

Surds & Indices: Basic formulas

Index
Power

$2^{\textcircled{3}}$, $3^{\textcircled{4}}$

(घात एवं करणी: मूलभूत सूत्र)

$$\sqrt[3]{16} \rightarrow \text{Surd} \checkmark$$

$$\sqrt{7} = 7^{\frac{1}{2}}$$

Surd \checkmark \rightarrow Irrational

अपरिमेय

$$\sqrt[4]{27} \nearrow$$

$$\sqrt{16} = 4$$

Surd \times Integer \checkmark

Basic formulas:

$$0^n = 0$$

$$\text{Eg } 0^5 = 0$$

$$0 = 0 \times 0 \times 0 \times 0 \times 0$$

$$0^7 = 0$$

$$0^{1000} = 0$$

$$a^0 = 1$$

$$a^1 = a$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\text{Eg. } 3^0 = 1, 4^0 = 1, 1000^0 = 1$$

$$\text{Eg } 5^1 = 5, 10^1 = 10, 500^1 = 500$$

$$\text{Eg. } 2^{-3} = \frac{1}{2^3} = \frac{1}{8}, \frac{3^{-4}}{7} = \frac{1}{7 \times 3^4}, \frac{5^4}{7^2} = \frac{7^{-2}}{5^4}$$

$$\text{Eg } \sqrt[2]{16} = 16^{\frac{1}{2}}, \sqrt[3]{27} = 27^{\frac{1}{3}}, \sqrt[4]{16} = 16^{\frac{1}{4}}$$

$$a+a+a+\dots \text{ n बार} = n \cdot a$$

$$5+5+5+\dots \text{ 10 बार} = 10 \times 5 = 50$$

$$a \times a \times a \times \dots \text{ n बार} = a^n$$

$$2 \times 2 \times 2 = 2^3$$

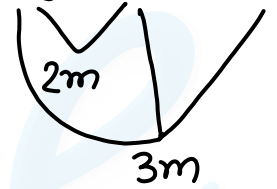
$$2 \times 2 \times 2 \times 2 = 2^4$$

$$a^{10} \times a^5 = a^{10+5} = a^{15}$$

$a \times a \times a \dots$ 10 बार

$a \times a \times a \dots$ 5 बार

coaching center

$$(a^m)^n = a^m \times a^m \times a^m \times \dots \quad n \text{ टाई}$$

$$= a^{m \times n}$$

$$(5^{10})^3 = 5^{10} \times 5^{10} \times 5^{10}$$
$$= 5^{10+10+10} = 5^{10 \times 3}$$

Basic formulas:

$a \times a \times a \dots$ m ताई $a^m \times a^n = a^{m+n}$
 $a \times a \times a \dots$ n ताई $\frac{a^m}{a^n} = a^{m-n}$
 $a \times a \times a \dots$ m ताई
 $a \times a \times a \dots$ n ताई $(a^m)^n = a^{m \times n}$
 $a^m \times a^{-n} = a^{m-n}$
 $a^{m^n} = a^{(m^n)}$
 $(ab)^n = a^n \times b^n$

Eg: $5^3 \times 5^6 = 5^9$

Eg $\frac{5^{17}}{5^{10}} = 5^{17-10} = 5^7$

Eg $(2^3)^4 = 2^{12}$, $(3^5)^7 = 3^{35}$, $(2^6)^2$

Eg: $2^{3^4} = 2^{(3^4)} = 2^{81} \rightarrow (2^4)^3 = (2^2)^6$

Eg $(6)^{10} = (2 \times 3)^{10} = 2^{10} \times 3^{10}$

Basic formulas:

If $a^m = a^n$, then

$$m = n \quad a \neq 0, 1, -1$$

If $a^m = b^m$, then

$a = \pm b$, if m is even

$a = b$, if m is odd

$$a^9 = 2^9$$

$$a = 2$$

$$a \neq -2$$

$$\underline{a}^{10} = \underline{2}^{10} = 1024$$

$$a = 2$$

$$a = -2$$

coaching center

Things to memorize:

$$\sqrt{2} = 1.414$$

$$\sqrt{3} = 1.732$$

$$\sqrt{5} = 2.236$$

- $2^{12}, 3^6, 5^5, 6^4$
- **Squares up to 30**
- **Cubes up to 12, (15, 21)**
- $\sqrt{2}, \sqrt{3}, \sqrt{5}$

coaching center

I. Recognize the rational number:

परिमेय संख्या बताइए:

~~a) $\sqrt[4]{27}$~~

~~b) $\sqrt[3]{16}$~~

~~c) $\sqrt{11}$~~

~~d) $\sqrt[5]{243} = 3$~~

e1

coaching center

2. Which of the following is a surd?

निम्न में से कौनसी करणी/अपरिमेय संख्या है?

a) $\sqrt[4]{81}$
3

b) $\sqrt[5]{32}$
2

c) $\sqrt[6]{729}$
3

~~d) $\sqrt[3]{25}$~~

e1

coaching center

3. Find $\sqrt{\sqrt{\sqrt{2}}}$

a) $2^{\frac{1}{3}}$

b) $2^{\frac{1}{4}}$

c) $2^{\frac{1}{6}}$

~~d) $2^{\frac{1}{8}}$~~

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$(a^m)^n = a^{mn}$$

$$\left((a^m)^n\right)^p = a^{mnp}$$

$\left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = 2^{\frac{1}{8}}$

coaching center

4. Find the value of $\sqrt[4]{\sqrt[3]{\sqrt{a}}}$?

~~a) $a^{\frac{1}{24}}$~~

b) $a^{\frac{1}{7}}$

c) $a^{\frac{1}{8}}$

d) $a^{\frac{1}{12}}$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\left(\left(a^{\frac{1}{2}} \right)^{\frac{1}{3}} \right)^{\frac{1}{4}} = a^{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}} = a^{\frac{1}{24}}$$

coaching center

$$a \times b \times c = a \times c \times b \\ = b \times c \times a$$

5.
$$\left[\sqrt[14]{\sqrt[3]{\sqrt{\left(5 \sqrt{\left(\sqrt[3]{5^6}\right)^2}\right)^{15}}}}\right]^7$$

a) 5^2

b) 5^4

c) 5^8

d) 5^{12}

$$(5^2)^3 = 5^{2 \times 3} = 5^6$$

$$\left(\left(\left(5^6 \right)^{\frac{1}{3}} \right)^2 \right)^{\frac{1}{5}}$$

$$5^{\cancel{6} \times \cancel{2} \times \cancel{15} \times \frac{1}{\cancel{3}} \times \frac{1}{\cancel{5}} \times \frac{1}{\cancel{3}} \times \frac{1}{\cancel{14}} \times 7}$$

$$= 25$$

$$\sqrt[3]{\sqrt{5}} = 5^{\frac{1}{2} \times \frac{1}{3}} = 5^{\frac{1}{6}}$$

coaching center

6. $\left[\sqrt[3]{\sqrt[6]{5^9}} \right]^4$ $\left[\sqrt[3]{\sqrt[6]{5^9}} \right]^4$

a) 5^2

~~b) 5^4~~

c) 5^8

d) 5^{12}

\square^2

$5^{\cancel{9} \times 4 \times \frac{1}{6} \times \frac{1}{3} \times 2}$

=

coaching center

$$7. \frac{3^0 + 3^{-1}}{3^{-1} - 3^0}$$

~~a) -2~~

b) -1

c) 1

d) 2

$$3^{-1} = \frac{1}{3}$$

$$\frac{1 + \frac{1}{3}}{\frac{1}{3} - 1} = \frac{\frac{4}{3}}{\frac{1}{3} - 1} = \frac{4}{3} \times \frac{3}{-2} = -2$$

coaching center

$$8. \frac{(xy)^{a-b} (xy)^{b-c}}{(xy)^{a-c}} = ?$$

~~a) 1~~

b) 0

c) xy

d) xy^{abc}

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^p} = a^{m-p}$$

$$xy^{a-b+b-c-a+c} \\ = xy^0 = 1$$

$$\frac{a^q \times a^m \times a^n}{a^r \times a^p} = a^{m+n+q-p-r}$$

coaching center

9. The value of $(x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \cdot (x^{a+b})^{a-b}$ ($x \neq 0$) is

$(x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \cdot (x^{a+b})^{a-b}$ ($x \neq 0$) का मान है :

