

20. For how many integer values of 'p' the roots of the equation $x^2 - px + 12 = 0$ are not real?

अगर समीकरण $x^2 - px + 12 = 0$ के मूल वास्तविक नहीं हैं तो P के कितने पूर्णांक मान हो सकते हैं?

a) 14

b) 6

~~c) 13~~

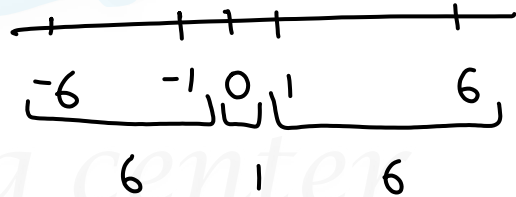
d) 7

$$p^2 - 48 < 0$$

$$p^2 < 48$$

$\sqrt{-ve}$

$$b^2 - 4ac < 0$$



coaching center

21. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ has equal roots, then which one of the following is correct?

$17x^2 - 20x + 8 = 0$ यदि समीकरण $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ के मूल बराबर हैं, तो निम्न में से कौन-सा ठीक है ?

$400 = 4 \times 17 \times 8$

a) $ab = cd$

~~b) $ad = bc$~~

c) $a^2 + c^2 = b^2 + d^2$

d) $ac = bd$

$b^2 - 4ac = 0$

OR
 $b^2 = 4ac$

~~$4(ac + bd)^2 = 4(a^2 + b^2)(c^2 + d^2)$~~

$\Rightarrow \cancel{a^2c^2} + \cancel{b^2d^2} + 2abcd = \cancel{a^2c^2} + a^2d^2 + b^2c^2 + \cancel{b^2d^2}$

$\Rightarrow a^2d^2 + b^2c^2 + 2abcd = 0$

$\Rightarrow (ad - bc)^2 = 0$

$\Rightarrow ad = bc$

$\frac{a}{b} = \frac{c}{d}$
 $\frac{a}{b} - \frac{c}{d} = 0$

21. If the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$ are equal, where $(a, b, c > 0)$ which one of the following is correct ?

यदि समीकरण $(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$ के मूल बराबर हैं, जहां $(a, b, c > 0)$ है, तो निम्न में से कौन सा ठीक है ?

$$b^2 = 4ac$$

a) $a + b + c = 0$

b) $a + b = c$

~~c) $a = b = c$~~

d) $a - b = c$

$$4(b^3 + a^2c^2 - 2ab^2c) = 4(a^2c^2 - a^3b - bc^3 + abc^2)$$

$$\Rightarrow a^3 + c^3 + b^3 = 3abc$$

$$a = b = c \quad \text{OR} \quad a + b + c = 0$$

$$\times a, b, c > 0$$

23. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which of the following is true?

यदि समीकरण $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ के मूल बराबर हैं, तो निम्नलिखित में से कौन सा सही है?

a) $b = \frac{a+c}{ac}$

~~b) $\frac{2}{b} = \left(\frac{1}{a}\right) + \left(\frac{1}{c}\right)$~~

c) $2b = \left(\frac{1}{a}\right) + \left(\frac{1}{c}\right)$

d) $abc = ab + bc + ca$

$b^2 = 4ac$

$$b^2(c^2 + a^2 - 2ac) = 4ac(ab - b^2 - ac + bc)$$

$$\Rightarrow b^2c^2 + a^2b^2 - 2abc^2 = 4a^2bc - 4ab^2c - 4a^2c^2 + 4abc^2$$

$$\Rightarrow b^2c^2 + a^2b^2 + 4a^2c^2 - 4a^2bc + 2ab^2c - 4abc^2 = 0$$

$$\Rightarrow (bc)^2 + (ab)^2 + (2ac)^2 - 2 \cdot ab \cdot (-2ac)$$

$$\Rightarrow (bc + ab - 2ac)^2 = 0$$

$$bc + ab - 2ac = 0$$

$$\Rightarrow \frac{bc + ab}{abc} = \frac{2ac}{abc}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

coaching center

Conjugate pairs (संयुग्मी जोड़े):

