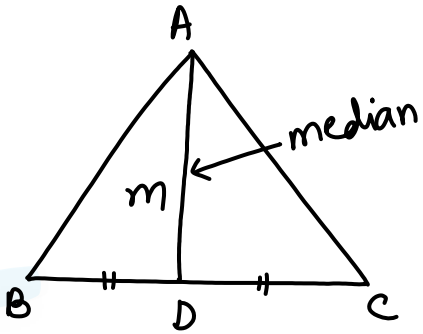
Apollonius theorem (अपोलोनियस प्रमेय):

Relation b/w 3 sides & 1 median

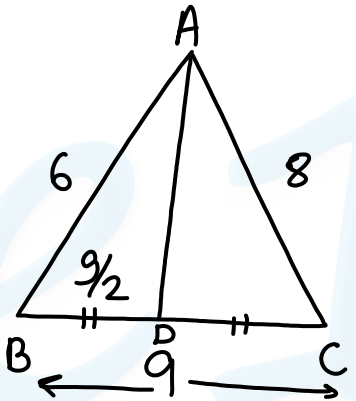
$$AB^2 + AC^2 = 2(BD^2 + m^2)$$



50

$36+64=2\left(AD^2+\frac{81}{4}\right)$

$\frac{119}{4}=AD^2$



6. In $\Delta ABC, AB = 6cm,\ AC = 8cm,$ and $BC = 9cm$. The length of median AD is:

ΔABC में, $AB = 6$ सेमी, $AC = 8$ सेमी, और $BC = 9$ सेमी है। माध्यिका AD की लम्बाई कितनी है?

a) $\frac{\sqrt{317}}{2}$ cm

b) $\frac{\sqrt{119}}{2}$ cm

c) $\frac{\sqrt{313}}{2}$ cm

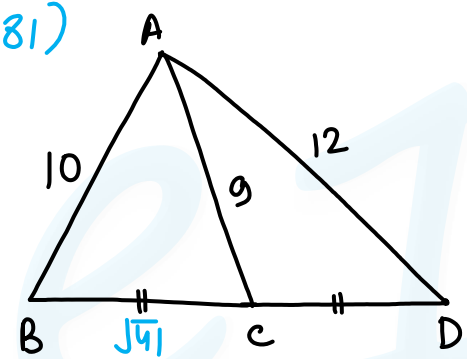
d) $\frac{\sqrt{115}}{2}$ cm

$100 + 144 = 2(BC^2 + 81)$

122

$41 = BC^2$

$\sqrt{41} = BC$



7. In ΔABC, C is the midpoint of BD. If $AB = 10\ cm$, $AD = 12\ cm$ and $AC = 9\ cm$, then $BD = ?$

ΔABC में, C, BD का मध्यबिंदु है, यदि $AB = 10$ सेमी, $AD = 12$ सेमी और $AC = 9$ सेमी है, तो $BD = ?$

a) $2\sqrt{41}$
b) $2\sqrt{10}$
c) $\sqrt{41}$
d) $\sqrt{10}$

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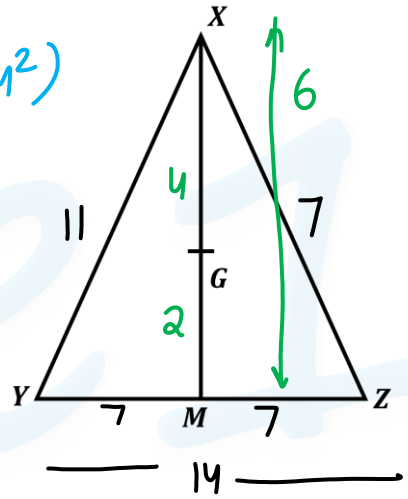
85

