2}^\circ+\sin^2 82\frac{1}{2}^\circ+\tan^2 2^\circ\cdot\tan^2 88^\circ\right)$ is:

$\left(\sin^2 7\frac{1}{2}^\circ+\sin^2 82\frac{1}{2}^\circ+\tan^2 2^\circ\cdot\tan^2 88^\circ\right)$ का मान है;

90° 90° 1^2

$$1+1$$

$$1+1=2$$

Q) What is the value of $\frac{\sin 43^\circ}{\cos 47^\circ}+\frac{\cos 19^\circ}{\sin 71^\circ}-8\cos^2 60^\circ$

$\frac{\sin 43^\circ}{\cos 47^\circ}+\frac{\cos 19^\circ}{\sin 71^\circ}-8\cos^2 60^\circ$ का मान बताइए:

90 90

$$1+1-8\times\left(\frac{1}{2}\right)^2$$

$$2-\overset{2}{\cancel{8}}\times\frac{1}{\cancel{4}}$$

$$2-2=0$$

Q) What is $2\,\text{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$ equal to?

$2\,\text{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$ किसके बराबर है?

$$2\,\text{cosec}^2 23 \cdot \cot^2 67 - \cot^2 67^\circ - (\sin^2 23 + \sin^2 67^\circ)$$

$$2\times \text{cosec}^2 23 \cdot \tan^2 23$$

$$2\times\frac{1}{\cancel{\sin^2 23}}\times\frac{\cancel{\sin^2 23}}{\cos^2 23}$$

$$2\sec^2 23^\circ$$

$$2\sec^2 23 - 1 - \underbrace{\cot^2 67^\circ}_{\tan^2 23^\circ}$$

$$\sec^2 23 + \sec^2 23 - \tan^2 23 - 1$$

$$\sec^2 23 + 1 - 1$$

$$\sec^2 23^\circ$$

1. What is the value of $\sin^2 21 + \sin^2 69$?

$\sin^2 21 + \sin^2 69$ का मान क्या है?

(a) 0
(b) 2
(c) 0.5
(d) 1

2. What is the value of $\frac{\cos 37^\circ}{\sin 53^\circ}$?

$\frac{\cos 37^\circ}{\sin 53^\circ}$ का मान क्या होगा?

(a) 1/2
(b) $1/\sqrt{2}$
(c) 0
(b) 1

3. What is the value of $\frac{4}{3}(\sin^2 35^\circ + \sin^2 55^\circ)$?

$\frac{4}{3}(\sin^2 35^\circ + \sin^2 55^\circ)$ का मान क्या है?

(a) 8/9
(b) 4/3
(c) 3/4
(d) 2/3

4. $\frac{(\cos 9^\circ + \sin 81^\circ)(\sec 9^\circ + \operatorname{cosec} 81^\circ)}{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}$

(a) $1/2$
(b) $4/3$
(c) 2
(d) 1

5. $(\sin 37^\circ \cos 53^\circ + \cos 37^\circ \sin 53^\circ)$ - $\frac{4\cos^2 37^\circ - 7 + 4\cos^2 53^\circ}{\tan^2 47^\circ + 4 - \operatorname{cosec}^2 43^\circ} = ?$

(a) 1
(b) -2
(c) 0
(d) 2

6. $\frac{\tan 13^\circ \tan 36^\circ \tan 45^\circ \tan 54^\circ \tan 77^\circ}{2\sec^2 60^\circ(\sin^2 60^\circ - 3\cos 60^\circ + 2)} = ?$

(a) $-\frac{1}{4}$
(b) $-\frac{1}{10}$
(c) $\frac{1}{10}$
(d) $\frac{1}{4}$

7. The value of $\tan^2 48^\circ - \operatorname{cosec}^2 42^\circ + \operatorname{cosec}(67 + \theta) - \sec(23^\circ - \theta)$ is

$\tan^2 48^\circ - \operatorname{cosec}^2 42^\circ + \operatorname{cosec}(67+\theta) - \sec(23^\circ - \theta)$ का मान ज्ञात कीजिए।

(a) -1
(b) 0
(c) 1
(d) -2

8. If $4(\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ) - \cos 90^\circ + y\tan^2 66^\circ \cdot \tan^2 24^\circ = \frac{y}{2}$ then the value of y is:

यदि $4(\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ) - \cos 90^\circ + y\tan^2 66^\circ \cdot \tan^2 24^\circ = \frac{y}{2}$ है, तो y का मान ज्ञात करें।

(a) 4
(b) -4
(c) 8
(d)-8

9. $\frac{1-\tan A}{1+\tan A} = \frac{\tan 3^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 87^\circ}{\tan 27^\circ \tan 39^\circ \tan 51^\circ \tan 60^\circ \tan 63^\circ}$, then the value of $\cot A$ is:

$\frac{1-\tan A}{1+\tan A} = \frac{\tan 3^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 87^\circ}{\tan 27^\circ \tan 39^\circ \tan 51^\circ \tan 60^\circ \tan 63^\circ}$ है, तो $\cot A$ का मान ज्ञात कीजिए।

(a) 3
(b) 2
(c) 1
(d) 4

10. $\tan(63^\circ - \theta) - \cot(27^\circ + \theta) + \frac{\operatorname{cosec}^2 70 - \tan^2 20}{\sec^2 37 - \cot^2 53} = ?$

(a) 2
(b) 3
(c) 0
(d) 1

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
D	D	B	B	D	C	A	D	B	D

Worksheet solution

Sol 1

$$\sin^2 21 + \sin^2 69$$

$$\sin^2 21 + \sin^2 (90^\circ - 21^\circ)$$

$$\sin^2 21 + \cos^2 21 = 1$$

Sol 2

$$\frac{\cos 37^\circ}{\sin 53^\circ} = \frac{\cos(90^\circ - 37^\circ)}{\sin 53^\circ} = \frac{\sin 53^\circ}{\sin 53^\circ} = 1$$

Sol 3

$$= \frac{4}{3}\left(\sin^2 35 + \underbrace{\sin^2 55}_{\cos^2(90-35)}\right)$$

$$= \frac{4}{3}\left(\sin^2 35 + \cos^2 35\right) = \frac{4}{3}$$

Sol 4

$$= \frac{(\cos 9^\circ + \sin 81^\circ)(\sec 9^\circ + \operatorname{cosec} 81^\circ)}{2\sin^2 63^\circ + 1 + 2\sin^2 27}$$

$$= \frac{[\cos 9^\circ + \sin(90^\circ - 9^\circ)][\sec 9^\circ + \operatorname{cosec}(90^\circ - 9^\circ)}{2\sin^2 63^\circ + 1 + 2\sin^2(90^\circ - 27^\circ)}$$

$$= \frac{(\cos 9^\circ + \cos 9^\circ)(\sec 9^\circ + \sec 9^\circ)}{1 + 2\sin^2 63^\circ + 2\cos 63^\circ}$$

$$= \frac{(2\cos 9^\circ)\cdot(2\sec 9^\circ)}{1 + 2(\sin^2 63^\circ + \cos^2 63^\circ)}$$

$$= \frac{2\cos 9^\circ \times \frac{2}{\cos 9^\circ}}{1+2} = \frac{4}{3}$$

$\boxed{\text{Sol 5}}$ $(\sin 37^\circ \cos 53^\circ + \cos 37^\circ \sin 35^\circ)$

$$- \frac{4\cos^2 37^\circ - 7 + 4\cos^2 53^\circ}{\tan^2 47^\circ + 4 - \operatorname{cosec}^2 43^\circ}$$

$A + B = 90^\circ$

तो $\sin A = \cos B$, $\sec B = \operatorname{cosec} A$

$$= \sin^2 37 + \cos^2 37 - \frac{4(\sin^2 37 + \cos^2 37) - 7}{-(\sec^2 47^\circ - \tan^2 47^\circ) + 4}$$

$$= 1 - \frac{4 - 7}{(-) + 4} = 1 + \frac{3}{3} = 2$$

$\boxed{\text{Sol 6}}$

$$\frac{\tan 13^\circ \cdot \tan 36^\circ \tan 45^\circ \tan 54^\circ \tan 77^\circ}{2\sec^2 60^\circ (\sin^2 60^\circ - 3\cos 60^\circ + 2)}$$

$A + B = 90^\circ$

तो

$\tan A \cdot \tan B = 1$

$$= \frac{\tan 13^\circ \cdot \tan 77^\circ \tan 36^\circ \tan 54^\circ \cdot \tan 45^\circ}{2\sec^2 60^\circ (\sin^2 60^\circ - 3\cos 60^\circ + 2)}$$

$$= \frac{1 \times 1 \times 1}{2 \times (2)^2 \left(\frac{\sqrt{3}}{2}\right)^2 - 3 \times \frac{1}{2} + 2}$$

$$= \frac{1}{2 \times 4\left(\frac{3}{4} - \frac{3}{2} + 2\right)} = \frac{1}{10}$$

Sol 7

$$\tan^2 48^\circ - \operatorname{cosec}^2 42^\circ + \operatorname{cosec}(67+\theta) - \sec(23^\circ-\theta)$$

$$= \tan^2 48^\circ - \operatorname{cosec}^2(90^\circ-48^\circ) + \operatorname{cosec}(90^\circ-(23^\circ-\theta)) - \sec(23^\circ-\theta)$$

$$= \tan^2 48^\circ - \sec^2 48^\circ + \sec(23^\circ-\theta) - \sec(23^\circ-\theta)$$

$$= -(\sec^2 48^\circ - \tan^2 48^\circ) + 0$$

$$= -1$$

Sol 8

$$4(\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ) - \cos 90^\circ + y \tan^2 66^\circ \cdot \tan^2 24^\circ = \frac{y}{2}$$

$$\operatorname{cosec}(90^\circ-\theta) = \sec\theta$$

$$\operatorname{cosec}(90^\circ-33^\circ) = \sec 33^\circ$$

$$4(\sec^2 33^\circ - \tan^2 33^\circ) - \cos 90^\circ + y \tan^2 66^\circ \cdot \tan^2 24 = \frac{y}{2}$$

$$(\because \tan^2 A \cdot \tan^2(90-A) = 1)$$

$$4 - 0 + y = \frac{y}{2}$$

$$y = -8$$

Sol 9

$$\frac{1-\tan A}{1+\tan A} = \frac{\tan 3^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 87^\circ}{\tan 27^\circ \tan 39^\circ \tan 51^\circ \tan 60^\circ \tan 63^\circ}$$

यदि $\tan A \cdot \tan B = 1$

$$A + B = 90^\circ$$

$$\frac{1-\tan A}{1+\tan A} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$\frac{1-\tan A}{1+\tan A} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$$

Apply componendo and dividendo:

$$\tan A = \frac{1}{2} \Rightarrow \cot A = 2$$

$\boxed{\text{Sol 10}}$

$$\tan(63^\circ - \theta) - \cot(27^\circ + \theta) + \frac{\operatorname{cosec}^2 70 - \tan^2 20}{\sec^2 37 - \cot^2 53}$$

$$\tan(90^\circ - \theta) = \cot\theta$$

$$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$$

$$= \tan[90^\circ - (27 + \theta)] - \cot(27^\circ + \theta) + \frac{\operatorname{cosec}^2(90 - 20) - \tan^2 20}{\sec^2(90 - 53) - \cot^2 53}$$

$$= \cot(27^\circ + \theta) - \cot(27^\circ + \theta) + \frac{\sec^2 20 - \tan^2 20}{\operatorname{cosec}^2 53 + \cot^2 53^\circ}$$

$$= 0 + 1 = 1$$

TRIGONOMETRY

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

* ⊕ $\boxed{\sin(A+B) + \sin(A-B) = 2 \sin A \cos B}$

* ⊖ $\boxed{\sin(A+B) - \sin(A-B) = 2 \cos A \sin B}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

* ⊕ $\boxed{\cos(A+B) + \cos(A-B) = 2 \cos A \cos B}$

* ⊖ $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

* $\boxed{+\cos(A-B) - \cos(A+B)] = +2 \sin A \sin B}$

* $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

* $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

* $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

* $\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

(A=B) * $\boxed{\sin 2A} = \sin A \cdot \cos A + \cos A \cdot \sin A$

↳ $2 \sin A \cdot \cos A$

↳ $= \frac{2 \tan A}{1 + \tan^2 A}$

* $\boxed{\cos 2A} = 1 - 2 \sin^2 A$

↳ $= 2 \cos^2 A - 1$

↳ $\cos^2 A - \sin^2 A$

↳ $= \frac{1 - \tan^2 A}{1 + \tan^2 A}$

* $\tan 2A = \left(\frac{2\tan A}{1-\tan^2 A}\right)$

* $\cot 2A = \left(\frac{\cot^2 A - 1}{2\cot A}\right)$

Q) Find $(\alpha+\beta)$, if $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$?