$$3+2\sqrt{2}$$

$$3-2\sqrt{2}$$

$$5+\sqrt{3}$$

$$5-\sqrt{3}$$

$$\sqrt{3}+2$$

$$\sqrt{3}-2$$

Irrational

$$2-3i, 2+3i$$

$$5+4i, 5-4i$$

Imaginary

coaching center

$$ax^2 + bx + c = 0$$

i) If a, b, c are rational (परिमेय)

then irrational roots occur in conjugate pairs (अपरिमेय)

ii) If a, b, c are real (वास्तविक)

then imaginary roots occur in conjugate pairs

coaching center

24. If a, b, c are rational numbers and $2 + \sqrt{3}$ is a root of the equation

$\rightarrow ax^2 + bx + c = 0$, then $\frac{b}{c} = ?$ $2 - \sqrt{3}$

अगर a, b, c परिमेय संख्याएं हैं और $2 + \sqrt{3}$, समीकरण $ax^2 + bx + c = 0$ का एक हल $2 + \sqrt{3}$ है तो $\frac{b}{c}$ का मान ज्ञात करें।

a) ~~-4~~

b) 4

c) $\sqrt{3}$

d) $-\sqrt{3}$

$$\left. \begin{aligned} \text{SoR} &= 2 + \sqrt{3} + 2 - \sqrt{3} = 4 \\ \text{P.o.R} &= (2 - \sqrt{3}) = 1 \end{aligned} \right\} x^2 - 4x + 1 = 0$$

coaching center

$$S_0 R = 10$$

$$P_0 R = 25 - 27 = -2$$

$$x^2 - 10x - 2 = 0$$

$$\frac{1 + 100 + 4}{-10 + 20 - 2} = \frac{105}{8}$$

25. A root of equation $ax^2 + bx + c = 0$ (where a , b and c are rational numbers) is $5 + 3\sqrt{3}$. What is the value of

$$\frac{a^2 + b^2 + c^2}{ab + bc + ca} ?$$

$$5 - 3\sqrt{3}$$

समीकरण $ax^2 + bx + c = 0$ (जहाँ a , b तथा c परिमेय संख्याएं हैं) का मूल $5 + 3\sqrt{3}$ है। तो $\frac{a^2 + b^2 + c^2}{ab + bc + ca}$ का मान क्या है?

a) $\frac{35}{3}$

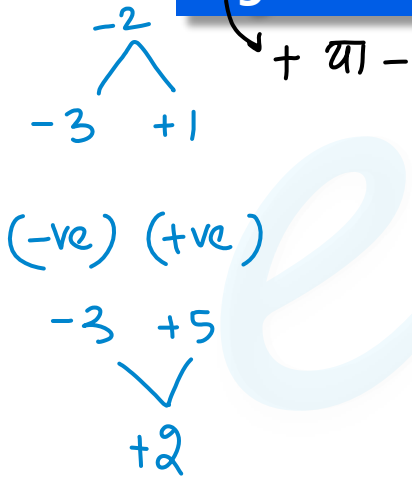
b) $\frac{37}{3}$

c) $-\frac{105}{8}$

d) $\frac{105}{8}$

coaching center

Sign of roots:



Por	Sor	Larger	Smaller
+	+	+	+
+	-	-	-
-	+	+	-
-	-	-	+

coaching center

26. What is the least integral value of k for which the equation $x^2 - 2(k-1)x + (2k+1) = 0$ has both the roots positive ?

k का न्यूनतम पूर्णांक मान क्या है जिसके लिए समीकरण $x^2 - 2(k-1)x + (2k+1) = 0$ के दोनों मूल धनात्मक हैं?

- a) 1 b) 4 c) $-\frac{1}{2}$ ~~d) 2~~

$$2k+1 > 0 \quad \& \quad 2(k-1) > 0$$
$$\Rightarrow k > -\frac{1}{2}$$
$$k > 1$$