$121 + 49 = 2(49 + XM^2)$

$36 = XM^2$

$6 = XM$



8. In triangle XYZ, G is the centroid. If $XY = 11cm, YZ = 14cm$ and XZ= 7cm, then what is the value (in cm) of GM?

त्रिभुज XYZ में, G केन्द्रक है| यदि $XY = 11cm, YZ = 14cm$ तथा XZ= 7cm है, तो GM का मान (cm में) क्या है?

a) 6 b) 4

c) 2 d) 3

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Acc. to apollonius theorem;

$PQ^2 + PR^2 = 2(PD^2 + QD^2)$

$\Rightarrow$ ~~$400 + 2500 = 2(PD^2 + 324)$~~ (1700)

$\Rightarrow PD^2 = 1700 - 324 = 1376$

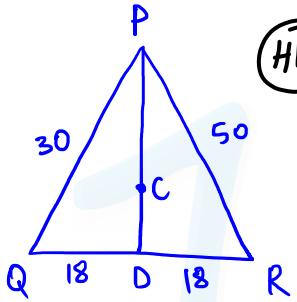
$PD = \sqrt{1376}$

4×344

4×86

$= 4\sqrt{86}$

$CD = \frac{1}{3} \times PD = \frac{1}{3} \times 4\sqrt{86}$



9. (HW) In triangle PQR, C is the centroid $PQ = 30cm$, $QR = 36\ cm$ and $PR = 50\ cm$. If D is the midpoint of QR, then what is the lenghth (in cm) of CD?

त्रिभुज PQR में, C केन्द्रक है| $PQ = 30\ cm$, $QR = 36\ cm$ तथा $PR = 50\ cm$ है| यदि D, QR का मध्यबिंदु है, तो CD की लम्बाई (cm में) क्या है?

a) $\frac{4\sqrt{86}}{3}$

b) $\frac{\sqrt{86}}{3}$

c) $\frac{5\sqrt{86}}{3}$

d) $\frac{5\sqrt{86}}{2}$

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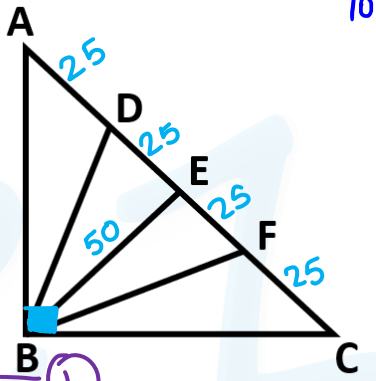
In $\triangle ABC$, $BE = 50$



$\triangle ABE$, $BD \rightarrow$ Median

$AB^2 + 2500 = 2(625 + BD^2)$ — (1)

$\triangle BEC$, $BF \rightarrow$ Median

$BC^2 + 2500 = 2(BF^2 + 625)$ — (2)

(1) + (2)

$10,000 + 5000 = 2(BD^2 + BF^2 + 1250)$

$7500 + 1250$

$BE^2 = 2500$

$+1250$

10. In the below figure, ΔABC is right angled, $\angle ABC = 90°$ and $AC = 100\ cm$. Also, $AD = DE = EF = FC$. Find the value of: $BD^2 + BE^2 + BF^2\ (in\ cm^2)$

निम्न आकृति में, ΔABC एक समकोण त्रिभुज है, $\angle ABC = 90°$ और $AC = 100\ cm$ है, और $AD = DE = EF = FC$ | $BD^2 + BE^2 + BF^2$ का मान ज्ञात करो |

a) $10,000\ cm^2$

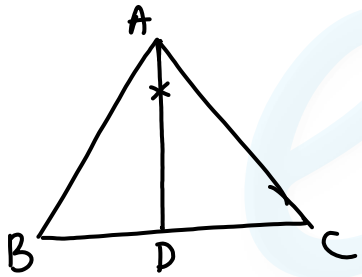
b) $5,000\ cm^2$

c) $8,750\ cm^2$

d) $12,500\ cm^2$

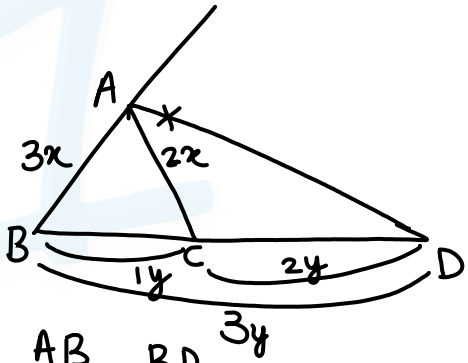
Exterior-angle bisector theorem:

