Symmetrical and Cyclic Expressions

Concept:

सारे variables equally treat

⇓

Most probably सारे variables equal होने पर eqn satisfy करती है।

$$\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$$

Put all variables equal.

$a=b=c=a$

$\left(\frac{a}{1-a}\right)\times 3=1$

$\Rightarrow 3a=1-a$

$\Rightarrow 4a=1 \Rightarrow a=\frac{1}{4}$

More examples:

$$\frac{m-a^2}{b^2+c^2}+\frac{m-b^2}{c^2+a^2}+\frac{m-c^2}{a^2+b^2}=3$$

$$x^2=y+z, y^2=z+x, z^2=x+y$$

Examples of not satisfying:

$$\frac{a^2-bc}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$$

ND

$a=b=c=a$

$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=1$$

$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=1$

$0+0+0=1$

$2=3$

$0=1$

477. If $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

अगर $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$ है तो $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ पता करें|

a) 1 b) 2 c) 3 ~~d) 4~~

$a=b=c$

$\left(\frac{a}{1-a}\right)\times 3 = 1$

$\Rightarrow 3a = 1-a$

$\Rightarrow 4a = 1$

$\Rightarrow a = \frac{1}{4}$

$\left(\frac{1}{\frac{3}{4}}\right)\times 3$

$= \frac{4}{3}\times 3$

478. If $\frac{a}{1-2a}+\frac{b}{1-2b}+\frac{c}{1-2c}=\frac{1}{2}$, then $\frac{1}{1-2a}+\frac{1}{1-2b}+\frac{1}{1-2c}$ is

अगर $\frac{a}{1-2a}+\frac{b}{1-2b}+\frac{c}{1-2c}=\frac{1}{2}$ तो $\frac{1}{1-2a}+\frac{1}{1-2b}+\frac{1}{1-2c}$ का मान:

a) -1 b) -4 c) 2 ~~d) 4~~

$a=b=c$

$\left(\frac{a}{1-2a}\right)\times 3=\frac{1}{2}$

$\Rightarrow 6a=1-2a$

$\Rightarrow a=\frac{1}{8}$

$\left(\frac{1}{\frac{3}{4}}\right)\times 3$

$=\frac{4}{3}\times 3=4$

479. $\frac{m-a^2}{b^2+c^2}+\frac{m-b^2}{c^2+a^2}+\frac{m-c^2}{a^2+b^2}=3$, then the value of m is:

$\frac{m-a^2}{b^2+c^2}+\frac{m-b^2}{c^2+a^2}+\frac{m-c^2}{a^2+b^2}=3$ है तो m पता करें|

a) $a^2+b^2=2a^2$ b) $a^2+b^2+c^2$

c) $a^2-b^2-c^2=-a^2$ d) $a^2+b^2-c^2$

$a=b=c$

$\frac{(m-a^2)3}{2a^2}=3$

$\Rightarrow m-a^2=2a^2$

$\Rightarrow m=3a^2$

480. If $\frac{m-3a^3}{b^3+c^3}+\frac{m-3b^3}{c^3+a^3}+\frac{m-3c^3}{a^3+b^3}=9$ then, $m=$?

यदि $\frac{m-3a^3}{b^3+c^3}+\frac{m-3b^3}{c^3+a^3}+\frac{m-3c^3}{a^3+b^3}=9$ है, तो $m=$?

a) $a^3+b^3+c^3$ b) $2a^3+2b^3+2c^3$

c) $3a^3+3b^3+3c^3$ d) 2

$a=b=c$

$\frac{(m-3a^3)}{2a^3}\times 3=9$

$\Rightarrow m-3a^3=6a^3$

$\Rightarrow m=9a^3$

481. If $\frac{x-a^2}{b+c}+\frac{x-b^2}{c+a}+\frac{x-c^2}{a+b}=4(a+b+c)$, then x is equal to

HW

अगर $\frac{x-a^2}{b+c}+\frac{x-b^2}{c+a}+\frac{x-c^2}{a+b}=4(a+b+c)$ है तो x किसके समान है?

a) $(a+b+c)^2$ b) $a^2+b^2+c^2$

c) $ab+bc+ca$ d) $a^2+b^2+c^2-ab-bc-ca$.