~~a) 1~~

b) 2

c) -1

d) 0

$$(x^m)^n \times (x^p)^q$$

$$= x^{mn} \times x^{pq}$$

$$= x^{mn+pq}$$

$$= x^{b^2-c^2+c^2-a^2+a^2-b^2}$$

$$= x^0 = 1$$

coaching center

$$10. \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = ?$$

~~a) 1~~

b) 0

c) x

d) x^{abc}

$(x^{a-b})^{a+b}$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0 = 1$$

coaching center

11. The value of $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = ?$

$\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$ का मान है :

~~a) 1~~

b) $\frac{1}{x^{abc}}$

c) $\frac{1}{x^{ab+bc+ca}}$

d) None of these

$$\frac{a-b}{ab} = \frac{a}{ab} - \frac{b}{ab}$$

$$(x^{a-b})^{\frac{1}{ab}}$$

$$= x^{\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}}$$

$$= x^{\frac{1}{b} - \frac{1}{a} + \frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c}}$$

$$= x^0 = 1$$

$$\frac{m^a}{n^a} = \left(\frac{m}{n}\right)^a$$

$$2. \frac{\left(x+\frac{1}{y}\right)^a \left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a \left(y-\frac{1}{x}\right)^b} = ?$$

a) $\left(\frac{y}{x}\right)^{a+b}$

~~b) $\left(\frac{x}{y}\right)^{a+b}$~~

c) $\left(\frac{y}{x}\right)^{a-b}$

d) $\left(\frac{x}{y}\right)^{a-b}$

$$\frac{\frac{a}{b}}{\frac{p}{q}} = \frac{a}{b} \times \frac{q}{p} = \frac{\left(\frac{xy+1}{y}\right)^a \left(\frac{xy-1}{y}\right)^b}{\left(\frac{xy+1}{x}\right)^a \left(\frac{xy-1}{x}\right)^b} = \frac{x^a x^b}{y^a y^b} = \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y}\right)^{a+b}$$

coaching center

13. If $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{2}} \div \left(\frac{p^6q^{-3}}{p^{-2}q^3}\right)^{\frac{1}{2}} = p^a q^b$, then the value of $a + b$, where p and q are different positive primes, is

यदि $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{2}} \div \left(\frac{p^6q^{-3}}{p^{-2}q^3}\right)^{\frac{1}{2}} = p^a q^b$ हो, तो $a + b$ का मान क्या है जिसमें p और q विभिन्न धनात्मक अभाज्य हैं?

a) 1

b) 0

~~c) -1~~

d) 2

$$\frac{p^{-2} \cdot q^2}{p^4 q^{-3}} = p^{-6} q^5 \quad \begin{array}{l} a = -6 \\ b = 5 \end{array}$$

coaching center

14. $4^{4x+1} = \frac{1}{64}$, then x is

a) $\frac{1}{2}$

~~b) -1~~

c) $-\frac{1}{2}$

d) $-\frac{1}{6}$

$$4^{4x+1} = \frac{1}{4^3} = 4^{-3}$$

$$\Rightarrow 4x+1 = -3$$

$$\Rightarrow x = -1$$

coaching center

15. If $5^{5x+5} = 1$, then x equals

a) 0

~~b) -1~~

c) 1

d) $-\frac{4}{5}$

$$= 5^0$$

$$\Rightarrow 5x+5 = 0$$

$$\Rightarrow x = -1$$

coaching center

16. If $27^{2x-1} = 243^3$, then x equals

~~a) 3~~

b) 6

c) 7

d) 9

$$(3^3)^{2x-1}$$

$$3^{3(2x-1)} = 3^{15}$$

$$x=3$$

coaching center

17. $(125)^x = 3125, x = ?$

a) $\frac{1}{5}$

b) $\frac{3}{5}$

~~c) $\frac{5}{3}$~~

d) $\frac{5}{7}$

$$5^{3x} = 5^5$$

$$x = \frac{5}{3}$$

coaching center

18. If $8^{3x-5} = \frac{1}{32^{7-4x}}$ then $x = ?$

अगर $8^{3x-5} = \frac{1}{32^{7-4x}}$ है तो $x = ?$

a) $\frac{16}{9}$

~~b) $\frac{20}{112}$~~ ~~c) $\frac{25}{13}$~~

d) 2

$$2^{9x-15} = 2^{20x-35}$$

$$\Rightarrow 20 = 11x$$

$$\Rightarrow x = \frac{20}{11}$$

coaching center

19. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then the value of x is
अगर $3^{x+y} = 81$ और $81^{x-y} = 3$ तो x का मान बताइए.

a) 42

$= 3^4$ b) $\frac{15}{8}$ $3^{4(x-y)} = 3^1$ $\frac{17}{8}$

d) 39

$$x+y=4$$

$$x-y = \frac{1}{4}$$

$$\frac{17}{4 \times 2} = \frac{17}{8}$$

coaching center

20. $2^{2x-y} = 16$ and $2^{x+y} = 32$, the value of xy is

$2^{2x-y} = 16$ और $2^{x+y} = 32$ है तो xy है

a) 2

b) 4

~~c) 6~~

d) 8

$$= 2^4$$

$$= 2^5$$

$$2x - y = 4$$

$$x + y = 5$$

$$3x = 9$$

$$x = 3$$

coaching center

$$a^m = \frac{1}{a^{-m}}$$
$$\left(\frac{a}{1}\right)^m = \left(\frac{1}{a}\right)^{-m}$$

$$\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$$
$$\frac{a^m}{b^m} = \frac{b^{-m}}{a^{-m}}$$

21. If $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$ then $a = ?$

अगर $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$ है तो $a = ?$

- a) -5 b) -6 ~~c) -7~~ d) 7

$$= \left(\frac{x}{y}\right)^{3a-17}$$

$$\Rightarrow 5a-3 = 3a-17$$

$$\Rightarrow 2a = -14$$

$$\Rightarrow a = -7$$

22. If $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{25}{9}\right)^{1-2x}$, then x is

अगर $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{25}{9}\right)^{1-2x}$ है तो x का मान है:

~~a) $-\frac{1}{4}$~~

b) $\frac{5}{4}$

c) $\frac{1}{4}$

d) $-\frac{5}{4}$

$$\frac{3}{5}^{-3} = \frac{3}{5}^{4x-2}$$

$$\Rightarrow -1 = 4x$$

$$-\frac{1}{4} = x$$

$$\left(\frac{3}{5} \cdot 2\right)^{1-2x}$$

$$\frac{3}{5}^{2-4x}$$

coaching center

23. If $(x^x)^{\frac{5}{4}} = x^{x^{\frac{5}{4}}}$ then $x = ?$

अगर $(x^x)^{\frac{5}{4}} = x^{x^{\frac{5}{4}}}$ है तो $x = ?$

a) $\frac{125}{64}$

~~b) $\frac{625}{256}$~~

c) $\frac{25}{16}$

d) $\frac{5}{4}$

$(2^3)^4 = 2^{12}$

$2^3^4 = 2^{81}$

$2^{3^{100}} = 2^{3^{100}}$

$x^{\left(\frac{5}{4}x\right)} = x^{\left(x^{\frac{5}{4}}\right)}$

$\Rightarrow \frac{5}{4}x = x^{\frac{5}{4}}$

$\Rightarrow \frac{5}{4} = x^{\frac{5}{4}-1} = x^{\frac{1}{4}}$

$x = \frac{625}{256}$

24. $x^{x\sqrt{x}} = (x\sqrt{x})^x$, then x equals

$x^{x\sqrt{x}} = (x\sqrt{x})^x$ है तो x का मान है

a) $\frac{4}{9}$

b) $\frac{2}{3}$

~~c) $\frac{9}{4}$~~

d) $\frac{3}{2}$

$$= (x^1 \cdot x^{\frac{1}{2}})^x$$

$$= (x^{\frac{3}{2}})^x$$

$$x^{x\sqrt{x}} = x^{\frac{3}{2}x}$$

$$\Rightarrow x\sqrt{x} = \frac{3}{2}x$$

$$\Rightarrow x = \frac{9}{4}$$

25. What are the possible solutions for x of the equation $x^{\sqrt{x}} = \sqrt[n]{x^x}$, where x and n are positive integers ?

समीकरण $x^{\sqrt{x}} = \sqrt[n]{x^x}$ के संभावित हल क्या हैं, जहां x और n धनात्मक पूर्णांक हैं ?

~~a) $0, n^2$~~

~~b) $1, n$~~

c) n, n^2

~~d) $1, n^2$~~

$$= (x^x)^{\frac{1}{n}}$$

$$x^{\sqrt{x}} = x^{\frac{x}{n}}$$

~~$\Rightarrow \sqrt{x} = \frac{x}{n}$~~

$$\Rightarrow \begin{aligned} n &= \sqrt{x} \\ n^2 &= x \end{aligned}$$

coaching center