(Interior) ABT :

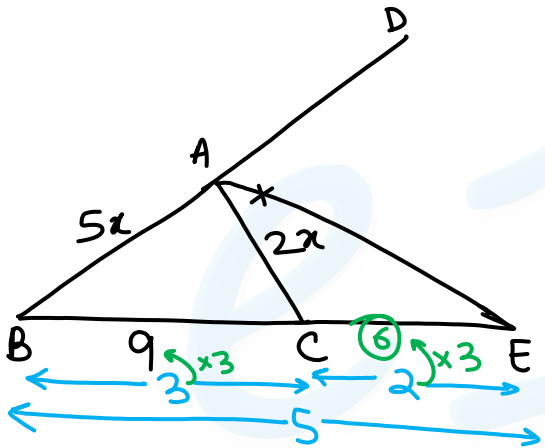


$$\frac{AB}{AC} = \frac{BD}{CD}$$

Ext ABT :



$$\frac{AB}{AC} = \frac{BD}{CD}$$



11. In a triangle ABC, $AB : AC = 5 : 2$, $BC = 9\ cm$, BA is produced to D, and the bisector of the Angle CAD meets BC produced at E. What is the length (in cm) of CE?

त्रिभुज ABC में, $AB : AC = 5 : 2$, $BC = 9\ cm$ हैं| BA को D तक बढ़ाया जाता है और कोण CAD का समद्विभाजक बढाई गई BC से E पर मिलता है| CE की लम्बाई (cm में) ज्ञात करें|

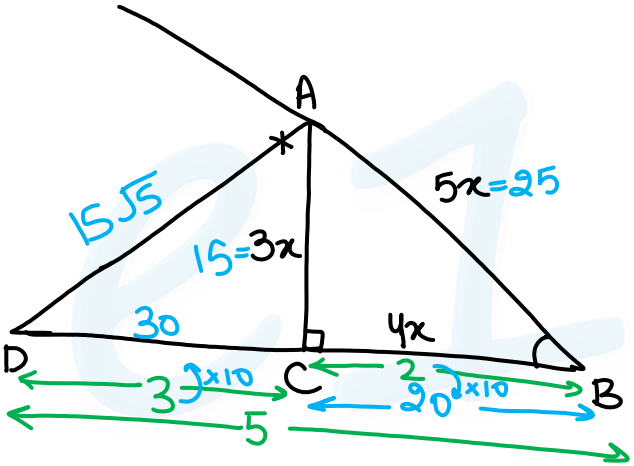
a) 9 b) 10

c) 3 d) 6

12. AB is the hypotenuse of the right angled triangle ΔABC. BC when produced meet the angle bisector of exterior $\angle A$ at D. Find the length of AD if $\sin B = 0.6$ and $BC = 20\ cm$.

AB समकोण त्रिभुज ΔABC का कर्ण है| BC बढ़ाने पर बाह्य $\angle A$ के समदविभाजक से D पर मिलती है. AD की लम्बाई बताइए अगर $sin\,B = 0.6$ और $BC = 20\ cm$ है.