On putting $a=b=c=a$

$\left(\frac{x-a^2}{2a}\right)\times 3=4(3a)$

$\Rightarrow x-a^2=8a^2$

$\Rightarrow x=9a^2$

only option a gives $9a^2$

on putting $a=b=c\neq a$

482. If $\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=(a+b+c)$ find the value of x ?

(HW) यदि $\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=(a+b+c)$ हो तो x का मान ज्ञात करो |

a) ab b) $bc+ca$

c) $ab+bc$ d) $ab+bc+ca$

On putting $a=b=c=a$

$$\frac{x-a^2}{2a}\times 3=3a$$

$$\Rightarrow x-a^2=2a^2$$

$$\Rightarrow x=3a^2$$

On putting $a=b=c=a$

only option d gives $3a^2$

483. If $xy + yz + zx = 1$ then the value of $\frac{1+y^2}{[(x+y)(y+z)]}$ is

अगर $xy + yz + zx = 1$ तो $\frac{1+y^2}{[(x+y)(y+z)]}$ का मान:

a) -1 b) 1 c) 2 d) 4

$x=y=z$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

$x=y=2$

$$\frac{1+x^2}{4x^2} = \frac{\frac{4}{3}}{\frac{4}{3}} = 1$$

484. If $XY + YZ + XZ = 1$ then find the value of $\frac{x+y}{1-xy} + \frac{y+z}{1-yz} + \frac{z+x}{1-zx}$.

अगर $XY + YZ + XZ = 1$ तो $\frac{x+y}{1-xy} + \frac{y+z}{1-yz} + \frac{z+x}{1-zx}$ का मान:

a) $xyz = x^3 = \frac{1}{3\sqrt{3}}$

b) $\frac{1}{xyz}$

c) $xy + yz + zx$

d) $x^2 + y^2 + z^2$

$x=y=z$

$3x^2=1$

$x^2=\frac{1}{3}$

$x=\pm\frac{1}{\sqrt{3}}$

$x=y=z$

$\frac{2x \times 3}{1-x^2} = \frac{6 \times \frac{1}{\sqrt{3}}}{\frac{2}{3}}$

$= \pm 3\sqrt{3}$

485. If $ab + bc + ca = abc$ then $\frac{b+c}{bc(a-1)} + \frac{c+a}{ca(b-1)} + \frac{a+b}{ab(c-1)}$ is equal to

अगर $ab + bc + ca = abc$ तो $\frac{b+c}{bc(a-1)} + \frac{c+a}{ca(b-1)} + \frac{a+b}{ab(c-1)}$ का मान:

a) -1 b) 1 c) $\frac{1}{3}$ d) 0

$a = b = c$

$3a^2 = a^3$

$3 = a$

$\frac{6}{9 \times 2} \times 3 = 1$

48. The value of x which satisfies the equation $\frac{x+a^2+2c^2}{b+c}+\frac{x+b^2+2a^2}{c+a}+\frac{x+c^2+2b^2}{a+b}=0$ is

x का मान बताइए जो समीकरण $\frac{x+a^2+2c^2}{b+c}+\frac{x+b^2+2a^2}{c+a}+\frac{x+c^2+2b^2}{a+b}=0$ को संतुष्ट करता है ।

a) $(a^2+b^2+c^2)$
b) $-(a^2+b^2+c^2)$
c) $(a^2+2b^2+c^2)$
d) $-(a^2+b^2+2c^2)$

$a=b=c$

$\left(\frac{x+3a^2}{2a}\right)\times 3=0$

$\Rightarrow x=-3a^2$

$x = y = 2$

४४7. If $x^2 = y + z,\ y^2 = z + x,\ z^2 = x + y$, then the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$ is

अगर $x^2 = y + z,\ y^2 = z + x,\ z^2 = x + y$ है तो $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$ का मान:

a) -1 b) 1 c) 2 d) 4

$x^2 = 2x$

$\frac{1}{3} \times 3 = 1$

488. Find the value of xyz, if $x^2 + y^2 = z + 1, y^2 + z^2 = x + 1$ and $z^2 + x^2 = y + 1$.

अगर $x^2 + y^2 = z + 1, y^2 + z^2 = x + 1$ और $z^2 + x^2 = y + 1$, तो xyz का मान क्या होगा?