$P < 0$ = +ve

$S < 0$ = +ve



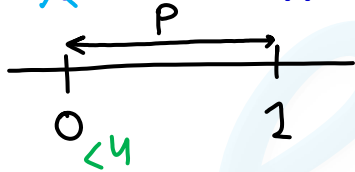
27. If $0 < p < 1$, then roots of the equation $(1 - p)x^2 + 4x + p = 0$ are
 अगर $0 < p < 1$ है तो समीकरण $(1 - p)x^2 + 4x + p = 0$ के मूल हैं:

a) Imaginary

c) Real and of opposite sign

b) Real and both positive

d) Real and both negative



$$16 - 4p(1-p) = +ve$$

\downarrow \downarrow
 < 1 < 1
 $4x^{\frac{1}{2}} x^{\frac{1}{2}}$

both
-ve

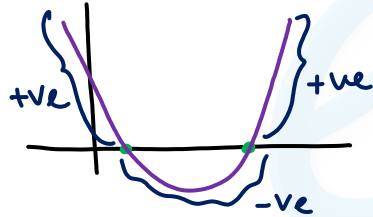
$$P \cdot R = \frac{p}{1-p} = \frac{+ve}{+ve} = +ve$$

$$S \cdot R = \frac{-4}{1-p} = \frac{-ve}{+ve} = -ve$$

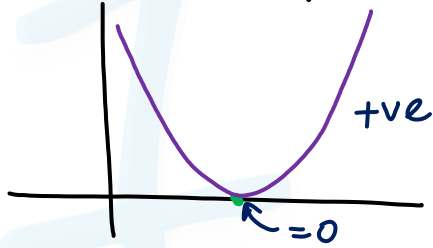
coaching center

$$ax^2 + bx + c, \quad a = +ve$$

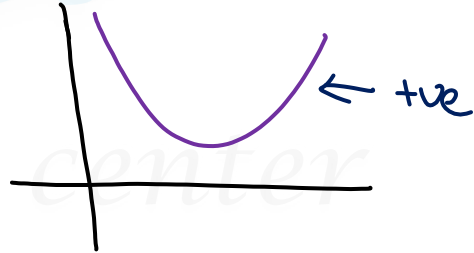
i) Real & Unequal



ii) Real & equal



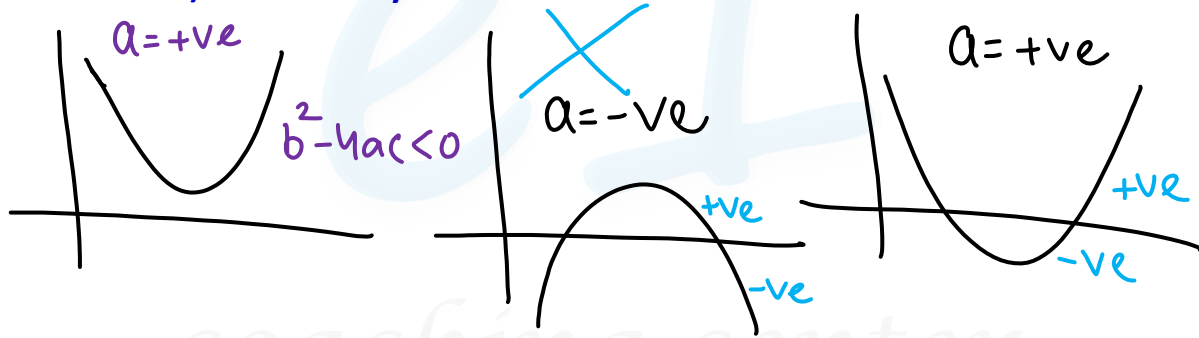
ii) Imaginary & Unequal



28. The value of the quadratic polynomial $ax^2 + bx + c$ is always positive if :

द्विघाती बहुपद $ax^2 + bx + c$ का मान हमेशा धनात्मक होगा यदि,

- ~~a) a is positive and $b^2 - 4ac < 0$~~
- b) a is positive and $b^2 - 4ac \geq 0$
- c) a can be any real number and $b^2 - 4ac < 0$
- d) a can be any real number and $b^2 - 4ac \geq 0$



coaching center

29. One-fourth of a herd of cows is in the forest. Twice the square root of the herd has gone to mountains and the remaining 15 are on the banks of a river. The total number of cows is:

गायों के एक झुण्ड का एक चौथाई हिस्सा जंगल में है। झुण्ड के वर्गमूल का दुगना पहाड़ों में गया है और बाकी की बची 15 गाय नदी के किनारे पर हैं। कुल गायों की संख्या ज्ञात करें।

$$x$$

$$\frac{1}{4}x$$

$$2\sqrt{x}$$

$$15$$

- a) 6
 b) 100
 c) 63
 d) 36

forest mountains bank

$$100 \neq 25 + 20 + 15$$

$$36 = 9 + 12 + 15$$

$$x = \frac{1}{4}x + 2\sqrt{x} + 15$$

$$4x^2 = x^2 + 4x + 15$$

$$\Rightarrow 3x^2 - 4x - 15 = 0$$

$$x = \frac{4 \pm 14}{6} = 3$$

$$\sqrt{16 + 180}$$

Factor / Remainder theorem:
गुणनखंड / शेषफल परमेय

$$f(x) = 4x^3 - 2x^2 + 5x - 9$$

If $f(x)$ is divided by $(x-1)$ $\begin{matrix} \nearrow x-1=0 \\ \Rightarrow x=1 \end{matrix}$

$$\left| \frac{f(x)}{x-1} \right|_{\text{Remainder}} = f(1) = 4 - 2 + 5 - 9 = -2$$

$$f(x) = 7x^4 - 3x^3 + 2x^2 - 5x + 8$$

$$\left| \frac{f(x)}{x+2} \right|_{\mathbb{R}} = f(-2)$$

$$x+2=0$$

$$\Rightarrow x = -2$$

coaching center

$$x-1 \overline{) 4x^3 - 2x^2 + 5x - 9} \quad (4x^2 + 2x + 7)$$

$$\underline{-4x^3 + 4x^2}$$

$$2x^2 + 5x$$

$$\underline{-2x^2 + 2x}$$

$$7x - 9$$

$$\underline{-7x + 7}$$

$$-2$$

$$f(x-1) = 0$$

$$\Rightarrow x = 1$$

0

$$4x^3 - 2x^2 + 5x - 9 = (x-1)(4x^2 + 2x + 7) + (-2)$$

Remainder
↓

$$4 \overline{) 35} \begin{matrix} 8 \\ 3 \\ 3 \end{matrix}$$

$$\underline{32}$$

$$3$$

$$4 \times 8 + 3$$

$$5 \overline{) 35} \begin{matrix} 7 \\ 0 \end{matrix}$$

$$\underline{35}$$

$$0$$

$$5 \times 7 + 0$$

coaching center

Factor/remainder theorem:

$$x^3 - 2x^2 - 5x + 6$$

coaching center

1. When $x^3 + 3x^2 + kx + 5$ is divided by $x + 1$, then remainder is 3. what is the remainder if it is divided by $x - 2$?

जब $x^3 + 3x^2 + kx + 5$ को $x + 1$ से विभाजित किया जाता है, तो शेषफल 3 बचता है। यदि $x - 2$ से विभाजित करें तो क्या शेषफल बचेगा ?

a) 25

b) 32

~~c) 33~~

d) 4

$$x + 1 = 0$$

$$\Rightarrow x = -1$$

$$x - 2 = 0$$

$$\Rightarrow x = 2$$

$$x = 2$$

$$-1 + 3 - k + 5 = 3$$

$$\Rightarrow k = 4$$

$$8 + 12 + 8 + 5 = 33$$

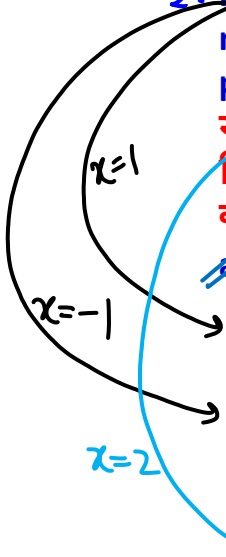
coaching center

2. If $x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $x - 1$ and $x + 1$ the remainders are 5 and 19 respectively. Find the remainder when this polynomial is divided by $x - 2$.

$x - 1 = 0 \Rightarrow x = 1$ $x + 1 = 0 \Rightarrow x = -1$

यदि $x^4 - 2x^3 + 3x^2 - ax + b$ को $x - 1$ और $x + 1$ द्वारा विभाजित किया जाता है तो शेषफल क्रमशः 5 और 19 बचता है। जब इस बहुपद को $x - 2$ के द्वारा विभाजित किया जाता है तो शेषफल ज्ञात करें।

- a) 10 ~~$x - 2 = 0$~~ b) 12 c) -1 d) 8



$1 - 2 + 3 - a + b = 5 \Rightarrow -a + b = 3$

$1 + 2 + 3 + a + b = 19 \Rightarrow a + b = 13$

~~$16 - 16 + 12 - 10 + 8 = 10$~~

coaching center

$$f\left(-\frac{2}{3}\right) = 15x - \frac{8}{9} - 14x \frac{4}{9} + \frac{8}{3} + 10$$

$$= \frac{-40 - 56 + 24 + 90}{9}$$

$$= \frac{18}{9} = 2$$

3. When $f(x) = 15x^3 - 14x^2 - 4x + 10$ is divided by $(3x + 2)$, then the remainder is:

जब $f(x) = 15x^3 - 14x^2 - 4x + 10$ को $(3x + 2)$, से विभाजित किया जाता है, तो _____ शेषफल प्राप्त होता है।

- ~~a) 2~~ b) -1 c) -2 d) 1

$$3x + 2 = 0$$

$$\Rightarrow x = -\frac{2}{3}$$

coaching center

4. If $3x^4 - 2x^3 + 3x^2 - 2x + 3$ is divided by $(3x + 2)$, then the remainder is

यदि $3x^4 - 2x^3 + 3x^2 - 2x + 3$ को $(3x + 2)$ से विभाजित किया जाता है, तो शेषफल होगा :

$$\begin{aligned} & 3 \times \frac{16}{81} + 2 \times \frac{8}{27} + 3 \times \frac{4}{9} + \frac{4}{3} + 3 \\ &= \frac{32 + 72 + 81}{27} \\ &= \frac{185}{27} \end{aligned}$$

$$x = -\frac{2}{3}$$

a) 0

b) $\frac{185}{27}$

c) $\frac{181}{25}$

d) $\frac{3}{4}$

$$3x + 2 = 0$$

$$x = -\frac{2}{3}$$

coaching center

5. If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are: $\overset{=0}{x=1}$ $\text{Rem} = 0$

यदि $(x + 2)$ और $(x - 1)$, $(x^3 + 10x^2 + mx + n)$ के गुणखंड हैं, तो m और n का मान ज्ञात करो।

$$x+2=0$$

$$\Rightarrow x = -2$$

a) $m = 5, n = -3$

b) $m = 17, n = -8$

~~c) $m = 7, n = -18$~~

d) $m = 23, n = 19$

$$-8 + 40 - 2m + n = 0 \Rightarrow -2m + n = -32$$

$$1 + 10 + m + n = 0 \Rightarrow m + n = -11$$

$$3m = 21$$

$$m = 7$$

6. If $(x + 1)$ and $(x - 1)$ are factor of $ax^3 + bx^2 + 3x + 5$. Find the value of $a - 2b$. $x = -1$ $Rem = 0$

यदि $(x + 1)$ और $(x - 1)$, $ax^3 + bx^2 + 3x + 5$ के गुणखंड है। $a - 2b$ का मान ज्ञात करो।

a) 1

b) -13

~~c) 7~~

d) 0

$$-a + b - 3 + 5 = 0 \Rightarrow -a + b = -2$$

$$a + b + 3 + 5 = 0 \Rightarrow a + b = -8$$

$$-3 + 10 = 7$$

coaching center

$$x+2=0$$
$$\Rightarrow x=-2$$

$$x-2=0$$
$$\Rightarrow x=2$$

(Hw)

7. If $px^3 + x^2 + 3x + q$ is exactly divisible by $(x+2)$ and $(x-2)$, then the value of p and q are:

यदि $px^3 + x^2 + 3x + q$, $(x+2)$ एवं $(x-2)$ से पूर्णतः विभाज्य है, तो p और q के मान _____ हैं।

a) $p = -\frac{3}{4}$ and $q = 4$

b) $p = \frac{3}{4}$ and $q = 4$

c) $p = \frac{3}{4}$ and $q = -4$

~~d) $p = -\frac{3}{4}$ and $q = -4$~~

$x = -2$
 $x = 2$

$$-8p + 4 - 6 + q = 0 \Rightarrow -8p + q = 2 \quad \text{--- (1)}$$

$$8p + 4 + 6 + q = 0 \Rightarrow 8p + q = -10 \quad \text{--- (2)}$$

$$\text{(1) + (2)} \rightarrow 2q = -8$$

$$q = -4$$

$$q = -4$$

$$8p = -6$$

$$p = -\frac{3}{4}$$