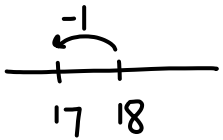
Cyclicity (when m^n is divided by some number):

$\frac{39^{22}}{7}$

$\frac{2019^{2019}}{7} \rightarrow 3^3 \xrightarrow{\times 6} 6$

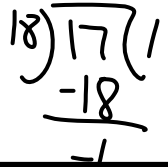
5 } odd
1 } even

$\frac{3^{21}}{5}$



$\frac{237^{237}}{8} \rightarrow 5^1$

$\frac{48^{978}}{13}$



$\frac{17^{136}}{18} \rightarrow (-1)^{\text{even}} \rightarrow 1$

? Power	Remainder

27/ Find the remainder: शेष ज्ञात करें:

a) $\frac{59^{637}}{58} \rightarrow (1)^{637} \rightarrow 1$

b) $\frac{96^{132}}{97} \rightarrow (-1)^{\text{even}} \rightarrow 1$

c) $\frac{181^{753}}{182} \rightarrow (-1)^{\text{odd}} \rightarrow -1/181$ ✓

d) $\frac{257^{1008}}{64} \rightarrow (1)^{1008} \rightarrow 1$

64) 257 (4
-256
1

e) $\frac{471^{344}}{118} \rightarrow (-1)^{\text{even}} \rightarrow 1$

f) $\frac{494^{139}+396^{137}}{99} \rightarrow (-1)^{\text{odd}} + 0 \rightarrow -1/98$

g) $\frac{7^{84}}{342} \rightarrow 1$; $\div 3$ → Rem = 0 | last 7, 49, 1

h) $\frac{31^{(31^{31})}}{32} \rightarrow (-1)^{\text{odd}} \rightarrow -1/31$

99) 494 (5
-495
-1

i) $\frac{153^{153}+153}{154} \rightarrow \overset{\text{odd}}{-1} + -1 \rightarrow -2/152$; 153

j) $\frac{88^{89}+26}{89} \rightarrow (-1)^{\text{odd}} + 26 \rightarrow -1+26 \rightarrow 25$

272 Find the remainder: शेष ज्ञात करें:

a) $\frac{9^{178}}{13} \to 9$

9, 3, 1

7, −1, −, 1 Rem = 0

b) $\frac{4^{2007}}{17} \to 4^3 \to \frac{64}{17} \to 13$

4, −1, 13, 1

c) $\frac{29^{120}}{13} \to 3^{0/last} \to 1$

3, 9, 1

d) $\frac{77^{100}}{10} \to 7 \to 1$

e) $\frac{10^{70}}{7} \to 3^4 \to \frac{81}{7} \to 4$

3, 2, −1, −3, −, 1

f) $\frac{2^{68}}{33} \to \frac{2^8}{33} \to \frac{256}{33} \to 25$

g) $\frac{16^{512}}{31} \to 2^{nd} \to 8$

16, 8, 4, 2, 1

h) $\frac{32^{32}}{7} \to 4^2 \to 2$

2, 4, 8, 16, 8, 16, 8

i) $\frac{5^{49}}{126} \to 5^1 \to 5$

2, 4, 1

3, 2, −1, −, 1

j) $\frac{3^{853}}{244} \to 3^3 \to 27$

$3^5 = 243 \to -1$

k) $\frac{50^{80}}{24} \to 2^{even} \to 16$

7, 49, 343, 2401, 1

l) $\frac{2^{50} \times 3^{90}}{7} \to \frac{2^2 \times 1}{7} \to 4$

5, 7, 35, −1, −, −, 1

m) $\frac{3^{50}+4^{50}}{20} \to 9+16 \to 25/5$

49, 3, 9, 7, 1

$(-1)^{odd} \to -1/4 \times (4) = 16$

n) $\frac{7^{656}}{400} \to 1$

o) $\frac{95^{75}}{18} \to 5^3 \to 35$

30×30×. 45 बार

p) $\frac{32^{32^{32}}}{7}$

q) $\frac{25^{25^{24^{23^{\cdot^{\cdot^{\cdot^{1}}}}}}}}{9}$

r) $\frac{15^{30^{45}}}{8} \to (-1)^{30^{45}} \to (-1)^{even} \to 1$

Part K

$\frac{50^{80}}{24} \rightarrow \frac{2^{80}}{24} \rightarrow \frac{8 \times 2^{77}}{24\ 3} \rightarrow (-1)^{odd} \rightarrow -1/2 \times 8 = 16$