a) 1 or $-\frac{1}{2}$ b) $1, -\frac{1}{8}$ c) $1, \frac{1}{8}$ d) $-1, \frac{1}{8}$

$x = y = z$

$x^3 =$

$2x^2 = x + 1$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow x = \frac{1 \pm 3}{4} = 1, \frac{-1}{2}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sqrt{1+8}$

४४१. If $x = \frac{a-b}{a+b}, y = \frac{b-c}{b+c}, z = \frac{c-a}{c+a}$ then find the value of $\frac{(1-x)(1-Y)(1-z)}{(1+x)(1+y)(1+z)}$

यदि $x = \frac{a-b}{a+b}, y = \frac{b-c}{b+c}, z = \frac{c-a}{c+a}$ हो तो $\frac{(1-x)(1-Y)(1-z)}{(1+x)(1+y)(1+z)}$ का मान ज्ञात करो |

a) 0 b) 1 c) 2 d) 3

$a=b=c$

$x=y=z$

$x=0 \quad y=0 \quad z=0$

$\frac{1}{1} = 1$

490. If $x = \frac{a}{b} + \frac{b}{a}$, $y = \frac{b}{c} + \frac{c}{b}$ and $z = \frac{c}{a} + \frac{a}{c}$, then what is the value of $xyz - x^2 - y^2 - z^2$?

यदि $x = \frac{a}{b} + \frac{b}{a}$, $y = \frac{b}{c} + \frac{c}{b}$ तथा $z = \frac{c}{a} + \frac{a}{c}$ है, तो $xyz - x^2 - y^2 - z^2$ का मान क्या है?

~~a) -4~~ b) 2 c) -1 d) -6

$a=b=c$

$x=y=z$

$x = 1+1 = 2$ $y = 2$ $z = 2$

$8 - 4 - 4 - 4$

$= -4$

491. If $\frac{x^2}{by+cz} = \frac{y^2}{ax+cz} = \frac{z^2}{ax+by} = 1$ Find $\frac{a}{x+a} + \frac{b}{y+b} + \frac{c}{z+c}$?

यदि $\frac{x^2}{by+cz} = \frac{y^2}{ax+cz} = \frac{z^2}{ax+by} = 1$ है, तो $\frac{a}{x+a} + \frac{b}{y+b} + \frac{c}{z+c}$ का मान ज्ञात करो |

a) 1 b) 2 c) 3 d) 4

$a = b = c$

$x = y = z$

$\frac{x^2}{2ax} = 1$

$x = 2a$

$\left(\frac{a}{2a+a}\right)^3$

$= \frac{a}{3a} \times 3 = 1$

492. If $\frac{x^2}{by+cz} = \frac{y^2}{ax+cz} = \frac{z^2}{ax+by} = 1$, Find $\frac{x}{x+a} + \frac{y}{y+b} + \frac{z}{z+c} = ?$

यदि $\frac{x^2}{by+cz} = \frac{y^2}{ax+cz} = \frac{z^2}{ax+by} = 1$ है, तो $\frac{x}{x+a} + \frac{y}{y+b} + \frac{z}{z+c}$ का मान ज्ञात करो |

a) 1 ~~b) 2~~ c) 3 d) 4

a=b=c

x=y=z

$\frac{x^2}{2ax} = 1$

$x = 2a$

$\frac{2a}{3a} \times 3$

Maxima and Minima

for all positive real nos, $AM \geq GM$, [AM=GM when Nos are equal]

$$AM = \frac{a+b}{2} = \frac{a+b+c}{3} = \frac{a+b+c+d}{4}$$

$$GM = (ab)^{\frac{1}{2}} = (abc)^{\frac{1}{3}} = (abcd)^{\frac{1}{4}}$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$a=4$, $b=1$

$AM = 2.5$

$GM = \sqrt{4 \times 1} = 2$

$2.5 \geq 2$

Concept: Sum of a number and its reciprocal

$x \rightarrow$ Real, positive

$$x + \frac{1}{x} \geq 2$$

Proof

AM $\geq$ GM

$$\frac{x + \frac{1}{x}}{2} \geq \sqrt{x \cdot \frac{1}{x}}$$

$$\Rightarrow x + \frac{1}{x} \geq 2$$

$$x = \frac{1}{x}$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1 = 1$$

Concept: Sum or product is fixed

① Sum = given, Prod (max) = ?