यदि $\tan\alpha = \frac{m}{m+1}$ और $\tan\beta = \frac{1}{2m+1}$ है, तो $(\alpha+\beta)$ ज्ञात कीजिये।

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$\frac{\frac{m}{m+1} + \frac{1}{(2m+1)}}{1-\frac{m}{m+1}\times\frac{1}{2m+1}} = \frac{\frac{2m^2+m+m+1}{\cancel{(m+1)(2m+1)}}}{\frac{(m+1)(2m+1)-m}{\cancel{(m+1)(2m+1)}}}$$

$$\frac{2m^2+2m+1}{2m^2+\cancel{m}+2m+1-\cancel{m}}$$

$$\frac{\cancel{2m^2+2m+1}}{\cancel{2m^2+2m+1}} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\alpha+\beta = 45^\circ$$

$$= \frac{\pi}{4}$$

IInd Method

Put = 1

$\tan\alpha = \frac{1}{2}$

$\tan\beta = \frac{1}{3}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{\cancel{5}}{\cancel{6}}}{\frac{\cancel{5}}{\cancel{6}}} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\alpha+\beta = 45^\circ$$

$$= \frac{\pi}{4}$$

Q) $\cos 18^\circ - \sin 18^\circ$

$\cos 18^\circ - \sin(90-72)$

$\cos 18 - \cos 72^\circ$

$2 \sin \frac{\cancel{90}^{45^\circ}}{\cancel{2}} \sin \frac{54}{2}$

$\left\{ \cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2} \right\}$

$2 \sin 45^\circ \sin 27^\circ$

$2 \times \frac{1}{\sqrt{2}} \times \sin 27$

$= \sqrt{2} \sin 27^\circ$

Q) $\sin 15^\circ + \sin 75^\circ = ?$

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$= 2 \sin 45^\circ \times \cos 30^\circ$

$\cancel{2} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{\cancel{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

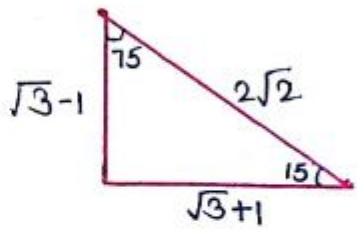
Q) $\sin 15^\circ + \sin 165^\circ = ?$

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$2 \times \sin 90^\circ \cdot \cos 75^\circ$

$2 \cos 75^\circ$

$\cancel{2} \times \frac{(\sqrt{3}-1)}{\cancel{2}\sqrt{2}}$



$\frac{\sqrt{3}-1}{\sqrt{2}} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{3-1}{\sqrt{2}(\sqrt{3}+1)}$

$\frac{2}{\sqrt{2}(\sqrt{3}+1)} = \frac{\sqrt{2}}{\sqrt{3}+1}$

Q) $\cot x\,(\sin 5x - \sin 3x) = ?$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cot x\,(2\cos 4x \cdot \sin x)$$

$$\frac{\cos x}{\cancel{\sin x}} \times 2\cos 4x \cdot \cancel{\sin x}$$

$$= 2\cos x \cdot \cos 4x$$

Q) $\dfrac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = ?$

$$\frac{\cancel{2}\sin 3\theta \cancel{\cos\theta}}{\cancel{2}\cos 3\theta \cancel{\cos\theta}}$$

$$= \tan 3\theta$$

Q) $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = ?$

$$\underbrace{\sin 10 + \sin 50}\ + \underbrace{\sin 20 + \sin 40^\circ}$$

$$2\sin 30^\circ \times \cos 20^\circ + 2\sin 30 \cos 10^\circ$$

$$2\sin 30^\circ\,[\cos 20 + \cos 10]$$

$$\cancel{2} \times \frac{1}{\cancel{2}}$$

$$= 2\cos 15^\circ \cdot \cos 5^\circ$$

Q) $\dfrac{\cos 7A + \cos 11A}{\cos 7A - \cos 11A} = ?$

$$\frac{\cancel{2}\cos 9A \cos 2A}{\cancel{2}\sin 9A \times \sin 2A}$$

$$\frac{\cot 9A}{\left(\dfrac{\sin 2A}{\cos 2A}\right)} = \frac{\cot 9A}{\tan 2A}$$

Q) What is the value of $[(\sin 7x - \sin 5x) \div (\cos 7x + \cos 5x)] - [(\cos 6x - \cos 4x) \div (\sin 6x + \sin 4x)] = ?$

$[(\sin 7x - \sin 5x) \div (\cos 7x + \cos 5x)] - [(\cos 6x - \cos 4x) \div (\sin 6x + \sin 4x)$ का मान क्या है?

$$[(2\cos 6x \cdot \sin x) \div (2\cos 6x \cos x)] - [(2\sin 5x \cdot \sin(-x)) \div (2\sin 5x \cdot \cos x)]$$

$$\left[\frac{\cancel{2\cos 6x} \cdot \sin x}{\cancel{2\cos 6x}\ \cos x}\right] - \left[\frac{\cancel{2\sin 5x}(-\sin x)}{\cancel{2\sin 5x}\ \cos x}\right]$$

$$\tan - (-\tan x)$$

$$\tan x + \tan x$$

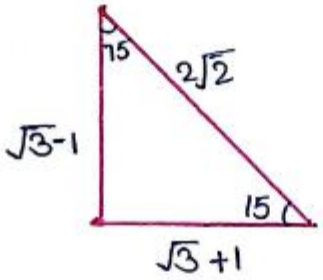
$$2\tan x$$

Q) $\tan 15^\circ - \cot 165^\circ = ?$

$$\tan 15^\circ - \cot(180 - 15)$$

$$\tan 15^\circ - (-\cot 15^\circ)$$

$$\boxed{\tan 15^\circ + \cot 15^\circ}$$



$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{3-1}$$

$$\frac{4 - \cancel{2\sqrt{3}} + 4 + \cancel{2\sqrt{3}}}{2} = \cancel{\frac{8}{2}}\ 4$$

1. $\sin 0^\circ \sin 30^\circ \cdot \sin 45^\circ \sin 60^\circ \sin 90^\circ = ?$
(a) 1
(b) 0
(c) $\frac{\sqrt{6}}{8}$
(d) 4

2. If $\cos 2A = \sin 75^\circ$, then the smallest positive value of A is :
यदि $\cos 2A = \sin 75^\circ$ है, तो A का सबसे छोटा धनात्मक मान क्या होगा?
(a) 15°
(b) 7.5°
(c) 30°
(d) 37.5°

3. If θ lies in the first quadrant and $\cos^2\theta - \sin^2\theta = \frac{1}{2}$, then find the value of $\tan^2 2\theta + \sin^2 3\theta$
यदि θ पहले चतुर्थांश में स्थित है और $\cos^2\theta - \sin^2\theta = \frac{1}{2}$ है, तो $\tan^2 2\theta + \sin^2 3\theta$ का मान ज्ञात करें।
(a) $\frac{7}{2}$
(b) 3
(C) 4
(d) $\frac{4}{3}$

4. The value of $\sin 2\theta(\tan\theta + \cot\theta)$ is :
$\sin 2\theta(\tan\theta + \cot\theta)$ का मान ज्ञात कीजिए।
(a) $\frac{2}{3}$
(b) 1
(c) $\frac{1}{2}$
(d) 2

5. What is the value of $\operatorname{cosec} 15^\circ \sec 15^\circ$?
$\operatorname{cosec} 15^\circ \sec 15^\circ$ का मान क्या है?
(a) 0.5
(b) 4
(c) 2
(d) 1

6. The value of $\frac{\sin 4\theta}{(1-\cos 4\theta)}$ is :
$\frac{\sin 4\theta}{(1-\cos 4\theta)}$ का मान क्या होगा?
(a) $\cot\theta$
(b) $\cot 2\theta$

(c) $\tan\theta$
(d) $\tan 2\theta$

7. $\frac{\sin 4\theta+\sin 2\theta}{\cos 4\theta+\cos 2\theta}=?$
(a) $\tan 3\theta$
(b) $\cot 3\theta$
(c) $\tan\theta$
(d) $\cot\theta$

8. What is the value of $(2\cos^2\theta-1)\left[\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right]$?
$(2\cos^2\theta-1)\left[\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right]$ का मान क्या है?
(a) 2
(b) 0
(c) $\frac{\sqrt{3}}{2}$
(d) 1

9. What is the value of $\sin 73^\circ+\cos 137^\circ$?
$\sin 73^\circ+\cos 137^\circ$ का मान क्या होगा?
(a) $\sin 13^\circ$
(b) $\cos 13^\circ$
(c) $\cos 18^\circ$
(d) $\sin 18^\circ$

10. Find the value of the following.
निम्न का मान ज्ञात कीजिए।
$\frac{\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta}{\cos 2\theta\cos-\sin 3\theta\sin 4\theta}$
(a) $\cot\theta$
(b) $\cot 2\theta$
(c) $\tan\theta$
(d) $\tan 2\theta$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
B	B	C	D	B	B	A	A	A	D

Worksheet solution

sol 1

$\sin 0^\circ \cdot \sin 30^\circ \sin 45^\circ \sin 60^\circ \cdot \sin 90^\circ$

$\sin 0^\circ = 0$

$\therefore$ All expression will become zero.

sol 2

$\cos 2A = \sin 75$

$\cos 2A = \cos 15$

$2A = 15$

$A = 7.5$

sol 3

$\cos^2\theta - \sin^2\theta = \frac{1}{2}$

$\cos 2\theta = \cos 60^\circ$

$2\theta = 60^\circ$

$\theta = 30^\circ$

$\therefore \tan^2 2\theta + \sin^2 3\theta$

$\tan^2 60^\circ + \sin^2 90^\circ$

$(\sqrt{3})^2 + 1 = 4$

sol 4

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

$\therefore \sin 2\theta (\tan\theta + \cot\theta)$

$= 2\sin\theta\cos\theta (\sec\theta \operatorname{cosec}\theta)$

$= 2$

sol 5

$= \operatorname{cosec} 15^\circ \cdot \sec 15^\circ$

$= \frac{1}{\sin 15^\circ} \cdot \frac{1}{\cos 15^\circ}$

इसे अंश और हर में गुणा करने पर,

$$= \frac{2}{2\sin 15^\circ \cos 15^\circ}$$

$$= \frac{2}{\sin 2\times 15^\circ} = \frac{2}{\sin 30^\circ} = 4$$

Sol 6

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{1-(1-2\sin^2\theta)}$$

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{1-1+2\sin^2 2\theta}$$

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{2\sin^2 2\theta} = \cot 2\theta$$

Sol 7

$$\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \frac{2\sin\frac{(4\theta+2\theta)}{2} \cdot \cos\frac{2\theta}{2}}{2\cos 3\theta \cdot \cos\theta}$$

$$= \frac{2\sin 3\theta \cdot \cos\theta}{2\cos 3\theta \cdot \cos\theta}$$

$$= \tan 3\theta$$

Sol 8

$$= (2\cos^2\theta - 1)\left[\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right]$$

$$= (2\cos^2\theta - 1)\left[\frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{(1-\tan\theta)\cdot(1+\tan\theta)}\right]$$

$$= (2\cos^2\theta - 1)\left[\frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{1-\tan^2\theta}\right]$$

$$= (2\cos^2\theta - 1) \times \frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$$

$$= \cos 2\theta \times \frac{2}{\cos 2\theta} = 2$$

$\boxed{\text{sol 9}}$ $\sin 73^\circ + \cos 137^\circ$

$$= \sin 73^\circ + \cos(90+47^\circ)$$

$$= 2\cos\frac{73+47^\circ}{2}\,\sin\frac{73^\circ-47^\circ}{2}$$

$$= 2\cos 60^\circ \cdot \sin\frac{26^\circ}{2}$$

$$= 2\times\frac{1}{2}\cdot\sin 13^\circ = \sin 13^\circ$$

$\boxed{\text{sol 10}}$ $$\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$$

$$= \frac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cdot\cos\theta - 2\sin 3\theta\cdot\sin 4\theta}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta}$$

$$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 7\theta}$$

$$= \frac{2\cos\frac{7\theta+3\theta}{2}\,\sin\frac{7\theta-3\theta}{2}}{2\cos\frac{3\theta+7\theta}{2}\,\cos\frac{3\theta-7\theta}{2}} = \frac{\cos 5\theta\cdot\sin 2\theta}{\cos 5\theta\cdot\cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

