

Divisibility of expressions

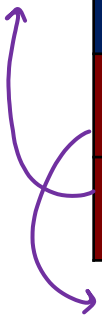
(व्यंजकों द्वारा विभाज्यता)

coaching center

Concept:

$531^{31} + 230^{31} \rightarrow 531 + 230 = 761$
 $\rightarrow n=1 (x-y)$
 $n=2 (x^2-y^2)=(x-y)(x+y)$

Expression	If n is odd	If n is even
$x^n - y^n$	$(x - y)$	$(x - y)(x + y)$
$x^n + y^n$	$(x + y)$	<i>can't say</i>



$$\begin{cases} 70^{51} - 30^{51} \rightarrow (70-30=40) \\ 70^{52} - 30^{52} \rightarrow (70-30)(70+30)=40 \times 100 \end{cases}$$

$n=2$

$$(x^2 + y^2) \times$$

$$(x-y) \times \text{Something}$$

$$(x+y) \times \text{"}$$

coaching center

$$\frac{x^n - y^n \rightarrow \text{even}}{\downarrow}$$
$$(x-y)(x+y)$$

195 What is the remainder if $17^{26} - 11^{26}$ is divided by 42?

$17^{26} - 11^{26}$ को 42 से विभाजित करने पर शेष संख्या ज्ञात करें।

a) 0

b) 1

c) 13

d) 14

$$(17+11)(17-11)$$
$$= \frac{28 \times 6^2}{14 \times 3}$$

$$14 \times 3$$

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$$\frac{x^n + y^n \rightarrow \text{even}}{\quad}$$

Can't say

$$(5^2)^{35} + (7^2)^{35}$$

$$= 25^{35} + 49^{35}$$

↓

$$25 + 49 = 74 \checkmark$$

196 What is the remainder when we divide $5^{70} + 7^{70}$ by 74?

जब हम $5^{70} + 7^{70}$ को 74 से भाग देते हैं तो शेषफल क्या होता है?

a) 7

b) 1

c) 0

d) 5

$$\frac{x^n + y^n \rightarrow \text{odd}}{x + y \checkmark}$$

coaching center

$$x^n - y^n$$

197 For every positive integer n , $5^{4n} - 4^{4n}$ is divisible by:
प्रत्येक धनात्मक पूर्णांक n के लिए, $5^{4n} - 4^{4n}$ विभाजित होगा -

a) 9

b) 41

c) 369

d) All the above

$$(5^4)^n - (4^4)^n$$

$$= 625^n - 256^n$$

$$625 - 256 = 369$$

$$9 \times 41$$

odd/even

$$(5^2)^{2n} - (4^2)^{2n}$$

$$= 25^{2n} - 16^{2n}$$

$$(25-16)(25+16)$$

$$= 9 \times 41 = 369$$

2n → even

$$\begin{aligned}
 & (2^6)^n - (4^2)^n \\
 &= 64^n - 16^n \rightarrow \text{odd/even} \\
 &\quad \downarrow \\
 & 64 - 16 = 48
 \end{aligned}$$

198 The expression $2^{6n} - 4^{2n}$, where n is a natural number is always divisible by

व्यंजक $2^{6n} - 4^{2n}$, जिसमें n एक प्राकृतिक संख्या है, हमेशा किससे विभाजित होगा?

a) 15

b) 18

c) 36

d) 48

$$\begin{aligned}
 & (2^3)^{2n} - 4^{2n} \\
 &= 8^{2n} - 4^{2n} \rightarrow \text{even} \\
 &\quad \downarrow \\
 & (8+4)(8-4) \\
 &= 12 \times 4 = 48
 \end{aligned}$$

$$1^5 = 1^{10} = 1^{100} = 1^n = 1$$

199 $(68)^n + 1$ is exactly divisible by 23 when n is?

$(68)^n + 1$, 23 विभाज्य है यदि n:

- a) Any natural number ~~b) Odd number~~
c) Even number d) Only prime number

$$68^n + 1^n$$

$$x^n + y^n \rightarrow \text{odd, तभी } x+y \checkmark$$

$$68+1 = 69 = 23 \times 3$$

coaching center

200 One of the factors of $(8^{2k} + 5^{2k})$, where k is an odd number, is:

$(8^{2k} + 5^{2k})$ का एक गुणखंड, जहां k एक विषम संख्या है:

- a) 86 b) 88 c) 84 ~~d) 89~~

$$\begin{aligned} & (8^2)^k + (5^2)^k \\ &= 64^k + 25^k \xrightarrow{k \rightarrow \text{odd}} \\ & \quad \downarrow \\ & \quad 89 \end{aligned}$$

power odd

coaching center

$$\begin{aligned}
 N &= 2^{48} - 1^{48} \\
 &\quad \downarrow \\
 &= (2^6)^8 - (1)^8 \\
 &= 64^8 - 1^8 \\
 &\quad \downarrow \\
 &= (64-1)(64+1) \\
 &\Rightarrow 63 \cdot 65 \\
 &\quad \quad \quad \downarrow \\
 &\quad \quad \quad 129
 \end{aligned}$$

201 $N = 2^{48} - 1$ and N is exactly divisible by two numbers between 60 and 70. What is the sum of those two numbers?