↓

when all variables equal.

5 5 2
10 1 1
12 0 0

Ques: $a+b+c=12$

4 4 4

$abc_{(max)} =$

$4 \times 4 \times 4 = 64$

$abc_{(min)} = -\infty$

$\frac{12}{3} = 4$

-7 -1

$a+b=8$

4 4

$ab = 16$

$-7 \times -1 = 7$

② Prod = given, $Sum_{(min)} = ?$

when all variables equal.

Ques: 125 = 5 × 5 × 5

$abc = 125$

$5 \times 5 \times 5$

$a+b+c_{(min)} =$

$5+5+5$

$= 15$

$a+b+c_{(max)} = \infty$

493. If x is a real number, find the minimum value of $x^2 + \frac{1}{x^2}$

यदि x एक वास्तविक संख्या है तो $x^2 + \frac{1}{x^2}$ का न्यूनतम मान है :

a) 1 b) -1 c) 2 d) $\frac{1}{2}$

494. If x is a real number, find the minimum value of $x^2 + \frac{1}{x^2+1}$

यदि x एक वास्तविक संख्या है तो $x^2 + \frac{1}{x^2+1}$ का न्यूनतम मान है :

a) 1 b) -1 c) 2 d) $\frac{1}{2}$

$$x^2 + \frac{1}{x^2+1} + 1 - 1$$

$$= (x^2+1) + \frac{1}{x^2+1} - 1$$

$$= 2 - 1 = 1$$

495. If a, b, c are positive real numbers, then the least value of $\frac{ab(a+b)+bc(b+c)+ca(c+a)}{abc}$ is :

यदि a, b, c तीन धनात्मक वास्तविक संख्याएं हैं, तो $\frac{ab(a+b)+bc(b+c)+ca(c+a)}{abc}$ का न्यूनतम मान होगा:

a) 1 b) 9 c) 6 d) 4

$\frac{p+q+r}{x} = \frac{p}{x} + \frac{q}{x} + \frac{r}{x}$

$\frac{ab(a+b)}{abc}$

$\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}$

$= \frac{a}{c} + \frac{b}{c} + \frac{b}{a} + \frac{c}{a} + \frac{c}{b} + \frac{a}{b}$

$2 + 2 + 2 = 6$

496. If a, b, c are all positive real numbers, then the minimum value of the expression $\frac{(a^2+a+1)(b^2+b+1)(c^2+c+1)}{abc}$ is:

यदि a, b, c तीन धनात्मक वास्तविक संख्याएं हैं, तो $\frac{(a^2+a+1)(b^2+b+1)(c^2+c+1)}{abc}$ का न्यूनतम मान है :

a) 3 b) 9 c) 27 d) 1

$a \quad abc \quad c$

b

$\left(a+\frac{1}{a}+1\right)\left(b+\frac{1}{b}+1\right)\left(c+\frac{1}{c}+1\right)$

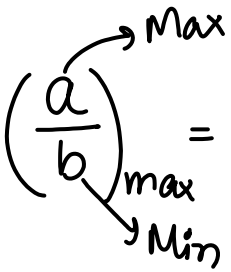
2

$= 3 \times 3 \times 3$

47. If x is a real number, find the maximum value of $\frac{x^8}{x^{16}+1}$

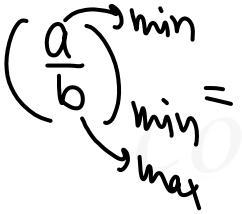
यदि x एक वास्तविक संख्या है तो $\frac{x^8}{x^{16}+1}$ का अधिकतम मान है :

a) 1 b) -1 c) 2 d) $\frac{1}{2}$



$= \frac{1}{x^8 + \frac{1}{x^8}} = \frac{1}{2}$

x^8 → min



498. For $x > 0$, what is the minimum value of $x + \frac{x+2}{2x}$?

HW $x > 0$ के लिए, $x + \frac{x+2}{2x}$ का न्यूनतम मान क्या होगा?