TRIGONOMETRY

- $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Halfangle

- $2\cos^2\theta = 1 + \cos 2\theta$

$$\boxed{\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}}$$

- $2\sin^2\theta = 1 - \cos 2\theta$

$$\boxed{\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}}$$

Ex:- $\cos 22\frac{1}{2}^\circ$

$$\cos 22\frac{1}{2}^\circ = \sqrt{\frac{1+\cos 45^\circ}{2}}$$

$$\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

- $\frac{\cos 2\theta}{1} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$\frac{1}{\cos 2\theta} = \frac{1+\tan^2\theta}{1-\tan^2\theta}$$

apply C & D Rule

$$\frac{1+\cos 2\theta}{1-\cos 2\theta} = \frac{1}{\tan^2\theta}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\boxed{\tan\theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}}$$

- $(45+\theta^\circ)$

$$\tan(45^\circ+\theta) = \frac{\tan 45 + \tan\theta}{1-\tan 45 \tan\theta} = \left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$$

$$\Downarrow$$

$$\frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\left\{\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right\}$$

$$\frac{(\cos\theta+\sin\theta)^2}{\cos^2\theta-\sin^2\theta}$$

$$\frac{1+2\sin\theta\cos\theta}{\cos 2\theta}$$

$$\frac{1+\sin 2\theta}{\cos 2\theta}$$

$$\boxed{\sec 2\theta + \tan 2\theta}$$

- $\tan(45+\theta) = \frac{1+\tan\theta}{1-\tan\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \sec 2\theta + \tan 2\theta$

- $\cot(45+\theta) = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \sec 2\theta - \tan 2\theta$

Q) $\tan 54^\circ = ?$

$$\tan(45^\circ+9^\circ)$$

$$\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}$$

Q) Simplify the expression: व्यंजक को सरल करें:

$$\frac{1-\tan^2(22.5^\circ)}{1+\tan^2(22.5^\circ)}$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\frac{1-\tan^2(22.5^\circ)}{1+\tan^2(22.5^\circ)} = \cos 45^\circ$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}$$

Q) Find the value of $\dfrac{\left[1+\tan^2\frac{2\pi-\theta}{2}\right]}{\left[2\cot\frac{\pi-\theta}{2}\right]}$?

$\dfrac{\left[1+\tan^2\frac{2\pi-\theta}{2}\right]}{\left[2\cot\frac{\pi-\theta}{2}\right]}$ का मान ज्ञात कीजिये।

$$\frac{1+\tan^2\left(\pi-\frac{\theta}{2}\right)}{2\cot\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}$$

$$\frac{\sec^2\left(\pi-\frac{\theta}{2}\right)}{2\tan\frac{\theta}{2}}$$

$$=\frac{\sec^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}}=\frac{1}{2\cos^2\frac{\theta}{2}}\times\frac{\cancel{\cos\frac{\theta}{2}}}{\sin\frac{\theta}{2}}$$

$$\frac{1}{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{1}{\sin\theta}=\operatorname{cosec}\theta$$

Q) The value of $\dfrac{\sin 2^\circ}{\sqrt{(2\cos^2 46^\circ-1)^2}}+\dfrac{\sin 4^\circ}{\sqrt{(1-2\sin^2 43^\circ)^2}}$ is

$\dfrac{\sin 2^\circ}{\sqrt{(2\cos^2 46^\circ-1)^2}}+\dfrac{\sin 4^\circ}{\sqrt{(1-2\sin^2 43^\circ)^2}}$ का मान है।

$$\frac{\sin 2^\circ}{\cos 92^\circ}+\frac{\sin 4^\circ}{\cos 86^\circ}$$

$$\Downarrow$$

$$\frac{\sin 2^\circ}{\cos(90+2)}+1$$

$$① \frac{\cancel{\sin 2}}{-\cancel{\sin 2}}+1$$

$$-1+1=0$$

Q) Consider the following निम्नलिखित पर विचार कीजिए :

(1) $\cos^4\theta - \sin^4\theta = \frac{2\tan\theta}{1-\tan^2\theta}, 0<\theta<\frac{\pi}{2}$ $\Rightarrow \underbrace{(\cos^2\theta+\sin^2\theta)}_{1}\underbrace{(\cos^2\theta-\sin^2\theta)}_{\cos 2\theta}$

$\cos 2\theta$

(2) $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}, 0<\theta<\frac{\pi}{2}$

(3) $\underbrace{\cos^2\theta - \sin^2\theta}_{(\cos 2\theta)} = \frac{1-\tan^2\theta}{1+\tan^2\theta}, 0<\theta<\frac{\pi}{2}$

Which of the above equations are identities?
उपर्युक्त समीकरणों में से कौन - सी सर्वसमिकाएँ हैं ?

(a) केवल 1 और 2
(b) केवल 2 और 3
(c) केवल 1 और 3
(d) 1, 2 और 3

Q) $\frac{1}{\underbrace{4}_{2\times 2}}[\sqrt{3}\cos 23^\circ - \sin 23^\circ] = ?$

$\frac{1}{2}\left[\frac{\sqrt{3}}{2}\cos 23^\circ - \frac{1}{2}\sin 23^\circ\right]$

$\frac{1}{2}\left[\cos 30^\circ \cos 23^\circ - \sin 30^\circ \sin 23^\circ\right]$

$\therefore \cos A \cos B - \sin A \sin B = \cos(A+B)$

$\frac{1}{2}\cos(30+23)$

$\frac{1}{2}\cos 53^\circ$

Q) Find the value of A for the given equation.
दिए गए समीकरण के लिए A का मान ज्ञात कीजिए।

$\tan A + \tan 2A + \tan 3A = \tan A \tan 2A \tan 3A$

$\tan A + \tan 2A = -\tan 3A + \tan A \tan 2A \tan 3A$

$\tan A + \tan 2A = -\tan 3A\,[1 - \tan A \tan 2A]$

$\frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A} = -\tan 3A$

$$\tan(A+2A) = -\tan 3A$$
$$\tan 3A = -\tan 3A$$
$$2\tan 3A = 0$$
$$\tan 3A = 0$$

By option

$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}}$

$$\tan \cancel{3} \times \frac{\pi}{\cancel{3}}$$
$$\underline{\tan \pi = 0}$$
$$\tan\left(\cancel{3} \times \frac{2\pi}{\cancel{3}}\right)$$
$$\underline{\tan 2\pi = 0}$$

Q) Find the value of $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$?

$\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ का मान ज्ञात कीजिये।

$$\tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right) = \frac{\tan\frac{2\pi}{5} - \tan\frac{\pi}{15}}{1 + \tan\frac{2\pi}{5}\tan\frac{\pi}{15}}$$

$$\left\{\begin{array}{l} \frac{2\pi}{5} + \frac{\pi}{15} = \frac{6\pi + \pi}{15} = \frac{7\pi}{15} \\ \frac{2\pi}{5} + \frac{\pi}{15} = \frac{6\pi - \pi}{15} = \frac{\cancel{5}\pi}{\cancel{15}_3} = \frac{\pi}{3} \end{array}\right\}$$

$$\tan\frac{\pi}{3} = \sqrt{3}\left(1 + \tan\frac{2\pi}{5}\tan\frac{\pi}{15}\right) = \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$$
$$\sqrt{3} + \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$$

$$\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \sqrt{3}$$

Q) Find the value of $\tan 250°$?

$\tan 250°$ का मान ज्ञात कीजिये।

$$\tan(180+70)$$
$$= \tan 70°$$
$$\tan(50+20) = \frac{\tan 50 + \tan 20}{1 - \tan 50 \tan 20}$$
$$\tan 70°(1 - \tan 50 \cdot \tan 20) = \tan 50 + \tan 20)$$
$$\tan 70° - \underbrace{\tan 70 \tan 50 \tan 20}_{1} = \tan 50 + \tan 20°$$
$$\tan 70 - \tan 50 = \tan 50 + \tan 20$$
$$\tan 70° = 2\tan 50 + \tan 20$$

TRIPLE ANGLE

- $\sin(A+B) = \sin A \cos B + \cos A \cdot \sin B$

$B=2A$ $\quad \sin 3A = \sin A \cos 2A + \cos A \sin 2A$

$\sin A(1-2\sin^2 A) + \cos A \cdot 2\sin A \cdot \cos A$

$\sin A - 2\sin^3 A + (1-\sin^2 A)\cdot 2\sin A$

$\sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A$

$$\boxed{\sin 3A = 3\sin A - 4\sin^3 A}$$

- $\sin 3A = 3\sin A - 4\sin^3 A$

$A=90+A$ $\quad \sin(270+3A^\circ)$

$-\cos 3A = 3\sin(90+A) - 4\sin^3(90+A)$

$3\cos A - 4\cos^3 A$

$$\boxed{\cos 3A = 4\cos^3 A - 3\cos A}$$

- $\tan 3\theta$

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \cdot \tan B}$$

$$\tan(\theta+2\theta) = \frac{\tan\theta + \tan 2\theta}{1-\tan\theta \cdot \tan 2\theta}$$

$\tan(3\theta)$

$$\frac{\tan\theta + \frac{2\tan\theta}{1-\tan^2\theta}}{1-\tan\theta \cdot \frac{2\tan\theta}{1-\tan^2\theta}}$$

$$\frac{\tan\theta - \tan^3\theta + 2\tan\theta}{1-\tan^2\theta - 2\tan^2\theta}$$

$$\boxed{\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}}$$

Q) $\dfrac{3\sin\theta - \sin 3\theta}{3\cos\theta + \cos 3\theta} = ?$

$$\frac{\cancel{3\sin\theta} - (\cancel{3\sin\theta} - 4\sin^3\theta)}{\cancel{3\cos\theta} + 4\cos^3\theta - \cancel{3\cos\theta}}$$

$$\frac{+\cancel{4}\sin^3\theta}{\cancel{4}\cos^3\theta} = \tan^3\theta$$

Ex:- $\sin 40^\circ \sin 20 \sin 80 = \frac{1}{4}\sin 60^\circ$

- $\sin(60^\circ - A)\sin A \sin(60^\circ + A) = \frac{1}{4}\sin 3A$
- $\cos(60^\circ - A)\cos A \cos(60^\circ + A) = \frac{1}{4}\cos 3A$
- $\tan(60^\circ - A)\tan A \tan(60^\circ + A) = \tan 3A$

Q) The value of $(\sin 10^\circ \sin 50^\circ \sin 70^\circ)$ is:

$(\sin 10^\circ \sin 50^\circ \sin 70^\circ)$ का मान है:

$\theta = 10^\circ$

$\sin\theta \cdot \sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$= \frac{1}{4}\sin 30^\circ$

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Q) The value of $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ is:

$\cos 20^\circ \underset{60-20}{\cos 40^\circ} \underset{60+20}{\cos 80^\circ}$ का मान है:

$\frac{1}{4}\cos 3\times 20$

$\frac{1}{4}\cos 60^\circ$

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Q) The value of $(\tan 20^\circ \tan 40^\circ \tan 80^\circ)$ is:

$(\underset{20}{\tan 20^\circ} \underset{60-20}{\tan 40^\circ} \underset{60+20}{\tan 80^\circ})$ का मान है:

$\tan 3\theta$

$\tan 3\times 20$

$\tan 60^\circ = \sqrt{3}$

Q) What is the value of $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \cdots\cdots \cos 177^\circ \cdot \cos 178^\circ \cdot \cos 179^\circ$

$\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \cdots \underset{\cos 90^\circ}{\downarrow} \cdots \cos 177^\circ \cdot \cos 178^\circ \cdot \cos 179^\circ$ का मान क्या है?

$\text{----}\times 0 \times \text{----}$

Ans = 0

Q) $\sin 1^\circ \cdot \sin 2^\circ \cdot \sin 3^\circ \cdots \cdot \sin 180^\circ$ is equal to -

$\sin 1^\circ \cdot \sin 2^\circ \cdot \sin 3^\circ \cdots \cdot \sin 180^\circ$ किसके बराबर है?

$\downarrow$

$\times 0$

Ans = 0

1. $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = ?$

(a) $2\sin 15^\circ \cos 5^\circ$

(b) $2\cos 15^\circ \sin 5^\circ$

(c) $2\sin 15^\circ \sin 5^\circ$

(d) $2\cos 15^\circ \cos 5^\circ$

2. The value of $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$ is:

$\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$ का मान है:

(a) 1

(b) -1

(c) 3

(d) $\frac{1}{\sqrt{3}}$

3. What is the average of the sixty terms given below?

$\cos^2 x, \cos^2 2x, \cos^2 3x, \ldots\ldots, \cos^2 30x, \sin^2 x, \sin^2 2x, \sin^2 3x \ldots \sin^2 30x$

नीचे दिए गए साठ पदों का औसत क्या होगा?