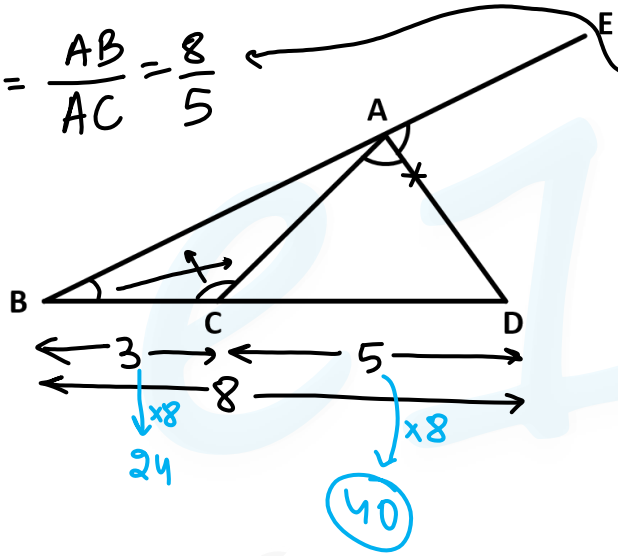
a) $15\sqrt{5}$ b) $3\sqrt{41}$
c) $10\sqrt{15}$ d) $15\sqrt{3}$



$$\sin B = \frac{AC}{AB} = \frac{6}{10} = \frac{3}{5}$$

Bhutesh Sir

$\frac{\sin\angle ACB}{\sin\angle ABC} = \frac{AB}{AC} = \frac{8}{5}$

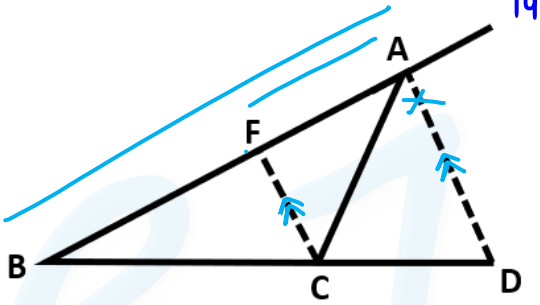


13. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If $\frac{\sin\angle ACB}{\sin\angle ABC} = \frac{8}{5}$ and $BC = 24\ cm$ then find the length of CD.

दी गयी आकृति में AD कोण EAC का सम द्विभाजक है जो बढाई गयी BC को D पर काटता है| अगर $\frac{\sin\angle ACB}{\sin\angle ABC} = \frac{8}{5}$ और $BC = 24\ cm$ है तो CD की लम्बाई बताओ.

a) $15\ cm$ b) $40\ cm$

c) $30\ cm$ d) $18\ cm$



14. The bisector of the exterior $\angle A$ of ΔABC intersects the side BC produced to D. Here CF is parallel to AD. Then

बाहरी कोण A का सम द्विभाजक बढाई गयी BC भुजा को D पर काटता है तो CF तथा AD समान्तर है तो

a) $\frac{AB}{AC} = \frac{BA}{FA}$ b) $\frac{AB}{AC} = \frac{CD}{BD}$

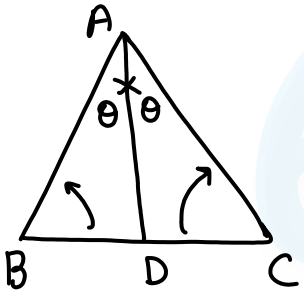
c) $\frac{AB}{AC} = \frac{BC}{CD}$ d) None of these

$\frac{AB}{AC} = \frac{BD}{CD}$

$\frac{BD}{CD} = \frac{BA}{FA}$

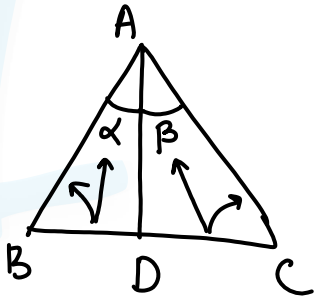
General angle bisector theorem:

$\triangle ABT$



$$\frac{BD}{DC} = \frac{AB}{AC}\,\frac{\sin\theta}{\sin\theta}$$

$\triangle GABT$



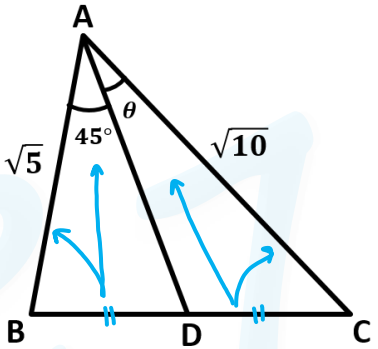
$$\frac{BD}{DC} = \frac{AB\sin\alpha}{AC\sin\beta}$$

15. In the given figure, AD is the median of ΔABC. Find the value of θ according to the values given.

दी गयी आकृति में, AD ΔABC की माध्यिका है| दी गयी जानकारी के अनुसार θ का मान बताओ.

a) $45°$ b) $30°$

c) $60°$ d) $15°$



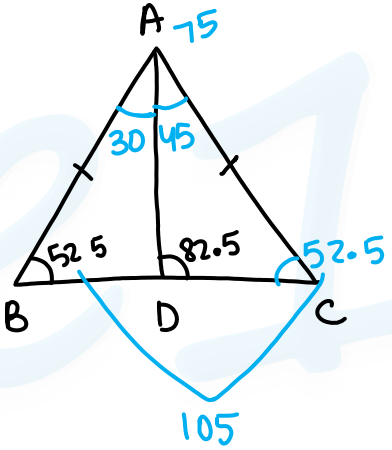
$$\frac{BD}{DC} = \frac{\sqrt{5}\sin 45}{\sqrt{10}\sin\theta}$$

$$\sin\theta = \frac{\sqrt{5}\cdot 1}{\sqrt{10}\cdot\sqrt{2}} = \frac{1}{2}$$

$\theta = 30°$

$$\frac{BD}{DC} = \frac{AB \sin 30}{AC \sin 45}$$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{1} = \frac{1}{\sqrt{2}}$$



16. In an isosceles ΔABC, $AB = AC$, $\angle B = 52.5°$ and $\angle ADC = 82.5°$ where D is a point on BC. Find $BD : DC$.

एक समद्विबाहु ΔABC में $AB = AC$, $\angle B = 52.5°$ और $\angle ADC = 82.5°$ जहाँ D भुजा BC पर एक बिंदु है. $BD : DC$ का मान बताइए.

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{3}}$

c) $\sqrt{2}$ d) $\sqrt{6}$

Bhutesh sir

17. In the given figure, PQRS is a sqare of side 8cm. $\angle PQO = 60^\circ$. What is the area (in cm^2) of the triangle POQ?

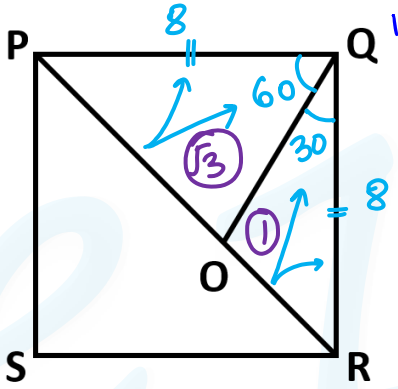
दी गई आकृति में , $PQRS$ $8cm$ भुजा वाला एक वर्ग है| $\angle PQO = 60^\circ$ है| त्रिभुज POQ का क्षेत्रफल (cm^2 में) क्या है?

a) $32\sqrt{3}$

b) $24[\sqrt{3}-1]$

c) $48[\sqrt{3}-1]$

d) $16[3-\sqrt{3}]$



$$\frac{PO}{OR} = \frac{PR \sin 60}{QR \sin 30}$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \frac{\sqrt{3}}{1}$$

$$\frac{(\sqrt{3}-1)}{2} \cdot \frac{\sqrt{3}}{\sqrt{3}+1} \times \frac{1}{2} \times 8 \times 8 = 16(3-\sqrt{3})$$