CF = 8

$2 \times 2 \times 2 \times 2 \ldots$ 80 बार

$2 \times 2 \times 2 \times \ldots$

3 बार 77 बार

Part (P)

$32^{32^{32}}$ $\quad (32^{32})$ की cyclicity से divide करने पर remainder = 1

$\frac{32}{7} \rightarrow \frac{4}{7} \rightarrow 4$

4, 2, 1

$\frac{32^{3^2}}{3} \rightarrow (-1)^{\text{even}} \rightarrow 1$

Part 2

$\frac{25^{25^{24^{23}}}}{9} \to 7^1 \to 7$

7, 4, 1 Cyclicity = 3

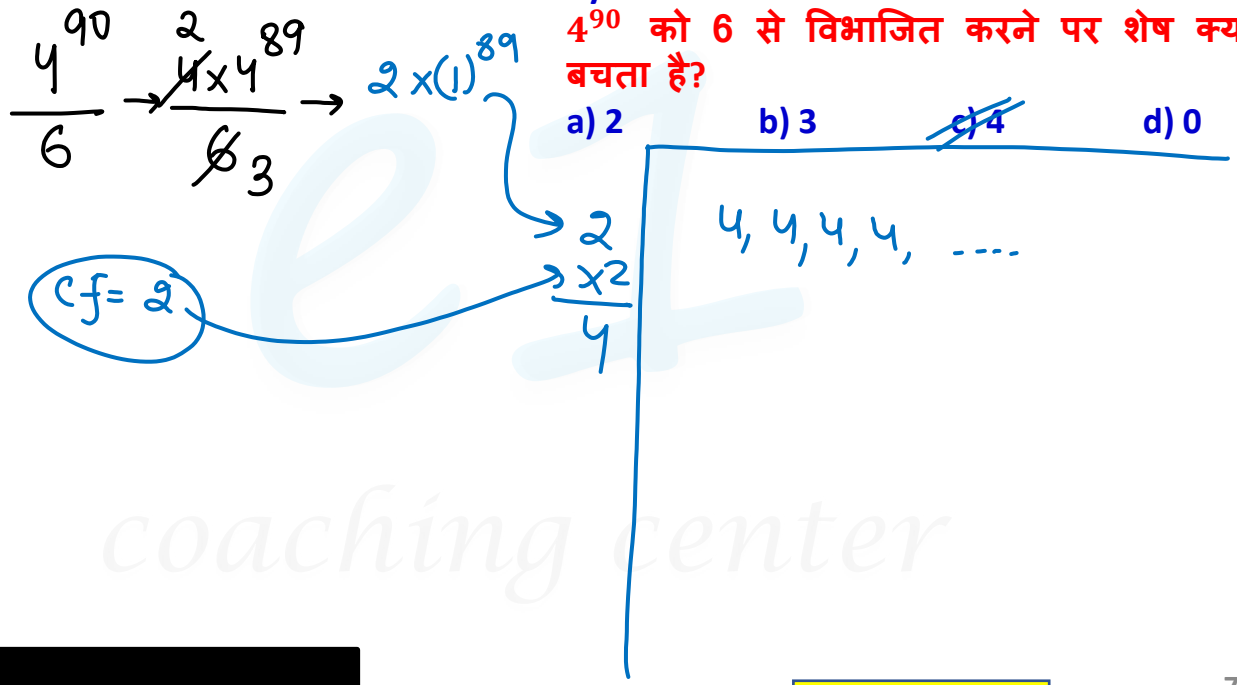
$\frac{25^{24^{23}}}{3} \to (1)^{24^{23^{22}}} \to 1$

$\left.\frac{\text{Power}}{\text{Cyclicity}}\right|_R = 1$

273 What is the remainder when 4^{90} is divided by 6?

4^{90} को 6 से विभाजित करने पर शेष क्या बचता है?

a) 2 b) 3 ~~c) 4~~ d) 0



Unit digit / Remainder

274 What is the last digit of $113^{106^{100}}$?

$113^{106^{100}}$ का अंतिम अंक-

a) 1 b) 3 c) 7 d) 9

$3^{106^{100}}$ 4 से divide, R = ?

