

Rare identities:

$$\begin{aligned}(a+b+c)^3 &= a^3+b^3+c^3+3(a+b)(b+c)(c+a) \\ &= a^3+b^3+c^3+3\left[a^2(b+c)+b^2(c+a)+c^2(a+b)\right] \\ &\quad +6abc\end{aligned}$$

coaching center

$$(a+1)(b+1) = ab + a + b + 1$$

$$(a-1)(b-1) = ab - a - b + 1$$

coaching center

508. Simplify the following expression

$$a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a) \leftarrow \begin{aligned} & \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) + \left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \right) + (6 \times 6 \times 6) + \\ & 3 \left(\frac{1}{2} + \frac{3}{2} \right) \left(\frac{1}{2} + 6 \right) \left(6 + \frac{3}{2} \right) \end{aligned}$$

$$= (a+b+c)^3$$

$$= \left(\frac{1}{2} + \frac{3}{2} + 6 \right)^3 = 8^3 = 512$$

निम्नलिखित व्यजक को हल कीजिए ।

$$\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) + \left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \right) + (6 \times 6 \times 6) + 3 \left(\frac{1}{2} + \frac{3}{2} \right) \left(\frac{1}{2} + 6 \right) \left(6 + \frac{3}{2} \right)$$

a) 64

b) 521

c) 256

d) 512

coaching center

$$\underbrace{(x+y)^{\frac{1}{3}}}_a + \underbrace{(y+z)^{\frac{1}{3}}}_b + \underbrace{(z+x)^{\frac{1}{3}}}_c = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$(x+y) + (y+z) + (z+x) = 3(x+y)^{\frac{1}{3}}$$

$$\times (y+z)^{\frac{1}{3}} \times (z+x)^{\frac{1}{3}}$$

$$\Rightarrow 2(x+y+z) =$$

$$\Rightarrow 8(x+y+z)^3 = 27(x+y)(y+z)(z+x)$$

509. If $(x+y)^{\frac{1}{3}} + (y+z)^{\frac{1}{3}} = -(z+x)^{\frac{1}{3}}$, then $(x^3 + y^3 + z^3)$ can be expressed as:

अगर $(x+y)^{\frac{1}{3}} + (y+z)^{\frac{1}{3}} = -(z+x)^{\frac{1}{3}}$, तो $(x^3 + y^3 + z^3)$ को किस रूप में व्यक्त किया जा सकता है:

- a) $(x+y)(y+z)(z+x)$ b) $\frac{1}{8}xyz$
~~c) $\frac{3}{8}(x+y)(y+z)(z+x)$~~ d) $3xyz$

$$\boxed{x^3 + y^3 + z^3} + 3(x+y)(y+z)(z+x) = \frac{27}{8} (x+y)(y+z)(z+x)$$

$$= \left(\frac{27}{8} - 3 \right) \boxed{}$$

$$= \frac{3}{8} \boxed{}$$

coaching center

510. If $A = \frac{(0.1)^3 + (0.2)^3 + (0.3)^3 + 3(0.005 + 0.016 + 0.027) + 0.036}{(0.1)^2 + (0.2)^2 + (0.3)^2 + 0.04 + 0.06 + 0.12}$, Then the value of $60A$ is:

यदि $A = \frac{(0.1)^3 + (0.2)^3 + (0.3)^3 + 3(0.005 + 0.016 + 0.027) + 0.036}{(0.1)^2 + (0.2)^2 + (0.3)^2 + 0.04 + 0.06 + 0.12}$ तो, $60A$ का मान है:

a) 20

b) 60

c) 36

d) 30

a

b

c

$.1^2(2+3)$

$6 \times .1 \times .2 \times .3$

$6abc$

$$A = \frac{a^3 + b^3 + c^3 + 3[a^2(b+c) + b^2(c+a) + c^2(a+b)] + 6abc}{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$$

$$= \frac{(a+b+c)^3}{(a+b+c)^2} = a+b+c = .6$$

$$60 \times .6 = 36$$

511. A and B are positive integers, If $A + B + AB = 65$, then what is the difference between A and B ($A, B < 15$)?

A तथा B धनात्मक पूर्णांक हैं। यदि $A + B + AB = 65$ है, तो A तथा B के मध्य अंतर क्या है? ($A, B < 15$)

a) 3

b) 4

~~c) 5~~

d) 6

$$\boxed{AB + A + B + 1}$$

$$(A+1)(B+1)$$

$$A, B = 10, 5$$

$$(A+1)(B+1) = 66$$

$$\times 1 \times 66$$

$$\times 2 \times 33$$

$$\times 3 \times 22$$

$$6 \times 11$$

$$11 \times 6$$

52. A and B are positive integers, If $A + B + AB = 23$, then how many values can $(A - B)$ have?

A तथा B धनात्मक पूर्णांक हैं। यदि $A + B + AB = 23$ है, तो $(A - B)$ के कितने मान हो सकते हैं?

a) 3

b) 6

~~c) 8~~

d) 4

A B

$$0 - 23 = -23$$

$$A - B = (A + 1) - (B + 1)$$

$$(A + 1)(B + 1) = 24$$

$$\begin{array}{l} 1 \times 24 = -23 \\ 2 \times 12 \\ 4 \times 3 \times 8 \\ 4 \times 6 = -2 \\ 6 \times 4 = 2 \end{array}$$

$$(a+b+c)(ab+bc+ca) - abc$$
$$= (a+b)(b+c)(c+a)$$

formula

$$c=0$$

$$(a+b)ab$$

$$b=0$$

$$(a+c)ac$$

513. Solve the following $(a+b+c)(ab+bc+ca) - abc = ?$

निम्नलिखित को हल करें

$$(a+b+c)(ab+bc+ca) - abc = ?$$

~~a)~~ $(a+b)(b+c)(c-a)$

~~b)~~ $(a-b)(b-c)(c-a)$

~~c)~~ $(a+b)(b-c)(c+a)$

~~d)~~ $(a+b)(b+c)(c+a)$

coaching center

e1

coaching center

514. If $a + b + c = 0$ then the value of $(a + b)(b + c)(c + a) + abc$ is equal to:

यदि $a + b + c = 0$ है, तो $(a + b)(b + c)(c + a) + abc$ का मान बराबर है :

a) -1

b) 1

$$\begin{array}{c} \cancel{c} \neq 0 \\ \underbrace{-c} \times \underbrace{-a} \times -b \\ \text{d) 18} \end{array}$$

$$a + b = -c$$

$$b + c = -a$$

$$-abc + abc = 0$$

coaching center

515. If $a + b + c = 0$ then the value of $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$

is

यदि $a + b + c = 0$ तो $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$ का मान निकालें

~~a) 0~~

b) 1

c) 3

d) 2

$$\begin{aligned} & \frac{1}{ac} + \frac{1}{ab} + \frac{1}{bc} \\ &= \frac{a+b+c}{abc} = \frac{0}{abc} = 0 \end{aligned}$$

coaching center

516. If $a + b + c = 0$, then $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$

अगर $a + b + c = 0$ है तो $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$ का मान:

a) 8

b) -3

~~c) 9~~

d) 0

$$(-1-1-1)(-1-1-1)$$

$$= -3 \times -3 = 9$$

coaching center

517. The coefficient of x^3y in $(x - 2y) \times (5x + y)^3$ is:

$(x - 2y) \times (5x + y)^3$ में x^3y का गुणांक (coefficient) ज्ञात करें।

a) 75

b) -150

c) 250

~~d) -175~~

$3 \times 25 \times x^2y$

(i) $x \times x^2y$

$1 \times 75 = 75 x^3y^3$

$(2a-3b)(a^3+3a^2b)$

a^3b

(ii) $y \times x^3$

$-2 \times 125 = -250$ } -175

$a \times a^2b = a^3b$
 $2 \times 3 = 6a^3b$

$b \times a^3 = a^3b$
 $-3 \times 1 = -3a^3b$

coaching center

518. What should be added to $x^4 + 8x^3 + 26x^2 + 40x + 15$ to obtain a perfect square

$x^4 + 8x^3 + 26x^2 + 40x + 15$ में क्या जोड़े ताकि एक पूर्ण वर्ग मिले .

~~a) 5~~

~~b) 10~~

~~c) 15~~

~~d) 21~~

$$x=0$$

$$15$$

$$x=1$$

$$1+8+26+40+15=90$$

coaching center

519. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$ then $c + \frac{1}{a}$ is equal to

अगर $a + \frac{1}{b} = 1$ और $b + \frac{1}{c} = 1$ है तो $c + \frac{1}{a}$:

a) 0

b) $\frac{1}{2}$

b +

~~c) 1~~

d) 2

$$c = 1$$

$$b + 1 = 1$$

X

$$b = 0$$

$$c = \frac{1}{2}$$

$$b + 2 = 1$$

$$b = -1$$

$$a - 1 = 1$$

$$c + \frac{1}{a} = \frac{1}{2} + \frac{1}{2} = 1$$

$$a = 2$$

Q 20. If a, b, c are non-zero, $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then the value of abc is

अगर a, b, c गैर शून्य हैं, $a + \frac{1}{b} = 1$ और $b + \frac{1}{c} = 1$, तो abc का मूल्य:

~~a) -1~~

b) 3

c) -3

d) 1

$$c = \frac{1}{2}$$

$$b + 2 = 1$$

$$b = -1$$

$$a - 1 = 1$$

$$a = 2$$

$$\begin{aligned} &\downarrow \\ &2 \times -1 \times \frac{1}{2} \\ &= -1 \end{aligned}$$

coaching center

521. If $a^2 + b^2 + c^2 = 16$, $x^2 + y^2 + z^2 = 25$ and $ax + by + cz = 20$, then the value of $\frac{a+b+c}{x+y+z} = \frac{4+0+0}{5+0+0} = \frac{4}{5}$

यदि $a^2 + b^2 + c^2 = 16$, $x^2 + y^2 + z^2 = 25$ और $ax + by + cz = 20$ हो, तो $\frac{a+b+c}{x+y+z}$ का मान निकालें ? $y=0$

a) $\frac{3}{5}$

b) $\frac{5}{3}$

~~c) $\frac{4}{5}$~~

d) $\frac{5}{4}$

$ax = 20$

$c = 0$

$z = 0$

$a^2 = 16$

$x = 5$

$b = 0$

$a = 4$

coaching center

Q22. If $\frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$ and $a - b + c \neq 0$, then which one of the following relation is true ?

यदि $\frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$ और $a - b + c \neq 0$ है, तो तो निम्न में से कौन सा कथन सत्य है ?

~~a) $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$~~

~~b) $\frac{1}{a} = \frac{1}{b} - \frac{1}{c}$~~

~~c) $\frac{1}{b} = \frac{1}{a} + \frac{1}{c}$~~

~~d) $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$~~

$b=c=1$

$a+1+a-1 = 1$

$1 = 2+1$

$1 = 2+1$

$2a = 1$

$a = \frac{1}{2}$

coaching center

Q23. If $ax + by = 1$ and $bx + ay = \frac{2ab}{a^2+b^2}$ then $(x^2 + y^2)(a^2 + b^2)$ is equal to

यदि $ax + by = 1$ और $bx + ay = \frac{2ab}{a^2+b^2}$ हो तो $(x^2 + y^2)(a^2 + b^2)$ किसके बराबर है ?

~~a) 1~~

b) 2

c) 0.5

d) 0

$$y=0$$

$$b=0$$

$$ax=1$$

$$0=0$$

$$\begin{aligned}x^2 \times a^2 &= (xa)^2 \\ &= (1)^2 = 1\end{aligned}$$

$$y=0$$

$$a=0$$

$$0=1$$

coaching center

524. If $\sqrt{\frac{x}{y}} = \frac{10}{3} - \sqrt{\frac{y}{x}}$ and $x - y = 8$, then the value of xy is equal to

यदि $\sqrt{\frac{x}{y}} = \frac{10}{3} - \sqrt{\frac{y}{x}}$ और $x - y = 8$ है, तो xy का मान बराबर है :

a) 36

b) 24

c) 16

~~d) 9~~

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3} = 3\frac{1}{3} = 3 + \frac{1}{3}$$

$$\sqrt{\frac{x}{y}} = 3, \frac{1}{3}$$

$$\frac{x}{y} = \frac{9}{1} \quad \frac{1}{9} \quad x$$

$$x = 9$$

$$y = 1$$

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3}$$

$$a = \sqrt{\frac{x}{y}}$$

$$\Rightarrow a + \frac{1}{a} = \frac{10}{3}$$

$$\sqrt{\frac{x}{y}} = 3, \frac{1}{3}$$

$$\textcircled{3a} \Rightarrow 3a^2 - 10a + 3 = 0$$

$$a = \frac{10 \pm 8}{6} = 3, \frac{1}{3}$$

$$\sqrt{100 - 36}$$

525. If $\sqrt{\frac{x}{y}} = \frac{24}{5} + \sqrt{\frac{y}{x}}$ and $x + y = 26$, then what is the value of xy ?

यदि $\sqrt{\frac{x}{y}} = \frac{24}{5} + \sqrt{\frac{y}{x}}$ और $x + y = 26$ है, तो xy का मान क्या होगा ?

a) 5

b) 15

~~c) 25~~

d) 30

$$\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \frac{24}{5} = 5 - \frac{1}{5}$$

$$\Rightarrow \sqrt{\frac{x}{y}} = 5$$

$$\Rightarrow \frac{x}{y} = \frac{25}{1}$$

$$x = 25$$

$$y = 1$$

526. If $3\sqrt{\frac{1-a}{a}} + 9 = 19 - 3\sqrt{\frac{a}{1-a}}$, then what is the value of a ?

यदि $3\sqrt{\frac{1-a}{a}} + 9 = 19 - 3\sqrt{\frac{a}{1-a}}$ है, तो a का मान क्या है?

a) $\frac{3}{10}, \frac{7}{10}$

~~b) $\frac{1}{10}, \frac{9}{10}$~~

c) $\frac{2}{5}, \frac{3}{5}$

d) $\frac{1}{5}, \frac{4}{5}$

$$\cancel{3}\sqrt{\frac{1-a}{a}} + \cancel{3}\sqrt{\frac{a}{1-a}} = \frac{10}{3} = 3\frac{1}{3} = 3 + \frac{1}{3}$$

$$\sqrt{\frac{1-a}{a}} = 3, \frac{1}{3} \Rightarrow \frac{1-a}{a} = 9, \frac{1}{9}$$

$$\frac{1-a}{a} = 9$$

$$\Rightarrow 1-a = 9a \Rightarrow a = \frac{1}{10}$$

$$\frac{1-a}{a} = \frac{1}{9} \Rightarrow 9-9a = a \Rightarrow \frac{9}{10} = a$$

527. The value of expression $x^4 - 17x^3 + 17x^2 - 17x + 17$ at $x = 16$ is
अगर $x = 16$ है तो $x^4 - 17x^3 + 17x^2 - 17x + 17 = ?$

- a) 0 ~~b) 1~~ c) 2 d) 3

$$= x^4 - 16x^3 - \cancel{x^3} + 16x^2 + \cancel{x^2} - 16x - x + 17$$

$$= 16^4 - 16 \times 16^3$$

$$-16 + 17 = 1$$

coaching center

528. If $x = 11$, then the value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1 = ?$

अगर $x = 11$ है तो $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1 = ?$

a) 5

~~b) 10~~

c) 15

d) 20

$$= x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$$

$$11 - 1 = 10$$

coaching center

529. Expand $x^2 + 2x + 3$ about $x = -2$.

$x = -2$ के संदर्भ में $x^2 + 2x + 3$ का प्रसार करें।

$$x = -2$$
$$\Rightarrow (x+2) = 0$$

अथ तर्क

~~$(x-2)^2 - 2(x+2) + 3$~~

~~$(x+2)^2 + 2(x+2) + 3$~~ $4+4+3$

~~$(x+2)^2 - 2(x+2) + 3$~~ $4-4+3$

~~$(x-2)^2 - 2(x-2) - 3$~~

$$x = 0 \quad 0+0+3 = 3$$

coaching center