Concept (2nd part of $a^3 \pm b^3$):

I II

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

i) II part = 0 & ask a^3+b^3 / a^3-b^3

ii) II part ⇒ Values put in place of a/b

107. If $a^2 + b^2 + ab = 0$, then $(a^3 - b^3)$ is equal to

अगर $a^2 + b^2 + ab = 0$ है, तो $(a^3 - b^3)$:

a) 0 b) 1

c) $(a + b)^3$ d) $a^2b^4 + a^4b^2$

$(a-b)(a^2+ab+b^2)$

$(\quad) \times 0 = 0$

108. If $\frac{a}{b}+\frac{b}{a}=1, a\neq 0, b\neq 0$ the value of a^3+b^3 is

अगर $\frac{a}{b}+\frac{b}{a}=1, a\neq 0, b\neq 0$ तो a^3+b^3 का मान:

a) 0 b) 1 c) -1 d) 2

$$\frac{a^2+b^2}{ab}=1$$

$$\Rightarrow a^2+b^2-ab=0$$

१७. If $a = \frac{b^2}{b-a}$, then the value of $a^3 + b^3$ is

अगर $a = \frac{b^2}{b-a}$, तो $a^3 + b^3$ का मान:

a) 6ab b) 0 c) 1 d) 2

$$ab - a^2 = b^2$$

$$0 = a^2 + b^2 - ab$$

110. If $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ then the value of $x^3 - y^3$ is

अगर $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ तो $x^3 - y^3$ का मान:

a) 0 b) 1 c) -1 d) 2

$$\frac{1}{x+y} = \frac{x+y}{xy}$$

$$\Rightarrow xy = x^2+y^2+2xy$$

$$\Rightarrow 0 = x^2+y^2+xy$$

$$(x-y)(\quad)$$

III. If $a^4 + b^4 = a^2b^2$, then $(a^6 + b^6)$ equals

अगर $a^4 + b^4 = a^2b^2$ है, तो $(a^6 + b^6)$:

a) 0 b) 1

c) $a^2 + b^2$ d) $a^2b^4 + a^4b^2$

$a^4 + b^4 - a^2b^2 = 0$

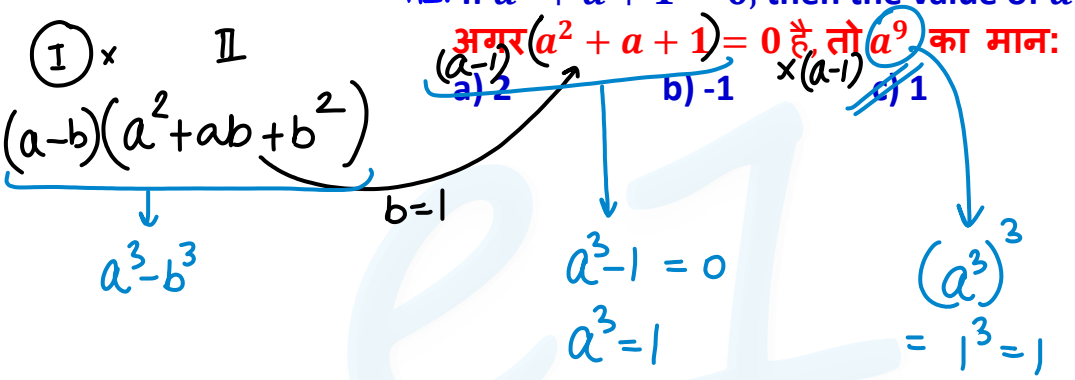
$= (a^2)^3 + (b^2)^3$

$= (a^2 + b^2)(a^4 + b^4 - a^2b^2)$

12. If $a^2 + a + 1 = 0$, then the value of a^9 is

अगर $a^2 + a + 1 = 0$ है, तो a^9 का मान:

a) 2 b) -1 c) 1 d) 0



113. If $a^2 + a + 1 = 0$, then the value of $a^5 + a^4 + 1$ is

अगर $a^2 + a + 1 = 0$ है, तो $a^5 + a^4 + 1$ का मान:

a) 1 b) 0 c) $a + 1$ d) a^2

$(a-b)(a^2+ab+b^2)$ $b=1$

$(a-1)\times$ $\times(a-1)$

$a^3 - 1 = 0$

$a^3 = 1$

$a^3 \times a^2 + a^3 \times a + 1$

$= a^2 + a + 1$

$= 0$

114. If $a^3 + 3a^2 + 9a = 1$, then what is the value of $a^3 + \left(\frac{3}{a}\right)$?

यदि $a^3 + 3a^2 + 9a = 1$ हो, तो $a^3 + \left(\frac{3}{a}\right)$ का मान क्या है?