$= 3^4 \rightarrow 81$

$(2\times53)^{100} \quad 106^{100} \Big|_{R} = 0/4$

$= 4, R = 1$

$273^{157} \rightarrow 3^{157} \rightarrow 3^1 \rightarrow 3$

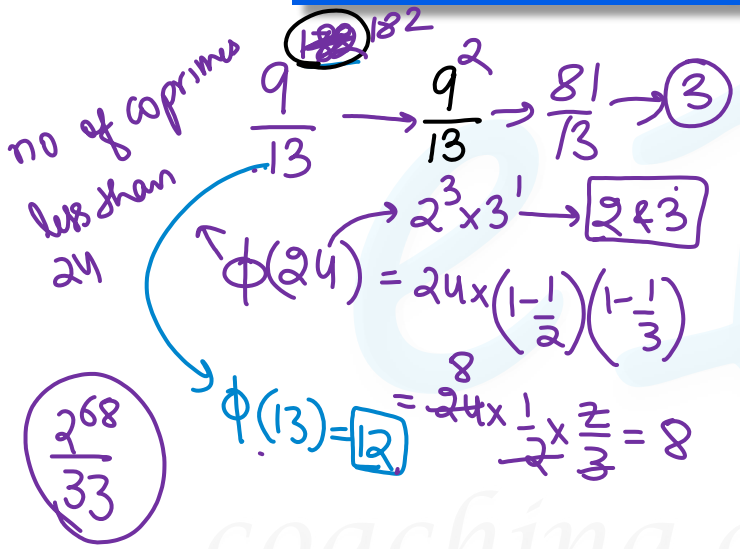
Cyclicity = 4

0, 1, 5, 6 → Same

4, 9 → 2 (o/e)

2, 3, 7, 8 → 4

Totient method / Eular's function:



Totient → ?

Cyclicity का multiple होता है, तो उससे divide करके Rem देख सकते हैं।

$\phi(180) = \cancel{180} \overset{12}{} \times \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5} = 48$

$180 = 2^2 \times 3^2 \times 5$

$\phi(100) = \cancel{100} \times \frac{1}{2} \times \frac{4}{5} = 40$

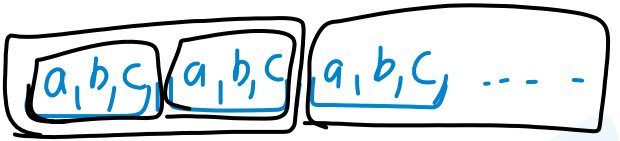
$100 = 2^2 \times 5^2$

$\phi(37) = \cancel{37} \times \frac{36}{\cancel{37}} = 36$

$\phi(23) = \cancel{23} \times \frac{22}{\cancel{23}}$

$\phi(17) = \cancel{17} \times \frac{16}{\cancel{17}} = 16$

$\phi(\text{Prime}) = \text{Prime} - 1$



74 terms

$\frac{74}{3} \Big| R \Rightarrow 2$

$\frac{74}{6} \Big| R \Rightarrow 2$

$\frac{2^{68}}{33} \rightarrow \frac{2^{8}}{33}$

$33 = 3 \times 11$

$\phi(33) = 33 \times \frac{2}{3} \times \frac{10}{11} = 20$

275 Find the remainder: शेष ज्ञात करें:

a) $\frac{72^{109}}{37} \to 35^{1} \to 35$

$36\overline{)109}\,(3$, -108, 1

$\phi(37)=36$

b) $\frac{54^{253}}{43} \to 11^{1}$

$\phi(43)=42$

$=36$, 11, 73, 4, x

c) $\frac{14^{222}}{23} \to 14^{2} \to \frac{196}{23} \to 12$

$\phi(23)=22$

d) $\frac{44^{74}}{76} \to \frac{44 \times 44^{73}}{76\ 19} \to 11 \times 6^{1} \to \frac{66}{19} \to 9$

$\phi(19)=18$

$42\overline{)169}\,(4$, -168, 1

e) $\frac{13^{169}}{49} \to 13^{1}$

$\phi(49)=49 \times \frac{6}{7}=42$

f) $\frac{37^{142}}{25} \to 12^{2} \to \frac{144}{25} \to 19$

$\phi(25)=25 \times \frac{4}{5}=20$

g) $\frac{23^{722}+24^{722}}{25} \to -2^{2}+(-1)^{\text{even}} \to 4+1=5$

$\phi(25)=20$

h) $\frac{913^{101^{100}}}{101} \to 4^{1} \to 4$

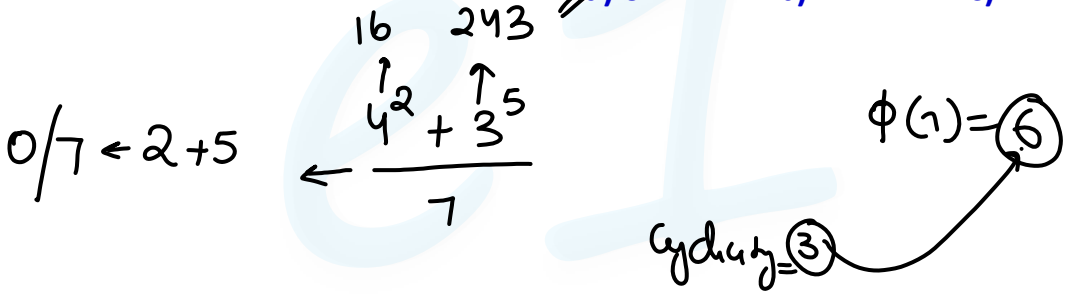
$\phi(101)=100$

$\frac{101^{100}}{100} \to (1)^{100} \to 1$

276 Find the remainder : $\frac{5555^{2222}+2222^{5555}}{7}=?$

शेषफल ज्ञात कीजिए : $\frac{5555^{2222}+2222^{5555}}{7}=?$

a) 0 b) 1 c) 2 d) 6



277 Find the last 2-digits of : 43^{362}

43^{362} के अंतिम दो अंक ज्ञात करें|

a) 19 b) 49 c) 39 d) 59

$\frac{43^{362}}{100} \rightarrow \frac{43^{2}}{100} \rightarrow \frac{1849}{100} \rightarrow 49$

Remainder on dividing by 100

$\phi(100) = 100 \times \frac{1}{2} \times \frac{4}{5} = 40$

$100 = 2^2 \times 5^2$

278 Find the last 3-digits of : 87^{2002}

87^{2002} के अंतिम 3 अंक ज्ञात करें|

a) 139 b) 269 c) 569 d) 369

$87^{2002} \rightarrow 87^{2} \rightarrow 7569$

$\frac{}{1000} \quad \frac{}{1000} \quad \frac{}{1000}$

Remainder on ÷ by 1000

$\phi(1000) = 100 \quad \phi \times \frac{1}{2} \times \frac{4}{5} = 400$

$1000 = 2^3 \times 5^3$

279 Find the last 3 digit of $63^{25} \times 25^{63}$.

$63^{25} \times 25^{63}$ के अंतिम 3 अंक ज्ञात कीजिए।

a) 375
b) 625
c) 125
d) None of these

$\frac{63^{25} \times 25^{63}}{1000} \rightarrow \frac{63^{25} \times 25 \times 25 \times 25^{61}}{1000}$

5

~~1000~~

~~40~~ 8

R — by 1000.

CF = 125

$(-1)^{odd} \times 5 \times (1)^{61} \rightarrow -1 \times 5 \rightarrow -5$ | $3 \times 125 = 375$