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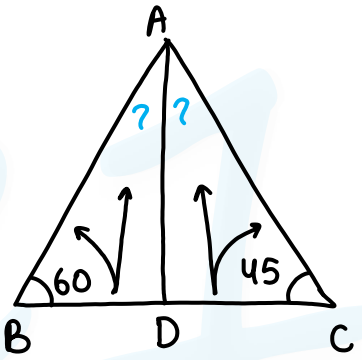
$$\frac{BD}{DC} = \frac{AB}{AC} \cdot \frac{\sin\angle BAD}{\sin\angle CAD} \rightarrow ?$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \square$$



Sine rule

$$\frac{AB}{AC} = \frac{\sin 45}{\sin 60} = \frac{1}{\sqrt{2}} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

18. In a triangle ABC, AD divides BC in the ratio 2:3. if $\angle B = 60°$ and $\angle C = 45°$ then , find the $\frac{\sin\angle BAD}{\sin\angle CAD}$.

त्रिभुज ABC में AD भुजा BC को 2:3 के अनुपात में बांटता है| अगर $\angle B = 60°$ और $\angle C = 45°$ है तो $\frac{\sin\angle BAD}{\sin\angle CAD} = ?$

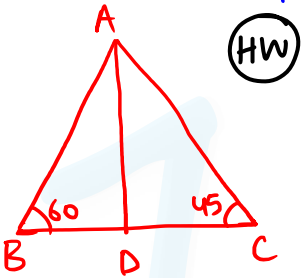
a) $\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{2}}{\sqrt{3}}$

c) $\frac{\sqrt{3}}{\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

Acc. to GABT:

$$\frac{BD}{CD} = \frac{AB \cdot \sin\angle BAD}{AC \cdot \sin\angle CAD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sin\angle BAD}{\sin\angle CAD}$$



Sine rule:

$$\frac{AB}{AC} = \frac{\sin 45}{\sin 60} = \frac{1}{\sqrt{2}} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

19. In a ΔABC, $\angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3 then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to

HW

ΔABC में $\angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{4}$ और D भुजा BC को 1 : 3 के अनुपात में अन्तःविभाजित करता है| $\frac{\sin\angle BAD}{\sin\angle CAD}$ पता करो|

60°

45°

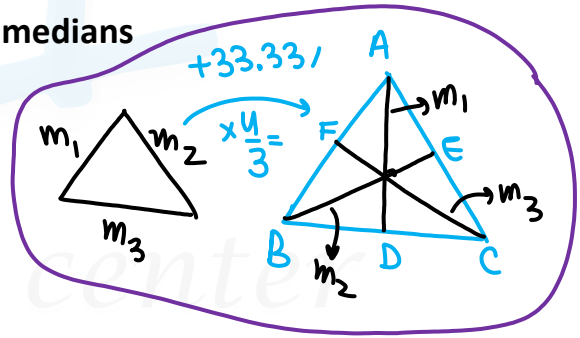
a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{3}}$

c) $\frac{1}{\sqrt{6}}$ d) $\sqrt{6}$

Some standard results:

1. Sum of lengths of all medians < sum of lengths of all sides
2. Area of triangle formed by sides is $= \frac{4}{3} \times$ Area of triangle formed by its medians
3. Sum of squares of sides $= \frac{4}{3} \times$ Sum of squares of medians
4. Sum of sides $< \frac{4}{3} \times$ Sum of medians

$AB^2+BC^2+CA^2=\frac{4}{3}\times[AD^2+BE^2+CF^2]$



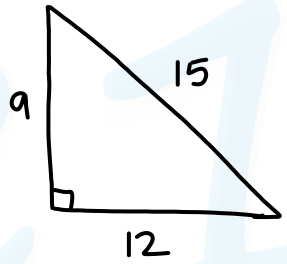
20. The length of three medians of a triangle are $9\ cm$, $12\ cm$ and $15\ cm$. The area (in sq. cm) of the triangle is