$N = 2^{48} - 1$ तथा N , 60 तथा 70 के बीच दो संख्या से पूर्णतः विभाजित है। उन संख्याओं का योग क्या है?

- a) 128 b) 256 c) 64 d) 512

250 & 300 के बीच में

$$\begin{aligned}
 &(28)^6 - (1)^6 \\
 &= 256^6 - 1^6 \\
 &= (256-1)(256+1)
 \end{aligned}$$

202 $2^{18} - 1$ is divisible by:
 $2^{18} - 1$ किससे विभाज्य है:

a) 11

b) 17

c) 13

~~d) 7~~

$$2^{18} - 1^{18}$$

↓

$$(2+1)(2-1) = 3$$

$$(2^3)^6 - 1^6$$

$$= 8^6 - 1^6$$

↓

$$(8-1)(8+1)$$

$$7 \times 9$$

203 Find the remainder when $11^{15} + 12^{15} + 13^{15} + 14^{15}$ is divided by 50?

जब $\underline{11}^{15} + \underline{12}^{15} + \underline{13}^{15} + \underline{14}^{15}$ को 50 से विभाजित किया जाता है तो शेष क्या बचेगा ?

- ~~a) 0~~ b) 1 c) 24 d) 26

$$11+12+13+14 = \underline{\underline{50}}$$

If ' n ' is odd and a, b, c, \dots, z are consecutive natural numbers then $(a^n + b^n + c^n + \dots + z^n)$ is divisible by $(a + b + c + \dots + z)$.

204. Which of the following statement(s) is/are TRUE ?

I. $1^{99} + 2^{99} + 3^{99} + 4^{99} + 5^{99}$ is exactly divisible by 5. $1+2+3+4+5=15$

II. $31^{11} > 17^{14}$

निम्नलिखित में से कौन सा/से कथन सत्य है/हैं?

I. $1^{99} + 2^{99} + 3^{99} + 4^{99} + 5^{99}$, 5 से पूर्णत विभाजित है।

II. $31^{11} > 17^{14}$

a) Only I

b) only II

c) Neither I nor II

d) Both I and II

$$\begin{array}{r} 31^{11} \\ \downarrow ++ \\ 32^{11} \end{array}$$

$$(2^5)^{11}$$

$$2^{55}$$

$$\begin{array}{r} 17^{14} \\ \downarrow -- \\ 16^{14} \end{array}$$

$$(2^4)^{14}$$

$$2^{56}$$

$$2^{55} < 2^{56}$$

coaching center

205. $23^9 + 24^9 + 25^9 + 26^9$ is divisible by:

$23^9 + 24^9 + 25^9 + 26^9$ निम्न में से किससे भाज्य है ?

~~a) 4 but not by 49~~

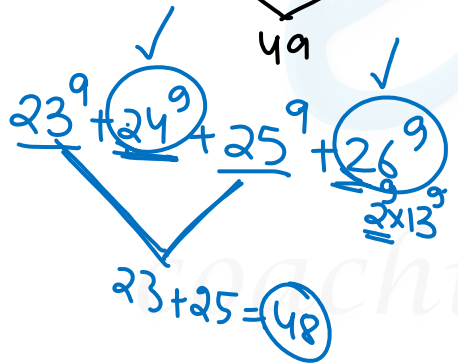
b) 49 but not by 4

~~c) both 4 and 49~~

~~d) neither by 4 nor by 49~~

$$23+24+25+26 = 98 = \underline{\underline{49}} \times \underline{\underline{2}}$$

इससे div की गारंटी है



206. $13^{15} - 7^{15} - 6^{15}$ is always divisible by:

$13^{15} - 7^{15} - 6^{15}$ निश्चित रूप से विभाजित है -

1) 6

2) 7

3) 13

4) All the above

iii) $13^{15} - 6^{15} - 7^{15}$

\downarrow \downarrow

$13 - 6 = 7$ 7

(7) ✓

i) $13^{15} - 7^{15} - 6^{15}$

$\underline{\underline{13}}$ ✓ $13^{15} - (7^{15} + 6^{15})$

\downarrow

$7 + 6 = 13$

(13) ✓

ii) $13^{15} - 7^{15} - 6^{15}$

\downarrow

$13 - 7 = 6$ ✓

(6) ✓

coaching center

$$55^5 + 17^5 - 72^5$$
$$= 55^5 - (72^5 - 17^5)$$

$72 - 17 = 55$

207. Given a number, $N = 55^5 + 17^5 - 72^5$, then which of the following is true?

दी गई संख्या, $N = 55^5 + 17^5 - 72^5$, तो निम्नलिखित में से कौन सा सत्य है?

- a) N is divisible by both 7 and 13
- ~~b) N is divisible by both 3 and 17~~
- c) N is divisible by 17 but not 3
- d) N is divisible by 11 but not 17

coaching center

$x^n + y^n$ is div by $x + y$ if n is odd.

odd multiple

$x^{15} + y^{15}$ " " " $x + y$.

Example:

odd multiple

$a^{15} + b^{15}$ is div by $a^3 + b^3$?

$(a^3)^5 + (b^3)^5$

$x^5 + y^5$ → $x + y$

x^5 ← x^3

y^5 ← y^3

coaching center

$$5 \times \left(\frac{x^{15} + y^{15}}{x^3 + y^3} \right) \checkmark \quad 5 \times \left(\frac{x^{40} + y^{40}}{x^8 + y^8} \right) \checkmark$$

$$\checkmark \left(\frac{x^{70} + y^{70}}{x^{10} + y^{10}} \right)$$

$$\frac{(x^7)^{10} + (y^7)^{10}}{x^{10} + y^{10}} \rightarrow \text{not divisible}$$

$$\frac{\underbrace{x^7}_a + \underbrace{y^7}_b}{a + b} \rightarrow \frac{a^{10} + b^{10}}{a + b}$$

coaching center

$$\frac{x^n + y^n}{x + y} \rightarrow \text{odd}$$

ques.

$$\frac{x^n - y^n}{x - y} \rightarrow \text{odd}$$

$$\frac{x^n - y^n}{(x - y)(x + y)} \rightarrow \text{even}$$

$$\frac{a^{10} + b^{10}}{a^2 + b^2} \times 5$$

$$\frac{a^{10} - b^{10}}{a^2 - b^2}$$

$$a^{10} - b^{10}$$

$$(a^5 - b^5)(a^5 + b^5)$$

coaching center

$$\frac{4^{16} + 1^{16}}{x}$$

$$\frac{10}{2}$$

An arrow labeled "की multiple $\times 3$ " points from the first fraction to the second.

$$4^{48} + 1^{48}$$

$$50$$

208 A number $4^{16} + 1$ is divisible by x . Which among the following is also divisible by x ?

एक संख्या $4^{16} + 1$, x से विभाज्य है। निम्नलिखित कौन-सी संख्या भी x से विभाज्य है?

- ~~a) $4^{96} + 1^{96}$~~
- ~~b) $4^{32} + 1$~~
- c) $4^8 + 1$
- ~~d) $4^{48} + 1$~~

$$\frac{m}{x}$$

$$\frac{mt}{x}$$

An arrow points from the first fraction to the second.

coaching center

$$\begin{array}{l} 2^{20} + 1^{20} \\ \hline \text{Whole no} \end{array} \xrightarrow{\times 3} 2^{60} + 1^{60}$$

209 Given that $2^{20} + 1$ is completely divisible by a whole number. Which of the following is completely divisible by the same number?

दिया गया है कि $2^{20} + 1$, किसी पूर्ण संख्या से पूर्णतः विभाज्य है। निम्न में से कौन सी संख्या उसी संख्या से पूर्णतः विभाज्य होगी?

- a) $2^{15} + 1$
- b) 5×2^{30}
- c) $2^{90} + 1$
- d) $2^{60} + 1^{60}$

coaching center

210 $(31^{20} - 1024)$ is not divisible by:

$(31^{20} - 1024)$ किससे विभाजित नहीं होता है?

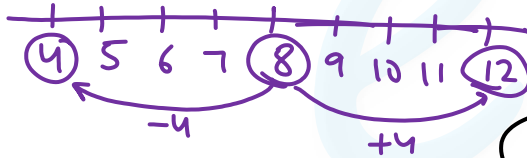
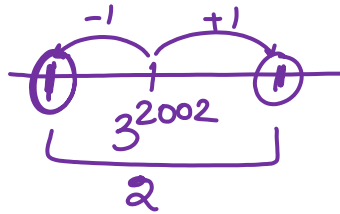
a) 137

b) 107

c) 9

~~d) 32~~

$$\begin{aligned} & 31^{20} - 2^{10} \\ &= (31^2)^{10} - 2^{10} \\ &= 961^{10} - 2^{10} \rightarrow \text{even} \\ &\quad \downarrow \\ & (961-2)(961+2) \\ &= 959 \times 963 \\ & \quad \text{7} \times 137 \quad 9 \times 107 \end{aligned}$$



21) Which of the following statement(s) is/are TRUE?

~~I. Highest common factor of $(3^{2002} - 1)$ and $(3^{2002} + 1)$ is 4~~

~~II. $(4^{84} - 1)$ is exactly divisible by 5.~~

निम्नलिखित में से कौन-सा/से कथन सत्य है/हैं?

I. $(3^{2002} - 1)$ तथा $(3^{2002} + 1)$ का महत्तम समापवर्तक 4 है।

II. $(4^{84} - 1)$, 5 से पूर्णतः विभाजित है।

a) Only I

c) Neither I nor II

~~b) only II~~

d) Both I and II

$$4^{84} - 1$$

$$(4-1)(4+1) = 3 \times 5$$

99%