$\cos^2 x, \cos^2 2x, \cos^2 3x, \ldots\ldots, \cos^2 30x, \sin^2 x, \sin^2 2x, \sin^2 3x \ldots \sin^2 30x$

(a) $\cos^2 x$

(b) 0.5

(c) 1

(d) $\cos^2 x \sin^2 x$

4. $\operatorname{cosec}(67^\circ + \theta) - \sec(23^\circ - \theta) + \cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ$

(a) 2

(b) 0

(c) 1

(d) 0.5

5. If $\sin x = \frac{1}{3}$, the value of $\sin 3x$ is:

यदि $\sin x = \frac{1}{3}$, तो $\sin 3x$ का मान है:

(a) $\frac{11}{23}$ (b) $\frac{13}{27}$

(c) $\frac{19}{27}$ (d) $\frac{23}{27}$

6. If $\cos x = \frac{1}{2}$, the value of $\cos 3x$ is:

यदि $\cos x = \frac{1}{2}$, तो $\cos 3x$ का मान है:

(a) $\frac{3}{2}$

(b) $\frac{-1}{2}$

(c) 1

(d) -1

7. If the value of $\left(\frac{1}{\cot 20^\circ}\frac{1}{\cot 40^\circ}\frac{1}{\cot 80^\circ}\right)$ is:

$\left(\frac{1}{\cot 20^\circ}\frac{1}{\cot 40^\circ}\frac{1}{\cot 80^\circ}\right)$ का मान है:

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{\sqrt{3}}{4}$

(c) $\sqrt{3}$

(d) $\frac{1}{\sqrt{3}}$

8. If the value of $\left(\frac{1}{\text{cosec } 10^\circ}\frac{1}{\text{cosec } 50^\circ}\frac{1}{\text{cosec } 70^\circ}\right)$

$\left(\frac{1}{\text{cosec } 10^\circ}\frac{1}{\text{cosec } 50^\circ}\frac{1}{\text{cosec } 70^\circ}\right)$ का मान है:

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{8}$

(d) $\frac{3}{4}$

9. If the value of $\left(\frac{1}{\sec 20^\circ}\frac{1}{\sec 40^\circ}\frac{1}{\sec 80^\circ}\right)$

$\left(\frac{1}{\sec 20^\circ}\frac{1}{\sec 40^\circ}\frac{1}{\sec 80^\circ}\right)$ का मान है:

(a) 1

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{8}$

10. The value of $(\tan 10^\circ \tan 50^\circ \tan 70^\circ)$ is:

$(\tan 10^\circ \tan 50^\circ \tan 70^\circ)$का मान है:

(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{3}{8}$ (d) $\frac{\sqrt{3}}{8}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
D	A	B	D	D	D	C	C	D	B

Worksheet solution

Sol 1

$\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ$

$= (\sin 50^\circ + \sin 10^\circ) + (\sin 40^\circ + \sin 20^\circ)$

$= 2\sin\left(\frac{50^\circ + 10^\circ}{2}\right)\cos\left(\frac{50^\circ - 10^\circ}{2}\right) + 2\sin\left(\frac{40^\circ + 20^\circ}{2}\right) \cdot \cos\left(\frac{40^\circ - 20^\circ}{2}\right)$

$= 2\sin 30^\circ \cdot \cos 20^\circ + 2\sin 30^\circ \cdot \cos 10^\circ$

$= 2\sin 30 [\cos 20^\circ + \cos 10^\circ]$

$= 2\sin 30 \left[2\cos\left(\frac{20^\circ + 10^\circ}{2}\right) \cdot \cos\left(\frac{20^\circ - 10^\circ}{2}\right)\right]$

$= 2 \times \frac{1}{2} [2\cos 15^\circ \cdot \cos 5^\circ] = 2\cos 15^\circ \cos 5^\circ$

Sol 2

$\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$

$= (\tan 5^\circ \cdot \tan 85^\circ)(\tan 25^\circ \cdot \tan 65^\circ) \cdot \tan 45^\circ$

$= (\tan 5^\circ \cdot \cot 5^\circ)(\tan 25^\circ \cdot \cot 25^\circ) \cdot \tan 45^\circ$

$= 1$

Sol 3

$\cos^2 x, \cos^2 2x, \cos^2 3x, \ldots, \cos^2 30x, \sin^2 x, \sin^2 2x, \sin^2 3x \ldots \sin^2 30x$

$= \frac{(\cos^2 x + \sin^2 x) + \ldots + (\cos^2 30x + \sin^2 30x)}{60}$

$= \frac{30}{60} = \frac{1}{2} = 0.5$

$\boxed{\text{sol4}}$ $\operatorname{cosec}(67^\circ+\theta) - \sec(23^\circ-\theta) + \cos 15^\circ \cos 35^\circ$

$\operatorname{cosec} 55^\circ \cdot \cos 60^\circ \cdot \operatorname{cosec} 75^\circ$

$\theta = 0^\circ$ रखने पर

$= \operatorname{cosec} 67^\circ - \sec(90^\circ - 67^\circ) + \cos 15^\circ \cos 35^\circ \cdot \frac{1}{\sin 55^\circ}$

$\times \cos 60^\circ \times \frac{1}{\sin 75^\circ}$

$= \operatorname{cosec} 67^\circ - \operatorname{cosec} 67^\circ + \cos 15^\circ \cdot \cos 35^\circ \frac{1}{\cos 35^\circ}$

$\times \frac{1}{2} \times \frac{1}{\cos 15^\circ}$

$= \frac{1}{2} = 0.5$

$\boxed{\text{sol5}}$ $\sin 3x = 3\sin x - 4\sin^3 x$

$= 3 \times \frac{1}{3} - 4\left(\frac{1}{3}\right)^3$

$= 1 - \frac{4}{27} = \frac{23}{27}$

$\boxed{\text{sol6}}$ $\cos 3x = 4\cos^3 x - 3\cos x$

$= 4 \times \left(\frac{1}{2}\right)^3 - 3 \times \frac{1}{2} = \left(\frac{1}{2} - \frac{3}{2}\right) = -1$

$\boxed{\text{sol7}}$ $\frac{1}{\cot 20^\circ} \cdot \frac{1}{\cot 40^\circ} \cdot \frac{1}{\cot 80^\circ}$

$= \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 80^\circ$

$= \tan 20^\circ \cdot \tan(60^\circ - 20^\circ) \cdot \tan(60^\circ + 20^\circ)$

$= \tan 3\theta = \tan 3 \times 20^\circ = \tan 60 = \sqrt{3}$

$\boxed{\text{sol 8}}$

$$\frac{1}{\operatorname{cosec} 10^\circ} \cdot \frac{1}{\operatorname{cosec} 50^\circ} \cdot \frac{1}{\operatorname{cosec} 70^\circ}$$

$$= \sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ$$

$$= \sin 10^\circ \cdot \sin(60^\circ - 10^\circ) \cdot \sin(60^\circ + 10^\circ)$$

$$= \frac{1}{4} \sin 3A$$

$$= \frac{1}{4} \times \sin 3 \times 10^\circ = \frac{1}{4} \sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$\boxed{\text{sol 9}}$

$$\frac{1}{\sec 20^\circ} \cdot \frac{1}{\sec 40^\circ} \cdot \frac{1}{\sec 80^\circ}$$

$$= \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

$$= \cos 20^\circ \cdot \cos(60^\circ - 20^\circ) \cos(60^\circ + 20^\circ)$$

$$= \frac{1}{4} \cos 3A$$

$$= \frac{1}{4} \cos 3 \times 20^\circ = \frac{1}{4} \cos 60^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$\boxed{\text{Sol 10}}$

$$\tan 10^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ$$

$$= \tan 10^\circ \cdot \tan(60 - 10^\circ) \cdot \tan(60 + 10^\circ)$$

$$= \tan 3A$$

$$= \tan 3 \times 10^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

TRIGONOMETRY

Q) Find: $\tan\left(\frac{5\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$

ज्ञात कीजिये: $\tan\left(\frac{5\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$?

$$\tan\left(\pi+\left(\frac{\pi}{4}+\theta\right)\right)-\tan\left(\frac{\pi}{4}-\theta\right)$$

$$\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$$

$$\sec 2\theta+\tan 2\theta-(\sec 2\theta-\tan 2\theta)$$

$$\cancel{\sec 2\theta}+\tan 2\theta-\cancel{\sec 2\theta}+\tan 2\theta$$

$$2\tan 2\theta$$

Q) If $A+B+C=\pi$, then the value of $\tan(A+B)+\tan C$ is:

यदि $A+B+C=\pi$, तो $\tan(A+B)+\tan C$ का मान है:

$$C=\pi-(A+B)$$

$$\tan C=\tan(\pi-(A+B))$$

$$\tan C=-\tan(A+B)$$

$$\tan C+\tan(A+B)=0$$

Q) If $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, the $(A+B+C)$ will be equal to:

यदि $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, तो $(A+B+C)$ बराबर होगा:

$$A+B+C=180^\circ/\pi$$

$$\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$$

$\boxed{A+B+C=180^\circ}$

$$A+B=180-C$$

$$\tan(A+B)=\tan(180-C)$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$$

$$\boxed{\tan A+\tan B+\tan C=\tan A\tan B\tan C}$$

$\boxed{A = B + C}$

$\tan A = \tan(B+C)$

$\downarrow$

$$\tan A = \frac{\tan B + \tan C}{1 - \tan B \tan C}$$

$\tan A - \tan A \tan B \tan C = \tan B + \tan C$

$$\boxed{\tan A - \tan B - \tan C = \tan A \tan B \tan C}$$

Ex :- $\tan 80^\circ - \tan 70^\circ - \tan 10 = ?$

$= \tan 80 \tan 70 \tan 10$

Q) If $\tan A - \tan B - \tan C = \tan A \tan B \tan C$, what is the value of A in terms of B and C?

यदि $\tan A - \tan B - \tan C = \tan A \tan B \tan C$ है, तो B और C के संदर्भ मे A का मान क्या है?

$A = B + C$

Q) $\tan 13x - \tan 9x - \tan 4x =$

$\tan 13x \cdot \tan 9x \cdot \tan 4x$

$\boxed{\begin{array}{l} a \sin x + b \cos x = m \\ a \cos x - b \sin x = n \end{array}}$

$$(a^2+b^2)\underbrace{(\sin^2 x + \cos^2 x)}_{1} = m^2 + n^2$$

$$\boxed{a^2 + b^2 = m^2 + n^2}$$

$a \sec x - b \tan x = m$

$a \tan x - b \sec x = n$

$$\boxed{a^2 - b^2 = m^2 - n^2}$$

$a \operatorname{cosec} x - b \cot x = m$

$a \cot x - b \operatorname{cosec} x = n$

$$\boxed{a^2 - b^2 = m^2 - n^2}$$

Q) If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then the value of x^2+y^2 is:

यदि $x = a\cos\theta + b\sin\theta$ तथा $y = a\sin\theta - b\cos\theta$ है, तो x^2+y^2 का मान ज्ञात करें।

$$a^2+b^2 = x^2+y^2$$

Q) यदि $x = m\sec A + n\tan A$ और $y = m\tan A + n\sec A$ है, तो x^2-y^2 का मान क्या है?

If $x = m\sec A + n\tan A$ and $y = m\tan A + n\sec A$, then what is x^2-y^2 equal to

$$m\sec A + n\tan A = x$$
$$m\tan A + n\sec A = y$$
$$m\sec A - (-n)\tan A = x$$
$$m\tan A - (-n)\sec A = y$$
$$m^2 - (-n)^2 = x^2 - y^2$$
$$m^2 - n^2 = x^2 - y^2$$

Q) If $a\cot\theta + b\,\mathrm{cosec}\,\theta = p$ and $b\cot\theta + a\,\mathrm{cosec}\,\theta = q$ then p^2-q^2 is equal to:

यदि $a\cot\theta + b\,\mathrm{cosec}\,\theta = p$ और $b\cot\theta + a\,\mathrm{cosec}\,\theta = q$ है, तो p^2-q^2 निम्नलिखित में से किसके बराबर होगा?

$$b\,\mathrm{cosec}\,\theta + a\cot\theta = p$$
$$b\cot\theta + a\,\mathrm{cosec}\,\theta = q$$
$$b\,\mathrm{cosec}\,\theta - (-a)\cot\theta = p$$
$$b\cot\theta - (-a)\,\mathrm{cosec}\,\theta = q$$
$$b^2 - a^2 = p^2 - q^2$$

Q) If $3\sin\theta + 5\cos\theta = 5$, then what is the value of $5\sin\theta - 3\cos\theta$?

यदि $3\sin\theta + 5\cos\theta = 5$, तो $5\sin\theta - 3\cos\theta$ का मान क्या है?

$3\cos\theta - 5\sin\theta = n$

$3^2 + \cancel{5^2} = \cancel{5^2} + n^2$

$\boxed{n = \pm 3}$

$5\sin\theta - 3\cos\theta$

$-(3\cos\theta - \sin\theta)$

-3

Q) If $\sin x + \cos x = \sqrt{2}\sin x$, then the value of $\sin x - \cos x$.

यदि $1\sin x + 1\cos x = \sqrt{2}\sin x$ है, तो $\sin x - \cos x$ का मान ज्ञात करें।

$1\sin x + 1\cos x = \sqrt{2}\sin x$

$1\cos x - 1\sin x = n$

$1^2 + 1^2 = (\sqrt{2}\sin x)^2 + n^2$

$2 - 2\sin^2 x = n^2$

$2(1 - \sin^2 x) = n^2$

$2\cos^2 x = n^2$

$\boxed{n = \sqrt{2}\cos x}$

$\sin x - \cos x$

$-(\cos x - \sin x)$

$-(\sqrt{2}\cos x)$

$-\sqrt{2}\cos x$

Q) If $\cos x + \sin x = \sqrt{2}\cos x$, what will be the value of $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$?

यदि $\cos x + \sin x = \sqrt{2}\cos x$ है, तो $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$ का मान क्या होगा?

$\sin x - \cos x = n$

$1^2 + 1^2 = (\sqrt{2}\cos x)^2 + n^2$

$2 - 2\cos^2 x = n^2$

$2(1 - \cos^2 x)$

$2\sin^2 x = n^2$

$\boxed{n = \sqrt{2}\sin x}$

$(\cos x - \sin x)^2 + (\cos x + \sin x)^2$

$(\sqrt{2}\sin x)^2 + (\sqrt{2}\cos x)^2$

$2\sin^2 x + 2\cos^2 x$

$2\underbrace{(\sin^2 x + \cos^2 x)}_{1}$

$2 \times 1 = 2$

Q) If $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$, Find the value of $(\cos\theta - \sin\theta)$.

यदि $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$ है, तो $(\cos\theta - \sin\theta)$ का मान ज्ञात कीजिए।

$$\cos\theta - \sin\theta = n$$

$$1^2 + 1^2 = \left(\frac{\sqrt{11}}{3}\right)^2 + n^2$$

$$2 = \frac{11}{9} = n^2$$

$$n^2 = 2 - \frac{11}{9} = \frac{7}{9}$$

$$n = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$$

Q) If $\sin\theta + \cos\theta = \frac{1}{29}$, find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.

यदि $\sin\theta + \cos\theta = \frac{1}{29}$ है, तो $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$ का मान ज्ञात कीजिए।

$$\cos\theta + \sin\theta = \frac{1}{29}$$

$$\sin\theta - \cos\theta = n$$

$$1^2 + 1^2 = \left(\frac{1}{29}\right)^2 + n^2$$

$$n^2 = \frac{1681}{841}$$

$$n = \sqrt{\frac{1681}{841}} = \frac{41}{29} \Rightarrow \sin\theta - \cos\theta$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$\frac{\frac{1}{\cancel{29}}}{\frac{41}{\cancel{29}}} = \frac{1}{41}$$

Q) If $\sin\theta - \cos\theta = \frac{7}{17}$, then find the value of $\sin\theta + \cos\theta$

यदि $\sin\theta - \cos\theta = \frac{7}{17}$ है, तो $\sin\theta + \cos\theta$ का मान ज्ञात कीजिए।

$$\cos\theta + \sin\theta = n$$

$$1^2 + 1^2 = \left(\frac{7}{17}\right)^2 + n^2$$

$$2 = \frac{49}{289} + n^2$$

$$n^2 = 2 - \frac{49}{289}$$

$$\frac{578-49}{289} = \frac{529}{289}$$

$$n = \sqrt{\frac{529}{289}} = \frac{23}{17}$$

Q) If $3\sin x + 4\cos x = 2$, then find the value of $3\cos x - 4\sin x$ is:

यदि $3\sin x + 4\cos x = 2$ तो $3\cos x - 4\sin x$ का मान ज्ञात करे।

$$3\cos x - 4\sin x = n$$

$$3^2 + 4^2 = 2^2 + n^2$$

$$9 + 16 = 4 + n^2$$

$$21 = n^2$$

$$n = \sqrt{21}$$

* Questions Based On Some Other Basic Identities like:-

(i) $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

$$= \sec A + \tan A$$

$$= \frac{1 + \sin A}{\cos A}$$

(ii) $\dfrac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \operatorname{cosec} A - \cot A$

$$= \frac{1 - \cos A}{\sin A}$$

(iii) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$$= \frac{1 + \cos A}{\sin A}$$

Q) If $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = p\sec\theta + q\tan\theta$ where $0 < \theta < \frac{\pi}{2}$ then what is the value of $p+q$?

यदि $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = p\sec\theta + q\tan\theta$ जहाँ $0 < \theta < \frac{\pi}{2}$ है तो $p+q$ का मान क्या है?

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$\sec\theta + \tan\theta$$

$$p = 1 \quad q = 1$$

$$p + q = 2$$

1. If $A+B+C=180^\circ$ and $\tan A=\frac{1}{2}$, $\tan B=3$, then what will be the value of $\tan C$?

यदि $A+B+C=180^\circ$ और $\tan A=\frac{1}{2}$, $\tan B=3$, तो $\tan C$ का मान क्या होगा?

(A) $-\frac{5}{3}$
(B) $\frac{5}{3}$
(C) $\frac{7}{2}$
(D) 7

2. If $\tan A=\frac{2}{3}$, $\tan B=\frac{3}{4}$, and $\tan C=k$, and $A+B+C=90^\circ$, then what will be the value of k?

यदि $\tan A=\frac{2}{3}$, $\tan B=\frac{3}{4}$, और $\tan C=k$, तथा $A+B+C=90^\circ$, तो k का मान क्या होगा?

(A) $17/6$
(B) $15/6$
(C) $12/5$
(D) $19/6$

3. If $\cot\theta+\cos\theta=x$ & $\cot\theta-\cos\theta=y$, then the value of (x^2-y^2) is:
यदि $\cot\theta+\cos\theta=x$ और $\cot\theta-\cos\theta=y$, तो (x^2-y^2) का मान है:

(a) $4\sqrt{xy}$ **(b)** $2xy$
(c) $2\sqrt{xy}$ **(d)** $4xy$

4. If $\tan x+\sin x=m$ and $\tan x-\sin x=n$, then (m^2-n^2) is equal to:

यदि $\tan x+\sin x=m$ और $\tan x-\sin x=n$, तो (m^2-n^2) बराबर है:

(a) $4\sqrt{mn}$ **(b)** $\sqrt{mn}$
(c) $2\sqrt{mn}$ **(d)** $4mn$

5. If $3\sin x + 5\cos x = 5$, then the value of $(3\cos x - 5\sin x)^2$ is:
यदि $3\sin x + 5\cos x = 5$, तो $(3\cos x - 5\sin x)^2$ का मान है:

(a) 2
(b) 4
(c) 5
(d) 9

6. If $a\cot\theta + b\operatorname{cosec}\theta = x^2$ and $b\cot\theta + a\operatorname{cosec}\theta = y^2$, then the value of $(x^4 - y^4)$ is:
यदि $a\cot\theta + b\operatorname{cosec}\theta = x^2$ और $b\cot\theta + a\operatorname{cosec}\theta = y^2$ है, तो $(x^4 - y^4)$ का मान है:

(a) $a^2 + b^2$
(b) $a^2 - b^2$
(c) $b^2 - a^2$
(d) $(a - b)^2$

7. If a and b are real numbers such that $a\cos\theta + b\sin\theta = 4$ and $a\sin\theta - b\cos\theta = 3$, then $(a^2 + b^2)$ is:

यदि a और b वास्तविक संख्याएँ हैं जैसे कि $a\cos\theta + b\sin\theta = 4$ और $a\cos\theta + b\sin\theta = 4$, तो $(a^2 + b^2)$ है:

(a) 7
(b) 12
(c) 25
(d) $\sqrt{12}$

8. The value of $\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is :

$\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ **का मान ज्ञात कीजिए।**

(a) 1
(b) -1
(c) 2
(d) -2

9. Find the value $\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1}\times\frac{\tan^2\theta(\operatorname{cosec}^2\theta-1)}{\sec\theta-\tan\theta}$

$\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1}\times\frac{\tan^2\theta(\operatorname{cosec}^2\theta-1)}{\sec\theta-\tan\theta}$ का मान है।

(a) 1
(b) 3
(c) 2
(d) 4

10. What is $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$ **equal to ?**

$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$ किसके बराबर है?

(a) 0
(b) 1
(c) $2\sin q$
(d) $2\cos q$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
D	A	A	A	D	C	C	A	A	A

Worksheet solution

Sol 1

$A+B+C=180^\circ$

$\tan A = \frac{1}{2}, \quad \tan B = 3$

$\tan(A+B) = -\tan C$

$$\frac{\tan A + \tan B}{1-\tan A \tan B} = -\tan C$$

$$\frac{\frac{1}{2}+3}{1-\frac{1}{2}\times 3} = -\tan C$$

$$-\tan C = \frac{7}{2}\times\frac{2}{-1}$$

$$\tan C = 7$$

Sol 2

$A+B+C=90^\circ$

$\tan A = \frac{2}{3}, \quad \tan B = \frac{3}{4}, \quad \tan C = K$

$\tan(A+B) = \tan C$

$$\frac{\tan A + \tan B}{1-\tan A \tan B} = K$$

$$\frac{\frac{2}{3}+\frac{3}{4}}{1-\frac{2}{3}\times\frac{3}{4}} = K$$

$$= \frac{\frac{8+9}{12}}{\frac{6}{12}} = \frac{17}{6}$$

Sol 3

$x^2-y^2 = (\cot\theta+\cos\theta)^2 \cdot (\cot\theta-\cos\theta)^2$

$= 4\cot\theta\cos\theta$

$= \frac{4\cos^2\theta}{\sin\theta}$

$xy = (\cot\theta+\cos\theta)(\cot\theta-\cos\theta)$

$= \cos^2\theta\cdot\left(\frac{1}{\sin^2\theta}-1\right)$

$= \cos^2\theta\cdot\frac{(1-\sin^2\theta)}{\sin^2\theta}$

$= \frac{\cos^4\theta}{\sin\theta}$

$\therefore \frac{\cos^2\theta}{\sin\theta} = \sqrt{xy}$

$(x^2-y^2) = 4\sqrt{xy}$

Sol 4

$$m^2 - n^2 = (\tan x + \sin x)^2 - (\tan x - \sin x)^2$$

$$= 4\tan x \sin x = \frac{4\sin^2 x}{\cos x}$$

$$mn = (\tan x + \sin x)(\tan - \sin x) = (\tan^2 x - \sin^2 x)$$

$$= \sin^2 x \cdot \left[\frac{1}{\sin^2 x} - 1\right]$$

$$= \frac{\sin^2 x \cdot (1 - \cos^2 x)}{\cos^2 x} = \frac{\sin^4 x}{\cos^2 x}$$

$$\therefore \frac{\sin^2 x}{\cos x} = \sqrt{mn}$$

$$(m^2 - n^2) = 4\sqrt{mn}$$

Sol 5

$$3\sin x + 5\cos x = 5 \quad \text{---(i)}$$

$$3\cos x - 5\sin x = a \quad \text{---(ii)}$$

squaring (i) and (ii) and adding

$$9(\sin^2 x + \cos^2 x) + 25(\cos^2 x + \sin x) = 25 + a^2$$

$$\therefore a^2 = 9$$

Alternative Method

$$a\sin x + b\cos x = m$$

$$a\cos x - b\sin x = n$$

$$a^2 + b^2 = m^2 + n^2$$

$\therefore \quad (3)^2 + (5)^2 = (5)^2 + a^2$

$a^2 = 9$

Sol 6 squaring and subtracting the given equation-

$(x^4 - y^4) = a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\operatorname{cosec}^2\theta + \cot^2\theta)$