a) 1 b) 2 c) $2\frac{1}{2}$ d) Cannot be determined

$$x + \frac{x}{2x} + \frac{2}{2x}$$

$$= \left(x + \frac{1}{x}\right) + \frac{1}{2}$$

min = 2

$$= 2 + \frac{1}{2}$$

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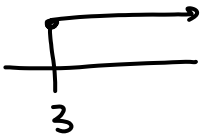
499. If x is a real number, find the minimum value of $9x + \frac{1}{4x}$

यदि x एक वास्तविक संख्या है तो $9x + \frac{1}{4x}$ का न्यूनतम मान है :

a) 1 b) 4 c) 2 d) 3

$AM \geq GM$



$$\frac{9x + \frac{1}{4x}}{2} \geq \sqrt{9x \times \frac{1}{4x}}$$

$$9x + \frac{1}{4x} \geq 3 \times 2$$

520. If $x + y + z = 24$, then the maximum value of xyz is

अगर $x + y + z = 24$ है तो xyz का अधिकतम मान ज्ञात करें|

a) 728 8 + 8 + 8   b) 64   c) 512   d) 400

Q. If $a+b+c+d=1$,then the maximum value of $(1+a)(1+b)(1+c)(1+d)$is

अगर $a+b+c+d=1$ है तो $(1+a)(1+b)(1+c)(1+d)$ का अधिकतम मान ज्ञात करें|

a) 1 b) $\left(\frac{1}{2}\right)^3$ c) $\left(\frac{3}{4}\right)^3$ d) $\left(\frac{5}{4}\right)^4$

$(1+a)+(1+b)+(1+c)+(1+d)=5$

$5/5$ $5/5$ $5/5$ $5/5$

502. If $a+b+c+d=2$, then the maximum value of $(1+a)(2+b)(3+c)(4+d)$ is _____ ?

यदि $a+b+c+d=2$ है, तो $(1+a)(2+b)(3+c)(4+d)$ का अधिकतम मान _____ है?

$1+\ 2+\ 3+\ 4+\ 10$

$3 \times 3 \times 3 \times 3 = 81$

a) 59.625 b) 81 c) 80 d) 64

$$\frac{(1+a)}{3}+\frac{(2+b)}{3}+\frac{(3+c)}{3}+\frac{(4+d)}{3}=12$$

$\frac{12}{4}=3$

SD3. If $x + y + z = 18$, the maximum value of $(x-2)(y+3)(z-4) =$?

यदि $x + y + z = 18$ है, $(x-2)(y+3)(z-4)$ का अधिकतम मान है:

a) 72 b) 125 c) 24 d) 216

$(x-2) + (y+3) + (z-4) = 15$

5 5 5

Q54. If $x + y + z = 21$ then find the maximum value of $(x-2)(y-1)(z+9)$.

यदि $x + y + z = 21$ है, तो $(x-2)(y-1)(z+9)$ का अधिकतम मान ज्ञात करो |

HW

a) 480 b) 729 c) 512 d) 216

$(x-2) + (y-1) + (z+9) = 21 - 2 - 1 + 9$

$9 \quad 9 \quad 9 \quad = 27$

505 If $abc = 125$ (where a, b, c are real numbers), then the minimum value of $a + b + c$ is

अगर $abc = 125$ है (जहां a, b, c वास्तविक संख्याएं हैं), तो $a + b + c$ का न्यूनतम मान क्या होगा?

a) 25 b) 15 c) 126 d) 100

506. If $(x-7)(y-10)(z-12)=1000$, then find the minimum value of $(x+y+z)$.

यदि $(x-7)(y-10)(z-12)=1000$ है, तो $(x+y+z)$ का न्यूनतम मान ज्ञात करो |

a) 49 b) 30 c) 59 d) None of these

$x-7=10$, $y-10=10$

$x=17$ $y=20$

$z-12=10$

$z=22$

$(x-7)+(y-10)+(z-12)$

$=x+y+z-29$

507. If $(x-5)(y+6)(z-8)=1331$, then the minimum value of $(x+y+z)$ is :

यदि $(x-5)(y+6)(z-8)=1331$ है, तो $(x+y+z)$ का न्यूनतम मान क्या होगा :

a) 40 b) 33 c) 19 d) Not unique

$x=16,\ y=5,\ z=19$