a) 31 b) 26 c) 28 d) 24

$(a^2+ab+b^2)\times(a-b)$

$b=3$

$(a-3)(a^2+3a+9) = \frac{1}{a}\times(a-3)$

$\Rightarrow a^3 - 27 = 1 - \frac{3}{a}$

$\Rightarrow a^3 + \frac{3}{a} = 28$

$a^3 = 1 - 3a^2 - 9a$

$+\ \frac{3}{a} = 3a^2 + 9a + 27$

$a^3 + \frac{3}{a} = 28$

SSC CGL T2, 17 Feb 2018

115. If $a + a^2 + a^3 - 1 = 0$, then what is the value of $a^3 + \frac{1}{a}$?

यदि $a + a^2 + a^3 - 1 = 0$ हो, तो $a^3 + \frac{1}{a}$ का मान क्या है?

a) 1 b) 4 c) 2 d) 3

$a^2 + ab + b^2$

$b = 1$

$a + a^2 + a^3 = 1$

$\Rightarrow (a-1)(1 + a + a^2) = \frac{1}{a} \times (a-1)$

$\Rightarrow a^3 - 1 = 1 - \frac{1}{a}$

$\Rightarrow a^3 + \frac{1}{a} = 2$

$a^3 = 1 - a - a^2$

$+ \frac{1}{a} = 1 + a + a^2$

$a^3 + \frac{1}{a} = 2$

SSC CGL T2, 19 Feb 2018

116. If $a + \frac{1}{a} + 1 = 0 (a \neq 0)$ then the value of $(a^4 - a)$ is:

अगर $a + \frac{1}{a} + 1 = 0 (a \neq 0)$ है तो $(a^4 - a)$ का मान:

a) 0 b) 1 c) -2a d) -1

$a^2 + ab + b^2$

$b = 1$

$a^2 + 1 + a = 0$

$(a-1)(a^2 + a + 1) = 0 \times (a-1)$

$a^3 - 1 = 0$

$a^3 = 1$

$1 = a^3 \times a - a$

$= a - a = 0$

$a^n \pm b^n$

i) $a+b=5$, $ab=3$, $a-b=?$

$a-b=\sqrt{25-4\times3}=\sqrt{13}$

ii) $a+b=7$, $a-b=3$, $ab=\frac{7^2-3^2}{4}=\frac{10\times\cancel{4}}{\cancel{4}}=10$

iii) $a-b=5$, $ab=2$, $a+b=\sqrt{25+4\times2}=\sqrt{33}$

$a+b=4,\ ab=2$

a^n+b^n

i) $a^2+b^2=(a+b)^2-2ab=16-4=12$

ii) $a^4+b^4=(a^2+b^2)^2-2a^2b^2=12^2-2\times4=144-8=136$

$(a^2)^2+(b^2)^2$

iii) $a^3+b^3=(a+b)^3-3ab(a+b)=64-3\times2\times4=40$

iv) $a^8+b^8=(a^4)^2+(b^4)^2=(a^4+b^4)^2-2a^4b^4$

v) $a^6+b^6=(a^3)^2+(b^3)^2=(a^3+b^3)^2-2a^3b^3$

$=(a^2)^3+(b^2)^3=(a^2+b^2)^3-3a^2b^2(a^2+b^2)$

$$a^5+b^5=(a^2+b^2)(a^3+b^3)-a^2b^2(a+b)$$

Proof:

$$(a^2+b^2)(a^3+b^3)=a^5+a^2b^3+b^2a^3+b^5$$

$$=a^5+b^5+a^2b^2(a+b)$$

$$\Rightarrow (a^2+b^2)(a^3+b^3)-a^2b^2(a+b)=a^5+b^5$$

$$a^7+b^7=(a^3+b^3)(a^4+b^4)-a^3b^3(a+b)$$

$$a^{11}+b^{11}=(a^5+b^5)(a^6+b^6)-a^5b^5(a+b)$$

Concept:

$a + b = 3$

$ab = 1$

$a^2 + b^2 = 3^2 - 2\times 1 = 7$

$a^3 + b^3 = 3^3 - 3\cdot 1\cdot 3 = 18$

$a^4 + b^4 = 7^2 - 2\times 1^2 = 47$

$a^5 + b^5 = 123$ ← $(a^3+b^3)(a^2+b^2) - a^2b^2(a+b)$

$= 18\times 7 - 1\times 3 = 126 - 3 = 123$

$a^6 + b^6 =$ $(a^2)^3 + (b^2)^3 \rightarrow 7^3 - 3\cdot 1^2\cdot 7 = 322$

$(a^3)^2 + (b^3)^2 \rightarrow 18^2 - 2\cdot 1 = 322$

$a^7 + b^7 =$ ← $(a^4+b^4)(a^3+b^3) - a^3b^3(a+b)$

$= 47\times 18 - 1\times 3$

$a^8 + b^8 = 47^2 - 2\cdot 1^4 =$

Power even

$a^2 - b^2 = (a+b)(a-b)$

$a^4 - b^4 = (a^2+b^2)(a+b)(a-b)$

$a^8 - b^8 = (a^4+b^4)(a^2+b^2)(a+b)(a-b)$

$a^6 - b^6 = (a^3+b^3)(a^3-b^3)$

$$a^3-b^3=(a-b)^3+3ab(a-b)$$

$$a^5-b^5=(a^2-b^2)(a^3+b^3)+a^2b^2(a-b)$$

↑ अगली power +ve

$$a^7-b^7=(a^4+b^4)(a^3-b^3)+a^3b^3(a-b)$$

$a^2-b^2=$

$a^3-b^3=$

$a^4-b^4=$

$a^5-b^5=$

$a^6-b^6=$

$a^7-b^7=$

$a^8-b^8=$

Concept:

$a + b = 3$ $\quad a-b=\sqrt{9-4}=\sqrt{5}$

$ab = 1$

$a^2 - b^2 =$ $\quad (a+b)(a-b)$ $\quad 3\times\sqrt{5}=3\sqrt{5}$

$a^3 - b^3 =$ $\quad 5\sqrt{5}+3\cdot1\sqrt{5}=8\sqrt{5}$

$a^4 - b^4 =$ $\quad (a^2+b^2)(a+b)(a-b)$ $\quad 7\times 3\times\sqrt{5}=21\sqrt{5}$

$a^5 - b^5 =$

$a^6 - b^6 =$ $\quad (a^3+b^3)(a^3-b^3) \rightarrow 18\times 8\sqrt{5}=144\sqrt{5}$

$a^7 - b^7 =$ $\quad (a^2)^3-(b^2)^3=(a^2-b^2)^3+3a^2b^2(a^2-b^2)$

$a^8 - b^8 =$

$(a^3+b^3)(a^2-b^2)+a^2b^2(a-b)$

$=(27-3\,13)(3\sqrt{5})+1\sqrt{5}=55\sqrt{5}$

$(a^4+b^4)(a^3-b^3)+a^3b^3(a-b)$

Practice:

$a+b$	ab	
5	2	$a^3+b^3=$
4	2	$a^4+b^4=$
$2\sqrt{7}$	3	$\sqrt{a^3-b^3}=$
5	3	$a^4-b^4=$
5		$a^3+b^3=80$

$125 - 3ab^3 5 = 80$

25 9 16

$a^2+b^2 = 16-4=12$

$a^2b^2 = 4$

$a-b = \sqrt{28-12} = 4$

$64+334$

$\sqrt{100}=10$

$(a^2+b^2)(a+b)(a-b)$

$= 19 \times 5 \times \sqrt{37}$