$$x^4 + x^2y^2 + y^4$$

Concept:

$$x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$$

Proof: [Based on $a^2-b^2=(a+b)(a-b)$]

$a^4-b^4=(a^2+b^2)(a^2-b^2)$

$a^3-b^3=(a^{\frac{3}{2}}+b^{\frac{3}{2}})(a^{\frac{3}{2}}-b^{\frac{3}{2}})$

$$x^4+x^2y^2+y^4=(x^2+y^2)^2-2x^2y^2+x^2y^2$$

$$=(x^2+y^2)^2-(xy)^2$$

$$=(x^2+y^2+xy)(x^2+y^2-xy)$$

$$\Rightarrow x^4+x^2y^2+y^4=(x^2+xy+y^2)(x^2-xy+y^2)$$

$$x^4+x^2y^2+y^4=(x^2+xy+y^2)(x^2-xy+y^2)$$

$$x^8+x^4y^4+y^8=(x^4+x^2y^2+y^4)(x^2-x^2y^2+y^4)$$

$$x^{16}+x^8y^8+y^{16}=(x^8+x^4y^4+y^8)(x^8-x^4y^4+y^8)$$

$$(x^4+x^2y^2+y^4)\times(x^4-x^2y^2+y^4)$$

$$(x^2+xy+y^2)(x^2-xy+y^2)$$

$$x^2+x^1y^1+y^2=\left(x+\sqrt{xy}+y\right)\left(x-\sqrt{xy}+y\right)$$

$$x^6+x^3y^3+y^6=\left(x^3+xy\sqrt{xy}+y^3\right)\left(x^3-xy\sqrt{xy}+y^3\right)$$

$$\left(x^4+x^2+1^4\right)=\left(x^2+x+1^2\right)\left(x^2-x+1\right)$$

$$\uparrow y=1$$

$$x^4+x^2y^2+y^4$$

Ques.

$$x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$$

$\Downarrow$ 21 $\quad$ $\Downarrow$ 7 $\quad$ $\Downarrow$? $\frac{21}{7} = 3$

$$x^2 + xy + y^2 = 7 \quad \text{—①}$$

$$-x^2 + xy - y^2 = 3 \quad \text{—②}$$

① − ②

$$2xy = 7 - 3 = 4$$

$$xy = \frac{4}{2} = 2 = \frac{\text{diff}}{2}$$

① + ②

$$2(x^2 + y^2) = 7 + 3 = 10$$

$$x^2 + y^2 = \frac{10}{2} = 5 = \frac{\text{Sum}}{2}$$

153. If $a^4 + a^2b^2 + b^4 = 8$ and $a^2 + ab + b^2 = 4$, then the value of ab is

अगर $a^4 + a^2b^2 + b^4 = 8$ और $a^2 + ab + b^2 = 4$ है तो ab का मान

a) -1 b) 0 c) 2 d) 1

$a^2 - ab + b^2 = 2 = \frac{8}{4}$

$2ab = 4 - 2 = 2$

$ab = 1$

154. If $x^4 + x^2y^2 + y^4 = 273$ and $x^2 - xy + y^2 = 13$ then the value of xy is:

अगर $x^4 + x^2y^2 + y^4 = 273$ और $x^2 - xy + y^2 = 13$ है तो xy का मान क्या होगा?

a) 4 b) 10 c) 6 d) 8

$x^2 + xy + y^2 = 21$

$\frac{273}{13}$

$xy = \frac{+ - -}{2} = \frac{21-13}{2} = 4$

155. If $x^4+x^2y^2+y^4=21$ and $x^2+xy+y^2=7$ then the value of $\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$ is:

अगर $x^4+x^2y^2+y^4=21$ और $x^2+xy+y^2=7$ है तो $\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$ का मान बताओ.

a) $\frac{5}{4}$ b) $\frac{7}{4}$ c) $\frac{7}{3}$ d) $\frac{5}{2}$

$x^2-xy+y^2=3$

$x^2+y^2=\frac{\text{Sum}}{2}=5$

$xy=\frac{+-(-)}{2}=2$

$\frac{x^2+y^2}{x^2y^2}=\frac{5}{4}$

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156. If $x^4 + y^4 + x^2y^2 = 117$ and $x^2 + y^2 - xy = 3\left(4 + \sqrt{3}\right)$, then the value of $(x^2 + y^2)$ will be:

यदि $x^4 + y^4 + x^2y^2 = 117$ और $x^2 + y^2 - xy = 3\left(4 + \sqrt{3}\right)$ है, $(x^2 + y^2)$ का ज्ञात करें|

a) 9 b) $6\sqrt{3}$ c) 12 d) $13\sqrt{3}$

$x^2+y^2+xy = 3(4-\sqrt{3})$

$\frac{117}{3(4+\sqrt{3})} = \frac{\cancel{9}\,3\ 117(4-\sqrt{3})}{\cancel{3}\times 13}$

$16-3$

$x^2+y^2 = \frac{\text{Sum}}{2} = \frac{3\times 8}{2} = 12$

157. If $x^4+x^2y^2+y^4=35$, $x^2+xy+y^2=7$ and $x,y>0$ then the value of x^3+y^3 is:

अगर $x^4+x^2y^2+y^4=35$, $x^2+xy+y^2=7$ और $x,y>0$ है तो x^3+y^3 का मान बताओ.

a) $10\sqrt{2}$ b) $22\sqrt{2}$ c) $16\sqrt{2}$ d) Can't say

$x^2-xy+y^2=5$

$x^2+y^2=\frac{\text{Sum}}{2}=6$

$+\quad 2xy=2$

$(x+y)^2=8$

$x+y=2\sqrt{2}$

$xy=1$

$x+y=\checkmark$

$xy=\checkmark$

$16\sqrt{2}-3\cdot1\cdot(2\sqrt{2})$

$=10\sqrt{2}$

158. If $x^8 + x^4y^4 + y^8 = 144$ and $x^4 + x^2y^2 + y^4 = 18$, then what is the value of $x^2 + y^2 =$?

अगर $x^8 + x^4y^4 + y^8 = 144$ और $x^4 + x^2y^2 + y^4 = 18$, है तो $x^2 + y^2$ का मान बताओ.

a) 23 b) $\sqrt{23}$ c) 5 d) $\sqrt{18}$

$x^4 - x^2y^2 + y^4 = 8$

$x^4 + y^4 = \frac{\text{Sum}}{2} = 13$

$+ \quad 2x^2y^2 = \frac{\oplus - \ominus}{2} = 5x = 10$

$(x^2 + y^2)^2 = \sqrt{23}$

159. If $x^8 + x^4y^4 + y^8 = 273$, $x^4 + x^2y^2 + y^4 = 21$ and $x, y > 0$, then what is the value of $(x + y)$?

अगर $x^8 + x^4y^4 + y^8 = 273$ और $x^4 + x^2y^2 + y^4 = 21$, $(x, y > 0)$ है तो $(x + y)$ का मान बताओ.

a) 6 b) 12 c) 3 d) Can't say

$x^4 - x^2y^2 + y^4 = 13$

$x^4 + y^4 = \frac{\text{Sum}}{2} = 17$

$+ \quad 2x^2y^2 = 8$

$(x^2 + y^2)^2 = \sqrt{25} = 5$

$2x^2y^2 = 8$

$xy = 2$

$+ \quad 2xy = 4$

$(x + y)^2 = \sqrt{9} = 3$

160. If $x^2 + xy + y^2 = 21$ and $x + \sqrt{xy} + y = 7$, then the value of $x - y$ is
अगर $x^2 + xy + y^2 = 21$ और $x + \sqrt{xy} + y = 7$ है तो $x - y$ का मान बताओ.

a) 9 b) 3 c) 5 d) Can't say

$x - \sqrt{xy} + y = 3$

$x + y = \frac{\text{Sum}}{2} = 5$

$\sqrt{xy} = \frac{⊕ - ⊖}{2} = 2$

$xy = 4$

$(x-y)^2 = 25 - 16 = 9$

$x - y = \pm 3$

161. If $x^2 + xy + y^2 = 84$ & $x - \sqrt{xy} + y = 6$, then find $x^3 + y^3$.

यदि $x^2 + xy + y^2 = 84$ और $x - \sqrt{xy} + y = 6$ है, तो $x^3 + y^3$ का मान ज्ञात करो |

a) 72 b) 520 c) 512 d) 600

$x + \sqrt{xy} + y = 14$

$x + y = 10$

$\sqrt{xy} = 4$

$xy = 16 < 8, 2$

$512 < 8$

$1000 - 316 \cdot 10$

$= 520$

162. If $x^4 + y^4 + x^2y^2 = 17\frac{1}{16}$ and $x^2 - xy + y^2 = 5\frac{1}{4}$, then one of the values of $(x - y)$ is:

यदि $x^4 + y^4 + x^2y^2 = 17\frac{1}{16}$ और $x^2 - xy + y^2 = 5\frac{1}{4}$ है, तो $(x - y)$ का मान बताइए|

a) $\frac{5}{2}$ b) $\frac{3}{4}$ c) $\frac{5}{4}$ d) $\frac{3}{2}$

$= \frac{21}{4}$

$x^2 + xy + y^2 = \frac{13}{4}$

$x^2 + y^2 = \frac{34}{4 \times 2} = \frac{17}{4}$

$-2xy = \oplus - \ominus = + \frac{8}{4} = -2$

$(x-y)^2 = \sqrt{\frac{25}{4}} = \frac{5}{2}$

163. If $a^4 + a^2b^2 + b^4 = 84$ and $a^2 + ab + b^2 = 14$, then find the value of $3a + 2b$.

यदि $a^4 + a^2b^2 + b^4 = 84$ और $a^2 + ab + b^2 = 14$ है, तो $3a + 2b$ का मान ज्ञात करो।

a) $7\sqrt{2}$ b) $8\sqrt{2}$ c) $9\sqrt{2}$ d) Can't say

$a^2 - ab + b^2 = 6$

a & b ~~are~~ Values ~~finite~~

① + ②

$a + b = \sqrt{18} = 3\sqrt{2}$

$a - b = \sqrt{2}$

① − ②

$2\sqrt{2}$ $\sqrt{2}$

$a^2 + b^2 = 10$ — ①

$2ab = 48$ — ②

$6\sqrt{2} + 2\sqrt{2}$

$= 8\sqrt{2}$

164. The value of $\frac{(4.6)^4+(5.4)^4+(24.84)^2}{(4.6)^2+(5.4)^2+24.84}$ is:

$\frac{(4.6)^4+(5.4)^4+(24.84)^2}{(4.6)^2+(5.4)^2+24.84}$ का मान क्या है?

a) 24.42 b) 24.24 c) 25.42 d) 25.48

$4.6=a$

$5.4=b$

$ab=(50-4)(50+4)$

$=2500-16$

$=2484$

$\oplus \times \ominus$

$$\frac{a^4+b^4+(ab)^2}{a^2+b^2+ab}=\frac{(a^2+ab+b^2)(a^2-ab+b^2)}{a^2+ab+b^2}$$

i) $(a+b)^2-3ab$

ii) $(a-b)^2+ab$

$.64+24.84=25.48$

165. If $16a^4 + 36a^2b^2 + 81b^4 = 91$ and $4a^2 + 9b^2 - 6ab = 13$, then what is the value of 3ab?

अगर $16a^4 + 36a^2b^2 + 81b^4 = 91$ और $4a^2 + 9b^2 - 6ab = 13$ है तो $3ab$ का मान बताओ.

a) -3 b) 5 c) $\frac{3}{2}$ d) $\frac{-3}{2}$

$4a^2 + 9b^2 + 6ab = 7$

$(2a)^4 + (2a)^2(3b)^2 + (3b)^4$

$a^4 + a^2b^2 + b^4$

$(+) - (-)$

$3+2ab = -\frac{6}{4} = -\frac{3}{2}$

166. If $16a^4 + 36a^2b^2 + 81b^4 = 91$ and $4a^2 + 9b^2 - 6ab = 13$, then what is the value of $\left(\frac{2}{5}a^2 + \frac{9}{10}b^2\right)$?