किसी त्रिभुज की मध्यिकाओं की लम्बाई क्रमश: $9\ cm$, $12\ cm$ तथा $15\ cm$ है| त्रिभुज का क्षेत्रफल क्या है?

a) $48\ cm^2$ b) $144\ cm^2$

c) $24\ cm^2$ d) $72\ cm^2$

$\frac{4}{3} \times \frac{1}{2} \times 12 \times 9$

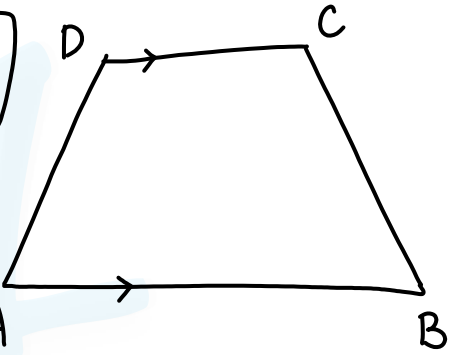


Sum of Squares of diagonals = a^2+b^2+2ab → type का

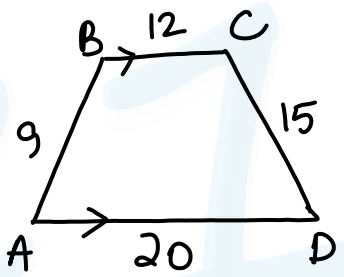
$$AC^2+BD^2 = AD^2+BC^2+2\,AB\cdot DC$$

(S) Square (P) Product

(S) Side वाली (P) Parallel



$$\begin{array}{r} 81 \\ 225 \\ +480 \\ \hline 786 \end{array}$$

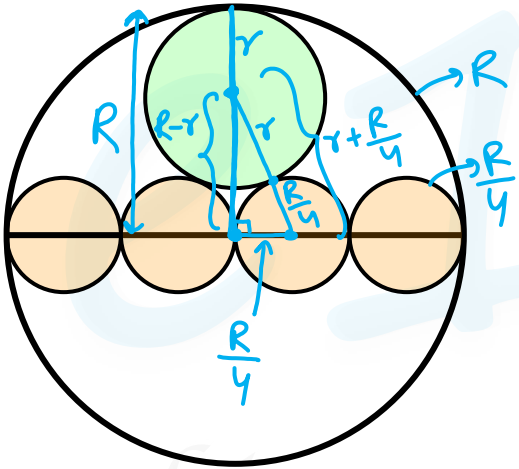


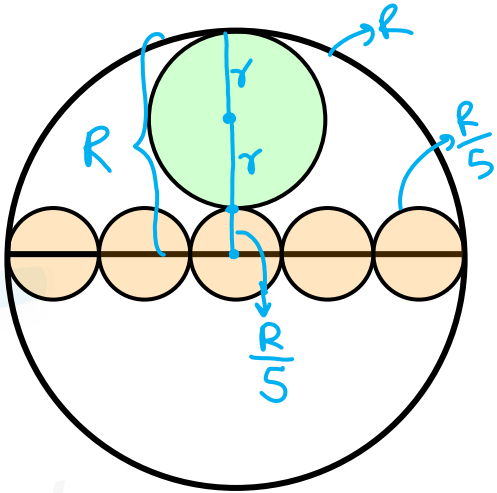
21. $ABCD$ is a trapezium of sides in which $BC \parallel AD$ and $AB = 9\ cm, BC = 12\ cm\ CD = 15\ cm\ DA = 20\ cm$. Find the sum of square of its diagonal.