$= a^2[\operatorname{cosec}^2\theta - 1 - \operatorname{cosec}^2\theta] + b^2[1 + \cot^2\theta - \cot^2\theta]$

$\therefore \quad x^4 - y^4 = b^2 - a^2$

Sol 7 According to Property -

$a^2 + b^2 = 3^2 + 4^2$

$a^2 + b^2 = 25$

Sol 8

$$\frac{\sin\theta + \cos\theta - 1}{\sin\theta - \cos\theta + 1} = \frac{\cos\theta}{1 + \sin\theta}$$

eqn मे मान रखने पर

$$\frac{\cos\theta}{1+\sin\theta} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$

$$\frac{\cos\theta}{1+\sin\theta} \times \frac{1+\sin\theta}{\cos\theta} = 1$$

Sol 9

$$\frac{\sin\theta + \cos\theta - 1}{\sin\theta - \cos\theta + 1} = \frac{\cos\theta}{1+\sin\theta}$$

equ में मान रखने पर –

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{\tan^2\theta(\operatorname{cosec}^2\theta - 1)}{\sec\theta - \tan\theta}$$

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{\tan^2\theta \times \cot^2\theta}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta + \tan\theta} \times \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec^2\theta - \tan^2\theta} = 1$$

Sol 10

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1+\sin\theta}{\cos\theta}$$

equation में मान रखने पर

$$= \frac{1+\sin\theta}{\cos\theta} - \frac{\sin\theta + 1}{\cos\theta}$$

$$= \sec\theta + \tan\theta - (\tan\theta + \sec\theta)$$

$$= \sec\theta + \tan\theta - \sec\theta - \tan\theta$$

$$= 0$$

TRIGONOMETRY

MINIMA & MAXIMA

	Min.	Max.
$\sin\theta$	-1	$+1$
$\cos\theta$	-1	$+1$
$\tan\theta$	$-\infty$	$+\infty$
$\cot\theta$	$-\infty$	$+\infty$
$\sec\theta$	$-\infty$	$+\infty$ (except -1 to $+1$)
$\operatorname{cosec}\theta$	$-\infty$	$+\infty$ (except -1 to $+1$)
$\sin^2\theta$	0	$+1$
$\cos^2\theta$	0	$+1$
$\tan^2\theta$	0	∞
$\cot^2\theta$	0	∞
$\sec^2\theta$	1	∞
$\operatorname{cosec}^2\theta$	1	∞

Q) $4 + \sin\theta < \begin{matrix} -1 \\ +1 \end{matrix}$

min. = ?

max. = ?

$4 - 1 = 3 \rightarrow$ min.

$4 + 1 = 5 \rightarrow$ max.

Q) $8 - \cos\theta < \begin{matrix} -1 \\ +1 \end{matrix}$

min. = ?

max. = ?

$8 - (-1) = 9 \rightarrow$ max.

$8 - (1) = 7 \rightarrow$ min.

Q) $4 + \sin^2\theta$ $\quad$ max = ?

$0 \quad 1$ $\quad$ min = ?

$4 + 0 = 4 \rightarrow$ min

$4 + 1 = 5 \rightarrow$ max.

Q) $7 + \tan^2\theta$

$0 \quad \infty$

min = ?

max = ?

$7 + 0 = 7 \rightarrow$ min

$7 + \infty = \infty \rightarrow$ max

$\boxed{x = \pm a \sin\theta \pm b\cos\theta}$

min. $-\sqrt{a^2+b^2}$

max. $+\sqrt{a^2+b^2}$

Q) Find the min. and max. value of the following expressions.
निम्नलिखित व्यंजकों का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

$7\sin\theta - 24\cos\theta$

min = $-\sqrt{7^2+24^2} = -\sqrt{625} = -25$

max. = $+\sqrt{7^2+24^2} = +25$

Q) The maximum value of $(2\sin\theta + 3\cos\theta)$ is :
$(2\sin\theta + 3\cos\theta)$ का अधिकतम मान ज्ञात कीजिए :

$+\sqrt{2^2+3^2}$

$+\sqrt{4+9}$

$= +\sqrt{13}$

Q) What is the maximum value of $7\cos A + 24\sin A + 32$
$7\cos A + 24\sin A + 32$ का अधिकतम मान क्या है ?

$\sqrt{7^2+24^2} + 32$

$+25 + 32$

57

• Min. value. $-25 + 32$

$= +7$

Q) Find the maximum value of $19\sin\theta + 6\cot\theta\sin\theta$
$19\sin\theta + 6\cot\theta\sin\theta$ का अधिकतम मान ज्ञात कीजिए।

$$19\sin\theta + 6 \times \frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta}$$

$$19\sin\theta + 6\cos\theta$$

$$\text{max.} = +\sqrt{19^2+6^2}$$

$$\sqrt{361+36}$$

$$\sqrt{397}$$

Q) Find the min. and max. value of $5\sin\theta + 12\cos\theta - 3$.

$5\sin\theta + 12\cos\theta - 3$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

$$\pm\sqrt{5^2+12^2}$$

$$\pm\sqrt{169}$$

$$\pm 13 - 3$$

$13 - 3$ → Max → 10

$-13 - 3$ → -16 → Min

Q) Find max & min value $8^{\sin\theta} \cdot 16^{\cos\theta}$

$8^{\sin\theta} \cdot 16^{\cos\theta}$ का अधिकतम और न्यूनतम मान ज्ञात करें:

$$(2^3)^{\sin\theta} \cdot (2^4)^{\cos\theta}$$

$$2^{3\sin\theta} \cdot 2^{4\cos\theta}$$

$$= 2^{3\sin\theta + 4\cos\theta}$$

$$2^{\pm\sqrt{3^2+4^2}} = 2^{\pm\sqrt{25}} = 2^{\pm 5}$$

2^5 → 32

2^{-5} → $\frac{1}{32}$

$\left\{2^{-5} = \frac{1}{2^5}\right\}$

Q) Find max & min value of $x^{12\sin\theta} \cdot x^{5\cos\theta}$

$x^{12\sin\theta} \cdot x^{5\cos\theta}$ का अधिकतम और न्यूनतम मान ज्ञात कीजिए।

$$x^{12\sin\theta + 5\cos\theta}$$

$$x^{\pm\sqrt{12^2+5^2}}$$

$$x^{\pm\sqrt{169}} = x^{\pm 13}$$

$$x^{13} \quad x^{-13}$$

Q) Find minimum and maximum value of $16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$

$16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$ का न्यूनतम और अधिकतम मान ज्ञात करें

$$(2^4)^{\sin\theta} \cdot (2^2)^{\sin\theta} \cdot 2^{\cos\theta} \cdot (2^7)^{\cos\theta}$$

$$2^{4\sin\theta} \cdot 2^{2\sin\theta} \cdot 2^{\cos\theta} \cdot 2^{7\cos\theta}$$

$$2^{6\sin\theta + 8\cos\theta}$$

$$2^{\pm\sqrt{6^2+8^2}}$$

$$2^{\pm\sqrt{100}} = 2^{\pm 10}$$

$$2^{-10} \quad 2^{+10}$$

Q) Find max. & min value of $\sin\theta + \cos(\theta + 30^\circ)$

$\sin\theta + \cos(\theta + 30^\circ)$ का अधिकतम और न्यूनतम मान ज्ञात कीजिए।

$$\sin\theta + \cos\theta \cdot \cos 30 - \sin\theta \cdot \sin 30^\circ$$

$$\sin\theta + \cos\theta \times \frac{\sqrt{3}}{2} - \sin\theta \times \frac{1}{2}$$

$$\left(1 - \frac{1}{2}\right)\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

$$\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

$$\pm\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\pm\sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$\pm\sqrt{\frac{\cancel{4}}{\cancel{4}}}\,1 \Rightarrow \pm 1$$

$$-1 \quad +1$$

Q) Find max & min value of $\cos\theta + \sin(\theta + 30^\circ)$.

$\cos\theta + \sin(\theta+30^\circ)$ का अधिकतम और न्यूनतम मान ज्ञात करें।

$$\cos\theta + \sin\theta \cdot \cos 30 + \cos\theta \cdot \sin 30^\circ$$

$$\cos\theta + \sin\theta \times \frac{\sqrt{3}}{2} + \cos\theta \times \frac{1}{2}$$

$$\left(1+\frac{1}{2}\right)\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

$$\frac{3}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

$$\pm\sqrt{\frac{9}{4}+\frac{3}{4}}$$

$$\pm\sqrt{\frac{12}{4}} \quad 3 \qquad \pm\sqrt{3}$$

$$\sqrt{3} \qquad -\sqrt{3}$$

* $-1 \leq \sin\theta \leq 1$

$$\downarrow$$

$$-1 < \sin n\theta \leq +1$$

$$\downarrow$$

$$-1 \leq \sin 2\theta \leq +1$$

Q) Find maximum, minimum value of $15\sin\left(\frac{\theta+\pi}{4}\right) + 20\cos\left(\frac{\theta+\pi}{4}\right)$.

$15\sin\left(\frac{\theta+\pi}{4}\right) + 20\cos\left(\frac{\theta+\pi}{4}\right)$ का अधिकतम, न्यूनतम मान ज्ञात कीजिए।

$$\frac{\theta+\pi}{4} = \alpha$$

$$15\sin\alpha + 20\cos\alpha$$

$$\pm\sqrt{15^2+20^2}$$

$$\pm\sqrt{625} = \pm 25$$

$$25 \qquad -25$$

$\boxed{a\sin^2\theta + b\cos^2\theta}$

If $a > b$

max = a

min = b

If $b > a$

max = b

min = a

Ex:- $10\sin^2\theta + 12\cos^2\theta$

max = 12

min = 10

Ex:- $8\sin^2\theta - 13\cos^2\theta$

$8\sin^2\theta + (-13)\cos^2\theta$

max = 8

min = −13

Q) Find minimum and maximum value of $Y = 3\sin^2\theta + 4\cos^2\theta$.

$Y = 3\sin^2\theta + 4\cos^2\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

$3\sin^2\theta + 3\cos^2\theta + \cos^2\theta$

$3(\underbrace{\sin^2\theta + \cos^2\theta}_{1}) + \cos^2\theta$

$3 + \cos^2\theta$

0 +1

3+0=3 3+1=4

Q) Find minimum and maximum value of $3\sin^2\theta - 5\cos^2\theta$.

$3\sin^2\theta - 5\cos^2\theta$ का न्यूनतम व अधिकतम मान ज्ञात कीजिए।

$3\sin^2\theta + (-5)\cos^2\theta$

max = +3

min = −5

Q) Find the max. & min value of $4^{\sin^2\theta} \times 8^{\cos^2\theta}$

$4^{\sin^2\theta} \times 8^{\cos^2\theta}$ का अधिकतम और न्यूनतम मान ज्ञात कीजिए।

$$(2^2)^{\sin^2\theta} \times (2^3)^{\cos^2\theta}$$

$$2^{2\sin^2\theta + 3\cos^2\theta}$$

$$2^3 \qquad 2^2$$

$$8 \qquad 4$$

Q) Find max & min value of $8^{\sin^2\theta}\, 16^{\cos^2\theta}$

$8^{\sin^2\theta}\, 16^{\cos^2\theta}$ का अधिकतम और न्यूनतम मान ज्ञात करें।

$$(2^3)^{\sin^2\theta} \cdot (2^4)^{\cos^2\theta}$$

$$2^{3\sin^2\theta + 4\cos^2\theta}$$

$$\text{max} = 2^4 = 16 \qquad \text{min} = 2^3 = 8$$

Q) Find the max. & min value of $9^{\sin^2\theta} \cdot 27^{\cos^2\theta}$.

$9^{\sin^2\theta} \cdot 27^{\cos^2\theta}$ का अधिकतम और न्यूनतम मान ज्ञात कीजिए।

$$(3^2)^{\sin^2\theta} \cdot (3^3)^{\cos^2\theta}$$

$$3^{2\sin^2\theta + 3\cos^2\theta}$$

$$\text{max.} = 3^3 = 27 \qquad \text{min} = 3^2 = 9$$

$\boxed{4\sin^2\theta + 3\cos^2\alpha}$

Max	min
$4\times1+3\times1$	$4\times0+3\times0$
$4+3$	$0+0$
$=7$	$=0$

$\boxed{\sin^{(even)}\theta + \cos^{(even)}\theta}$

Max = 1 min = $\theta = 45^\circ$

Ex: $\sin^2\theta + \cos^2\theta$

Max. = 1

Min. $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$$\frac{1}{2} + \frac{1}{2} = 1$$

Ex:- $\sin^4\theta + \cos^6\theta$

Max = 1 $\quad \theta = 45^\circ \Rightarrow$ min

$$\left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^6$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

$$\frac{1}{4} + \frac{1}{8}$$

$$\frac{2+1}{8} = \frac{3}{8}$$

$\boxed{AM \geq GM}$

$\boxed{a, b}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$a+b \geq 2\sqrt{ab}$$

$\boxed{x^2 = \frac{1}{x^2}}$

min. value $= 2\sqrt{ab}$

- $x \Rightarrow \sin\theta \Rightarrow a\sin^2\theta + b\,\text{cosec}^2\theta$ $\quad \boxed{a>b}$

  min. value $= 2\sqrt{ab}$

  If $\boxed{a<b}$ $\quad$ min. value $= (a+b)$

- $x = \cos\theta \Rightarrow a\cos^2\theta + b\sec^2\theta$ — if $a>b \Rightarrow$ min $= 2\sqrt{ab}$
  — if $a<b \Rightarrow$ min $= a+b$

- $a\tan^2\theta + b\cot^2\theta$

  Min $= 2\sqrt{ab}$

$Y = a\tan^2\theta + b\cot^2\theta \ (a>0, b>0)$

Min	Max
$2\sqrt{ab}$	∞

$Y = a\sin^2\theta + b\,\text{cosec}^2\theta \ (a>0, b>0)$

$Y = a\cos^2\theta + b\sec^2\theta \ (a>0, b>0)$

Min.^{m}	Min.^{m}	max^m
$2\sqrt{ab}\ (a \geq b$	$(a+b)\ (a \leq b)$	∞

Q) If $Y = 4\tan^2\theta + 9\cot^2\theta$ then find the value Y_{min}.

यदि $Y = 4\tan^2\theta + 9\cot^2\theta$ है, तो Y का न्यूनतम मान ज्ञात कीजिए?

$$min = 2\sqrt{4\times9}$$
$$2\sqrt{36}$$
$$2\times6 = 12$$

Q) If $Y = 8\tan^2\theta + 2\cot^2\theta$ then find the value of Y_{min} is:

यदि $Y = 8\tan^2\theta + 2\cot^2\theta$ है, तो Y का न्यूनतम मान ज्ञात कीजिए।

$$Y_{min} = 2\sqrt{ab}$$
$$2\sqrt{8\times2}$$
$$2\sqrt{16}$$
$$2\times4 = 8$$

Q) The least value of $8\,cosec^2\theta + 25\sin^2\theta$ is

$8\,cosec^2\theta + 25\sin^2\theta$ का न्यूनतम मान क्या होगा?

$$25\sin^2\theta + 8\,cosec^2\theta$$
$$min = 2\sqrt{25\times8}$$
$$2\sqrt{200}$$
$$2\times10\sqrt{2}$$
$$20\sqrt{2}$$

If $a > b \Rightarrow 2\sqrt{ab}$

Q) Find minimum value of $4\sec^2\theta + 1\cos^2\theta$

$4\sec^2\theta + 1\cos^2\theta$ का न्यूनतम मान ज्ञात कीजिए।

$$1\cos^2\theta + 4\sec^2\theta$$

$\underline{a<b}$ $\quad min = a+b$

$$min\text{-}value = 1+4 = 5$$

Q) Find minimum value of $4\sec^2\theta + 25\,cosec^2\theta$.

$4\sec^2\theta + 25\,cosec^2\theta$ का न्यूनतम मान ज्ञात कीजिए।

$$4(1+\tan^2\theta) + 25(1+\cot^2\theta)$$
$$4+4\tan^2\theta + 25+25\cot^2\theta$$
$$29 + \underbrace{4\tan^2\theta + 25\cot^2\theta}_{2\sqrt{4\times25}}$$
$$2\sqrt{100} = 2\times10 = 20$$

IInd Min value $= (\sqrt{4}+\sqrt{25})^2$

$$(2+5)^2$$
$$7^2 = 49$$

29+20
49

$\boxed{y = a\sec^2\theta + b\,\mathrm{cosec}^2\theta \ (a>0, b>0)}$

Min. $(\sqrt{a}+\sqrt{b})^2$ Max. ∞

• $\sin\theta \cdot \cos\theta$

$\frac{1}{2} \times 2\sin\theta \cdot \cos\theta$

$\frac{1}{2}\sin 2\theta \Rightarrow$ min $-\frac{1}{2}$, max $+\frac{1}{2}$

min -1, max $+1$

• $\sin^2\theta \cdot \cos^2\theta$

$\frac{1}{2^2} \times 2^2 \sin^2\theta \cdot \cos^2\theta$

$\frac{1}{4}\sin^2 2\theta$

$0 \quad 1$

min $= \frac{1}{4} \times 0 = 0$

max. $= \frac{1}{4} \times 1 = \frac{1}{4}$

• $\sin^3\theta \cdot \cos^3\theta$

$\frac{1}{2^3} \times 2^3 \sin^3\theta \cdot \cos^3\theta$

$\frac{1}{8}(\sin^3 2\theta)$

$-1 \quad +1$

Min $= \frac{-1}{8}$

Max $= +\frac{1}{8}$

$\boxed{\sin^n\theta \cdot \cos^n\theta}$

n = natural No.

- max. value → $\frac{1}{2^n}$
- min. value
  - n = even → 0
  - n = odd → $-\frac{1}{2^n}$

Q) Find the minimum and maximum value of $2\sin\theta \cdot \cos\theta$

$2\sin\theta \cdot \cos\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

$\sin 2\theta$

$-1 \quad +1$

Q) Find the minimum and maximum value of $Y = 32\sin^5\theta \cdot \cos^5\theta$.

$Y = 32\sin^5\theta \cdot \cos^5\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

$32(\sin^5\theta \cdot \cos^5\theta)$

max. $\frac{1}{2^5} = \frac{1}{32}$ $\quad$ min $-\frac{1}{2^5} = \frac{-1}{32}$

$\cancel{32} \times \frac{1}{\cancel{32}}$ $\quad$ $\cancel{32} \times \left(\frac{-1}{\cancel{32}}\right)$

$1 \quad -1$

Q) If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is:

यदि θ के किसी मान के लिए $A = \sin^2\theta + \cos^4\theta$ है, तो A का मान है।

$\sin^{even}\theta + \cos^{even}\theta$

max = 1

min = $\theta = 45^\circ$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^4$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \le A \le 1$

Q) The maximum value of $1 + \sin\left(\frac{x}{4} + \theta\right) + 2\cos\left(\frac{x}{4} - \theta\right)$ for real values of θ is:

θ के वास्तविक मान के लिए $1+\underbrace{\sin\left(\frac{x}{4}+\theta\right)}_{max=+1}+2\underbrace{\cos\left(\frac{x}{4}-\theta\right)}_{+1}$ का अधिकतम मान क्या होगा ?

$1+1+2\times1$
$=4$

- $\frac{x}{4}+\theta=90^\circ$ - $\theta+\theta=90$
  $2\theta=90$
  $\theta=45^\circ$
- $\frac{x}{4}-\theta=0^\circ$ = $\frac{x}{4}=\theta$

Q) $4\sin^2\theta+3\cos^2\theta+6\sin^2\alpha+7\cos^2\alpha+8\cos^2\beta$ (0, 1)

max value = ?
$4+7+8$
19

min value = ?
$3+6+8\times0$
$9+0$
9

Q) $\underbrace{\sin^2\theta+\cos^2\theta}_{1}+\tan^2\theta+\cot^2\theta+\sec^2\theta+\operatorname{cosec}^2\theta$

min = ? $1+\tan^2\theta+\cot^2\theta+1+\tan^2\theta+1+\cot^2\theta$

$3+2(\underbrace{1\tan^2\theta+1\cot^2\theta}_{2\sqrt{1\times1}})$
$=2$

$3+2\times2$
$=3+4=7$

Q) If $a\sin^3x+b\cos^3x=\sin x\cos x$ and $a\sin x=b\cos x$, then find the value of a^2+b^2, provided that x is neither 0° nor 90°

यदि $a\sin^3x+b\cos^3x=\sin x\cos x$ और $a\sin x=b\cos x$ हो, तो a^2+b^2 का मान ज्ञात कीजिए, बशर्ते कि x न तो 0° है और न ही 90° है।

$a\sin x\cdot\sin^2x+b\cos^3x=\sin x\cdot\cos x$

$b\cos x\cdot\sin^2x+b\cos^3x=\sin x\cdot\cos x$

$b\cos x(\underbrace{\sin^2x+\cos^2x}_{1})=\sin x\cdot\cos x$

$b\cancel{\cos x}=\sin x\cdot\cancel{\cos x}$

$\boxed{b=\sin x}$

$a \sin x = b \cos x$

$a \times \cancel{b} = \cancel{b} \cos x$

$\boxed{a = \cos x}$

Putting the value of $\sin x$ & $\cos x$ in original eq^n.

$a \times (b)^3 + b \times a^3 = a \times b$

$\cancel{ab}(a^2 + b^2) = \cancel{ab}$

$a^2 + b^2 = 1$

Q) If $m = a \sec A$ and $y = b \tan A$, then find the value of $b^2 m^2 - a^2 y^2 + \frac{a^2 y^2}{b^2 m^2} + \cos^2 A$.

यदि $m = a \sec A$ और $y = b \tan A$ है, तो $b^2 m^2 - a^2 y^2 + \frac{a^2 y^2}{b^2 m^2} + \cos^2 A$ का मान ज्ञात कीजिए।

$m^2 = a^2 \sec^2 A$

$y^2 = b^2 \tan^2 A$

$\frac{m^2}{a^2} = \sec^2 A \Rightarrow \frac{a^2}{m^2} = \cos^2 A$

$\frac{y^2}{b^2} = \tan^2 A$

$b^2 \times a^2 \sec^2 A - a^2 \times b^2 \tan^2 A + \cos^2 A \times \tan^2 A + \cos^2 A$

$b^2 a^2 \underbrace{(\sec^2 A - \tan^2 A)}_{1} + \underbrace{\sin^2 A + \cos^2 A}_{1}$

$b^2 a^2 + 1$

1. Find maximum and min . value of $3\sin\theta + 4\cos\theta$.

$3\sin\theta + 4\cos\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) 5,-5
(b) $-3, 3$
(c) 4,-4
(d) $1, -1$

2. Find maximum and minimum. value of $8\sin\theta + 6\cos\theta$.

$8\sin\theta + 6\cos\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) 8,-8
(b) 6,-6
(c) $10, -10$
(d) $5, -5$

3. Find maximum and minimum value of $\sin\theta + 12\cos\theta$.

$5\sin\theta + 12\cos\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $13, -13$
(b) $12, -12$
(c) $5, -5$
(d) $10, -10$

4. Find maximum and minimum value of $= \sin^6\theta + \cos^6\theta$.

$x = \sin^6\theta + \cos^6\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $1, \frac{1}{4}$
(b) $2, \frac{3}{4}$
(c) $1, -1$
(d) $2, -1$

5. Find maximum and minimum value of $x = \sin^6\theta + \cos^8\theta$.

$x = \sin^6\theta + \cos^8\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $1, \frac{3}{16}$
(b) $2, \frac{3}{16}$
(c) $1, -1$
(d) $3, \frac{1}{16}$

6. Find maximum and minimum value of $2 - \sin\theta\cos\theta$.

$2 - \sin\theta\cos\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $\frac{5}{2}, \frac{3}{2}$
(b) $\frac{1}{2}; \frac{-1}{2}$
(c) $1, -1$
(d) $\frac{5}{2}, \frac{-5}{2}$

7. Find maximum and minimum value of $3 + \sin^2\theta.\cos^2\theta$.

$3 + \sin^2\theta.\cos^2\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $\frac{13}{4}, \frac{-13}{4}$
(b) $\frac{13}{4}, 3$
(c) , -1 :
(d) $\frac{13}{4}, 0$

8. Find maximum and minimum value of $1 - \sin^4\theta.\cos^4\theta$.

$1 - \sin^4\theta.\cos^4\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $1, -1$
(b) $1, \frac{15}{16}$
(c) $1, \frac{-15}{16}$
(d) $\frac{15}{16}, \frac{-15}{16}$

9. Find maximum and minimum value of $6\sin\theta \cdot \cos\theta + 1 - 2\sin^2\theta$.

$6\sin\theta \cdot \cos\theta + 1 - 2\sin^2\theta$ का

अधिकतम और न्यूनतम मान ज्ञात कीजिए।

(a) $\sqrt{11}, -\sqrt{11}$
(b) $\sqrt{10}, -\sqrt{10}$,
(c) $1, -1$
(d) $7, -7$

10. Find maximum and minimum value of $1 + 8\sin\theta\cos\theta - 2\sin^2\theta$

$1 + 8\sin\theta\cos\theta - 2\sin^2\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें

(a) $\sqrt{17}, -\sqrt{17}$
(b) $\sqrt{15}, -\sqrt{15}$
(c) $1, -1$
(d) $10, -10$

11.Find maximum and minimum value of $\sin\theta\cos\theta + 2\cos^2\theta - 1$.

$4\sin\theta\cos\theta + 2\cos^2\theta - 1$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $\sqrt{6}, -\sqrt{6}$
(b) $\sqrt{5}, -\sqrt{5}$
(c) $2, -2$
(d) $\sqrt{8}, -\sqrt{8}$

12. Find the maximum value of $2 + \operatorname{cosec}^2\alpha$.

$2 + \operatorname{cosec}^2\alpha$ का अधिकतम मान ज्ञात कीजिए।

(a) -2
(b) 0
(c) -3
(d) ∞

13. Find the maximum value of $2 - \operatorname{cosec}^2\alpha$

$2 - \operatorname{cosec}^2\alpha$ का अधिकतम मान ज्ञात कीजिए

(a) 3
(b) -3
(c) 1
(d) 0

14. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is

$2\sin^2\theta + 3\cos^2\theta$ का न्यूनतम मान है

(a) 0
(b) 3
(c) 2
(d) 1

15. The minimum value of $\sin^2\theta + \cos^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$

$\sin^2\theta + \cos^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ का न्यूनतम मान है।

(a) 1
(b) 3
(c) 7
(d) 5

16. Maximum value of $(2\sin\theta + 3\cos\theta)$ is

$(2\sin\theta + 3\cos\theta)$ का अधिकतम मान है

(a) 2
(b) $\sqrt{13}$
(c) $\sqrt{15}$
(d) 1

17. The minimum value of $4\tan^2\theta + 9\cot^2\theta$ is equal to

$4\tan^2\theta + 9\cot^2\theta$ का न्यूनतम मान बराबर है

(a) 1
(b) 2
(c) 12
(d) 13

18. Find max&min value of $1 - \sin^2\theta\cos^2\theta$.

$1 - \sin^2\theta\cos^2\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) $2, -2$
(b) $1, \frac{3}{4}$
(c) 1,0
(d) $1, -1$

19. Find min value of $4\sin^2\theta + 9\text{cosec}^2\theta$.

$4\sin^2\theta + 9\text{cosec}^2\theta$ का न्यूनतम मान ज्ञात कीजिए।

(a) 12
(b) -12
(c) 9
(d) 4

20. Find min value of $9\cos^2\theta + 16\sec^2\theta$.

$9\cos^2\theta + 16\sec^2\theta$ का न्यूनतम मान ज्ञात कीजिए।

(a) -24
(b) 16
(c) 24
(d) 9

21. Find max&min value of $x = \sin^2\theta + \cos^4\theta$

$x = \sin^2\theta + \cos^4\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें

(a) $\frac{3}{4} \le x \le 1$
(b) $-1 \le x \le 1$
(c) $-\frac{3}{4} \le x \le \frac{3}{4}$
(d) $1 \le x \le 0$

22. Find max value of $\sin^4\theta + \cos^8\theta$.

$\sin^4\theta + \cos^8\theta$ का अधिकतम मान ज्ञात करें।

(a) 2
(b) 3
(c) 1
(d) 4

23. Find maximum and minimum value $10\sin\theta - 1$

$10\sin\theta - 1$ का अधिकतम और न्यूनतम मान ज्ञात करें

(a) $9, -11$
(b) 10,9
(c) $1, -1$
(d) 11,10

24. Find maximum and minimum value of $2\sin^2\theta - 3$?

$12\sin^2\theta - 3$ का अधिकतम और न्यूनतम मान ज्ञात करें?

(a) $12, -3$
(b) $9, -3$
(c) $10, -3$
(d) $1 - 1$

25. Find maximum and minimum. value of $4\sin^2\theta + 7\cos^2\theta + 5$

$\sin^2\theta + 7\cos^2\theta + 5$ का अधिकतम और न्यूनतम मान ज्ञात करें।

(a) 7,4 (b) 12,9
(c) 9,16 (d) 4,7

26. Find minimum value of $0\tan^2\theta + 15\sec^2\theta$:

$10\tan^2\theta + 15\sec^2$ का न्यूनतम मान ज्ञात करें:

(a) 15 (b) 25

(c) 10 (d) 0

27. Find maximum and minimum value of $\sin^2\theta \cdot \cos^2\theta$

$\sin^2\theta \cdot \cos^2\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें

(a) $\frac{1}{4}, 0$

(b) $\frac{1}{2}, \frac{-1}{2}$

(c) $\frac{1}{4}, \frac{-1}{4}$

(d) None of these

28. Find maximum and minimum value of $\sin^3\theta \cdot \cos^3\theta$

$\sin^3\theta \cdot \cos^3\theta$ का अधिकतम और न्यूनतम मान ज्ञात करें

(a) $\frac{1}{4}, \frac{-1}{4}$

(b) $\frac{1}{8}, \frac{-1}{8}$

(c) $2\sqrt{2}$

(d) $\frac{1}{2\sqrt{2}}$

29. Find maximum and minimum value $\sin^{113}\theta \cdot \cos^{113}\theta$.

$\sin^{113}\theta \cdot \cos^{113}\theta$ अधिकतम और न्यूनतम मान ज्ञात करें

(a) $\frac{1}{2^{113}}, \frac{-1}{2^{113}}$

(b) $1, -1$

(c) $2^{113}, -2^{113}$

(d) None of these

30. $\cos^3\theta + \sin^3\theta$ will be maximum when $\theta = ?$

$\cos^3\theta + \sin^3\theta$ अधिकतम होगा जब θ= ?

(a) 15°

(b) 25

(c) 45°

(d) 60°

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
A	C	A	A	A	A	B	B	B	A
11	12	13	14	15	16	17	18	19	20
B	D	C	C	C	B	C	B	A	C
21	22	23	24	25	26	27	28	29	30
A	C	A	B	B	A	A	B	A	C

Worksheet solution

Sol 1 $= 3\sin\theta + 4\cos\theta$

$$\text{Max} = \sqrt{a^2+b^2}$$
$$= \sqrt{3^2+4^2} = 5$$
$$\text{Min} = -\sqrt{a^2+b^2}$$
$$= -\sqrt{3^2+4^2} = -5$$

Sol 2 $8\sin\theta + 6\cos\theta$

$$\text{Max value} = \sqrt{8^2+6^2} = 10$$
$$\text{Min value} = -\sqrt{8^2+6^2} = -10$$

Sol 3 $5\sin\theta + 12\cos\theta$

$$= \sqrt{5^2+12^2} = \pm 13$$

Max value $= +13$

Min value $= -13$

Sol 4 $x = \sin^6\theta + \cos^6\theta$

Maximim $= 1$

For minimum value put $\theta = 45^\circ$

$$x = \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{4}$$

Minimum Value $= \frac{1}{4}$

$$= 1, \frac{1}{4}$$

Sol 5 $x = \sin^6\theta + \cos\theta$

Max $= 1$

For Minimum value Put $\theta = 45^\circ$

$$x = \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{3}{16}$$

$$1, \frac{3}{16}$$

Sol 6 $2 - \sin\theta\cos\theta$

$$\text{Min} = 2 - \frac{1}{2} = \frac{3}{2}$$
$$\text{Max} = 2 - \left(-\frac{1}{2}\right)$$
$$= \frac{5}{2}$$

Sol 7 $3 + \sin^2\theta\cos^2\theta$

$$\text{Max} = 3 + \frac{1}{2^2} = 3 + \frac{1}{4}$$
$$= \frac{13}{4}$$
$$\text{Min} = 3 + 0 = 3$$

Sol 8 $1 - \sin^4\theta\cos^4\theta$

$\boxed{\text{Power is even}}$

$$\text{Min} = 1 - \frac{1}{2^4} = \frac{15}{16}$$
$$\text{Max} = 1 - 0 = 1$$

Sol 9

$6\sin\theta\cos\theta + 1 - 2\sin^2\theta$

$3\sin 2\theta + \cos 2\theta$

$= \pm\sqrt{3^2+1^2} = \pm\sqrt{10}$

Max $= \sqrt{10}$

Min $= -\sqrt{10}$

Sol 10

$1 + 8\sin\theta\cdot\cos\theta - 2\sin^2\theta$

$4\sin 2\theta + \cos 2\theta$

$= \pm\sqrt{4^2+1^2} = \pm\sqrt{17}$

Max $= \sqrt{17}$

Min $= -\sqrt{17}$

Sol 11

$4\sin\theta\cos\theta + 2\cos^2\theta - 1$

$2\sin 2\theta + \cos 2\theta$

$= \pm\sqrt{2^2+1^2} = \pm\sqrt{5}$

Max $= \sqrt{5}$

Min $= -\sqrt{5}$

Sol 12

$2 + \operatorname{cosec}^2 a$

Maximum value of $\operatorname{cosec}^2 a = \infty$

Max value $= 2 + \infty = \infty$

Sol 13

$2 - \operatorname{cosec}^2 a$

Minimum value of $\operatorname{cosec}^2 a = 1$

Max value $= 2 - 1 = 1$

Sol 14

$2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$= 2 + \cos^2\theta$

Minimum value of $\cos^2\theta = 0$

Min. value $= 2 + 0 = 2$

Sol 15

$\sin^2\theta + \cos^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$

$= 1 + 1 + \tan^2\theta + 1 + \cot^2\theta + \tan^2\theta + \cot^2\theta$

$= 3 + 2(\tan^2\theta + \cot^2\theta)$

Min value $= 3 + 2\times 2\sqrt{1\times 1}$

$= 7$

Sol 16

$2\sin\theta + 3\cos\theta$

Max $= \sqrt{2^2+3^2}$

$= \sqrt{13}$

Sol 17

$4\tan^2\theta + 9\cot^2\theta$

Max value $= 2\times\sqrt{4\times 9}$

$= 12$

Sol 18

$1 - \sin^2\theta\cos^2\theta$

Max $= 1 - (0) = 1$

Max $= 1 - \left(\frac{1}{2^2}\right) = \frac{3}{4}$

Sol 19 $4\sin^2\theta + 9\operatorname{cosec}^2\theta$

$$\text{Min} = 2\sqrt{4\times 9}$$
$$= 12$$

Sol 20 $9\cos^2\theta + 16\sec^2\theta =$

$$\text{Min value} = 2\sqrt{9\times 16}$$
$$= 2\times 12 = 24$$

Sol 21 $x = \sin^2\theta + \cos^4\theta$

Max $= 1$

For Minimum value, put $\theta = 45^\circ$

$$x = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^4$$
$$= \frac{1}{2} + \frac{1}{4}$$
$$= \frac{3}{4} \le x \le 1$$

Sol 22 $\sin^4\theta + \cos^6\theta$

Power are even, than

Maximum vaule $= 1$

Sol 23 $10\sin - 1$

$$\text{Max} = 10\times 1 - 1 = 9$$
$$\text{Min} = 10(-1) - 1 = -11$$

Sol 24 $12\sin^2\theta - 3$

$$\text{Max} = 12\times 1 - 3 = 9$$
$$\text{Min} = 12(0) - 3 = -3$$

Sol 25 $4\sin^2\theta + 7\cos^2\theta + 5$

$$\text{Max value} = 7 + 5$$
$$= 12$$
$$\text{Min value} = 4 + 5$$
$$= 9$$

Sol 26 $10\tan^2\theta + 15\sec^2\theta$

$$= 10\tan^2\theta + 15(1 + \tan^2\theta)$$
$$= 15 + 25\tan^2\theta$$
$$\text{Min value} = 15 + 0$$
$$= 15$$

Sol 27 $\sin^2\theta \times \cos^2\theta$

$$\text{Max} = \frac{1}{2^2} = \frac{1}{4}$$
$$\text{Min} = 0$$

Sol 28 $\sin^3\theta \times \cos^3\theta$

$$\text{Max} = \frac{1}{2^3} = \frac{1}{8}$$
$$\text{Min} = \frac{-1}{2^3} = \frac{-1}{8}$$

Sol 29 $\sin^{113}\theta \cdot \cos^{113}\theta$

$$\text{Max} = \frac{1}{2^{113}}, \quad \text{Min} = \frac{-1}{2^{113}}$$

Sol 30 $\cos^3\theta + \sin^3\theta$

Maximum value at 45°