अगर $16a^4 + 36a^2b^2 + 81b^4 = 91$ और $4a^2 + 9b^2 - 6ab = 13$ है तो $\left(\frac{2}{5}a^2 + \frac{9}{10}b^2\right)$ का मान बताओ.

a) −3 b) 1 c) 2 d) 0

$4a^2+9b^2+6ab=7$

$\frac{4a^2+9b^2}{10} = \frac{10}{10} = 1$

$4a^2+9b^2 = \frac{\text{sum}}{2} = 10$

167. **Which one of the following is not a factor of this polynomial $x^8 + x^4 + 1$?**
निम्नलिखित में से कौन सा $x^8 + x^4 + 1$ बहुपद का गुणनखंड नहीं है?

a) $(x^2 + 1 + x)$
b) $(x^2 + 1 - x)$
c) $(x^4 + 1 - x^2)$
d) $(x^2 - 1 + x)$

$y=1$

$x^8 + x^4y^4 + y^8$

$(x^4 + x^2 + 1)(x^4 - x^2 + 1)$

$x^{16} + x^8 + 1 = (x^8 + x^4 + 1)(x^8 - x^4 + 1)$

$(x^2 + x + 1)(x^2 - x + 1)$

168. If $1 + 4x^2 + 16x^4 = 512, 1-2x + 4x^2 = 64$, then the value of $1 + 2x + 4x^2$ is:

यदि $1 + 4x^2 + 16x^4 = 512$ है और $1-2x + 4x^2 = 64$ है, तो $1 + 2x + 4x^2$ का मान कितना होगा?

$1+2x+4x^2 = 8$

a) 6 b) 8 c) 12 d) 10

169. If $16x^4 + 36x^2y^2 + 81y^4 = (4x^2 + 6xy + 9y^2)(Ax^2 + Bxy + Cy^2)$, then what is the value of $A - B + C$ is

अगर $16x^4 + 36x^2y^2 + 81y^4 = (4x^2 + 6xy + 9y^2)(Ax^2 + Bxy + Cy^2)$ है तो $A - B + C$ का मान बताओ.

a) 7 b) 19 c) 11 d) 10

$(4x^2 - 6xy + 9y^2)$

$4 + 6 + 9 = 19$

$A = 4$

$B = -6$

$C = 9$

170. If $a^4 - 7a^2b^2 + b^4 = 32$ and $a^2 - 3ab + b^2 = 4$. Find $\frac{a}{b} + \frac{b}{a}$.

Brain-storming

अगर $a^4 - 7a^2b^2 + b^4 = 32$ और $a^2 - 3ab + b^2 = 4$ है, तो $\frac{a}{b} + \frac{b}{a} =$

a) 2 b) 6 c) 18 d) 9

$a^4 + a^2b^2 + b^4 = (a^2+b^2)^2 - 2a^2b^2 + a^2b^2$

$- a^2b^2$

a^2+b^2
$= (a+b)^2 - 2ab$
$= (a-b)^2 + 2ab$

$(a^2+b^2)^2 - 2a^2b^2 - 7a^2b^2$

$= (a^2+b^2)^2 - (3ab)^2 \rightarrow (a^2+b^2+3ab)(a^2+b^2-3ab)$

$a^4 - 7a^2b^2 + b^4 = 32$

$a^2 - 3ab + b^2 = 4$

$a^2 + 3ab + b^2 = 8$

$\frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab} = \frac{6 \times 3}{2} = 9$

$a^2 + b^2 = \frac{\text{sum}}{2} = 6$

$3ab = \frac{2}{3}$

171. A factor of $a^4 - 11a^2b^2 + b^4$ is:

$a^4 - 11a^2b^2 + b^4$ का एक गुणनखंड है :

a) $(a^2 - b^2 - 3ab)$ b) $(a^2 + b^2 - 3ab)$

c) $(a^2 + b^2 + 3ab)$ d) $(a^2 - b^2 + 4ab)$

$a^4+b^4 - 11a^2b^2$ $\xrightarrow{a^2+b^2=(a+b)^2-2ab}$ $(a^2+b^2)^2 \underline{-2a^2b^2 - 11a^2b^2}$

$-13a^2b^2$

$a^2+b^2=(a-b)^2+2ab$

$(a^2-b^2)^2 \underline{+2a^2b^2 - 11a^2b^2}$

$-9a^2b^2$

$= (a^2-b^2)^2 - (3ab)^2$

$= (a^2-b^2+3ab)(a^2-b^2-3ab)$