$ABCD$ एक समलम्ब चतुर्भुज है जिसकी भुजा $BC \parallel AD$ और $AB = 9\ cm, BC = 12\ cm\ CD = 15\ cm\ DA = 20\ cm$ है| इसके विकरणों के वर्गों का जोड़ पता करो|

a) 576 b) 676
c) 786 d) 729

Some results for circles:





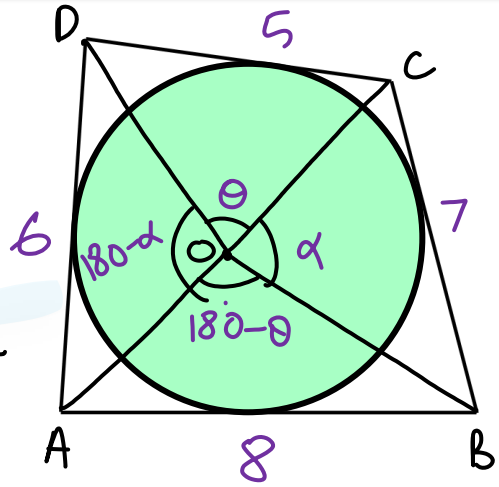
Tangential quadrilateral (स्पर्श रेखीय चतुर्भुज):

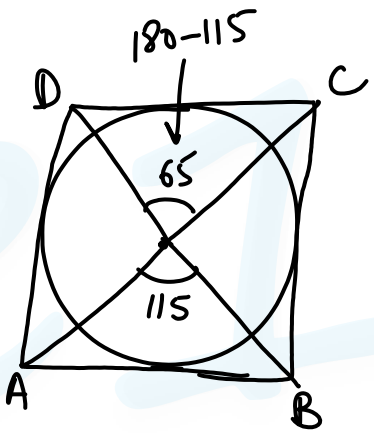
$AB+CD=AD+BC=\frac{\text{Perimeter}}{2}$

$\angle AOB+\angle COD=\angle BOC+\angle AOD=180$

$ar\,\Delta AOB+ar\,\Delta COD=ar\,\Delta AOD+ar\,\Delta BOC$

$=\frac{ar\,ABCD}{2}$





22. A circle of centre O inscribed in a quadrilateral $ABCD$ which touches all the sides of a quadrilateral. If $\angle AOB = 115°$. Find $\angle COD$.

चतुर्भुज $ABCD$ के अन्दर एक वृत्त इस प्रकार है कि यह चतुर्भुज की सभी भुजाओं को स्पर्श करता है| अगर $\angle AOB = 115°$ है तो $\angle COD$=?

a) 65° b) 115°

c) 130° d) 95°

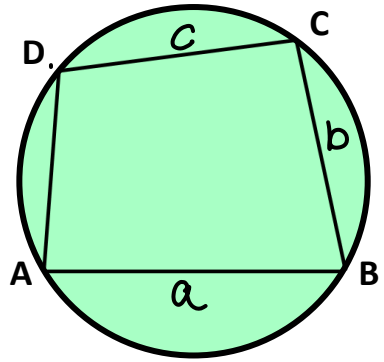
Brahmgupta's theorem:

Area of ABCD $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

Cyclic (चक्रीय)

$d = 0$

ar $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$



$S = \frac{a+b+c+d}{2}$

Ptolmey's theorem:

$$AC \times BD = AB \cdot DC + AD \cdot BC$$

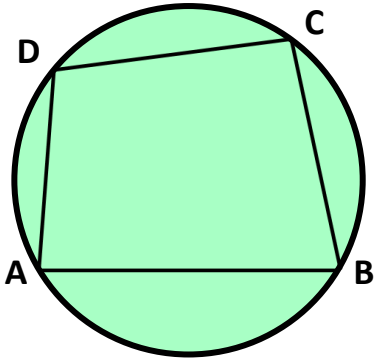
Prod of opp sides

prod of opp side

Cyclic (चक्रीय)

product of diagonals

Sum of prod of opp sides



Distance between Incenter & circumcenter:

$$d^2 = R^2 - 2 \cdot Rr$$

Euler's theorem: