Divisibility Rules

(विभाज्यता के नियम)

0 :

0 can't divide any number.

0 is divisible by every no. except 0.

$\frac{3}{0} = ND$ $\frac{0}{0} = ND$

$\frac{5}{0} = ND$

$\frac{N}{0} = ND$

$\frac{0}{5} = 0,$

$\frac{0}{3} = 0, \frac{0}{7} = 0$

1 :

Every no is divisible by 1.

$\frac{5}{1} = 5, \quad \frac{2}{1} = 2, \quad \frac{3}{1} = 3, \quad \frac{1}{1} = 1, \quad \frac{0}{1} = 0$

Divisibility by 2 & 5:

$2 \times 5 = 10$

0, 2, 4, 6, 8 ✓

$2^1 = 2$: if no. formed by last 1 digit, is div by 2./0

3572**1** (×), 2367**8** ✓, 237**0**,

$2^2 = 4$: if no. formed by last 2 digits, is div. by 4 /00

357**21** (×), 236**78** (×), 375**28**,

$2^3 = 8$: If no formed by last 3 digits is div. by 8/000

218513**721** (×), 34**576** ✓,

$2^4 = 16$: If no formed by last 4 digits is div by 16/0000

2202013**7 2864 2**

Proof:

② ¹

$$35271 = \boxed{35270} + 1 ✓$$

$$\underset{2 \times 5}{\underbrace{}} \quad \underset{2 \ 5}{\underset{\times \ \times}{1}}$$

② ²

$$46732 = \underline{467\underline{00}} + 32$$

$$10 \times 10 = \quad 2^2 = 4 \qquad \underset{4 \ 25}{\underset{\checkmark \ \times}{}}$$

$$2 \ 5 \ 2 \ 5 \qquad 5^2 = 25$$

Powers of 5:

$$5\overline{)2}\,^{0} \qquad \text{---} \quad 5/0$$
$$\underline{-0}$$
$$2$$

$5^1 = 5$: If no formed by last 1 digit is div by 5/0

357<u>2</u> ✗, 357<u>5</u>

$$\text{---} \quad \underline{00/25/50/75}$$
$$\uparrow$$

$5^2 = 25$: If no formed by last 2 digits is div by 25/00.

173<u>23</u> ✗, 1732<u>5</u> ✓, 1737<u>5</u> ✓,

$5^3 = 125$: If no formed by last 3 digits is div by 125/000

357<u>285</u> ✗, 657<u>375</u> ✓,

5

3: If sum of digits is div by 3.

57384 → 5+7+3+8+4 = 27

57384 ⇒ 0

175438 ⇒ 1

9: If sum of digits is div by 9.

$$357829431 \Rightarrow 3+5+7+8+2+9+4+3+1 = 42$$

357829431

6

Proof:

abcd

$= 1000a + 100b + 10c + d$

$= 999a + a + 99b + b + 9c + c + d$

$= \boxed{999a + 99b + 9c} + \boxed{a+b+c+d}$

$= 9(111a + 11b + c)$

9   9,3

5386

$= 5000 + 300 + 80 + 6$

Divisibility by 11:

1 3 5 7 2 8 3 4

22

11

diff = 22 - 11 = 11   (0, 11, 22, 33)

3 5 7 8 2 9 4 3

25

16

diff = 9

22

35 78 9 0 0 2 1 3 2   4

18

8

1 5 2 7 4 8 0 5 1   17/11 = 6 R

25

11-6=5
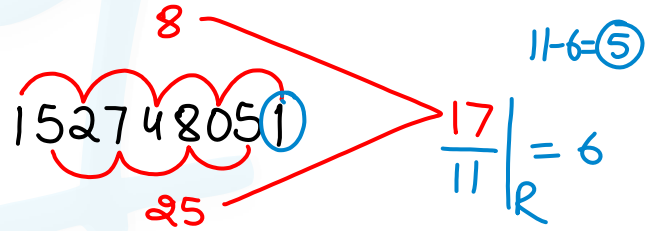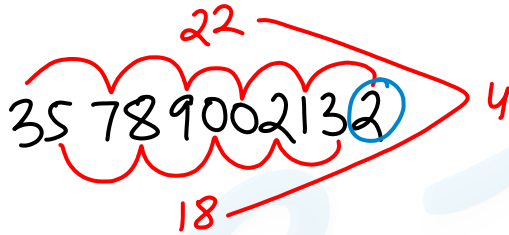
Divisibility by some other numbers (24, 88, 72 etc):

$24 : 24 = 8 \times 3$ (दोनों)

$2^3 \times 3^1$

9×8

11×8

22×4

$156 = 12 \times 13$

4×3

$12 = 6 \times 2 = 4 \times 3$

$2^2 \quad 3^1$

$11 \times 2 \rightarrow 2^2$

11×2^3

$45 = 9 \times 5$

11

Important divisibility cases:

347347
= 347×1001

- **1001** $= 7 \times 11 \times 13$
- **111** $= 37 \times 3$
- **231** $= 3 \times 7 \times 11$

$00723 = 723$

1001
× 132
____
132,132

10101
× 32
____
323232

$00\boxed{1001}$

- **abcabc** $= abc \times 1001$
- **ababab** $= ab \times 10101$
  $\times 10101$

1001
× 234
____
234234

712712 = 712×1001

$\dfrac{abab}{0\boxed{101}} = ab \times 101$

12

$$231231 = 231 \times 1001 = 3 \times 7 \times 11 \times 7 \times 11 \times 13 = 3 \times 7^2 \times 11^2 \times 13$$

$$111111 = 111 \times 1001 = 37 \times 3 \times 7 \times 11 \times 13$$

$$666666 = 666 \times 1001 = 3 \times 2 \times 37 \times 3 \times 7 \times 11 \times 13$$
$$6 \times 111$$

$$\overline{ab}ababab = ab \times 10101 01$$
$$01010101$$

Combined divisibility by 7, 11 & 13:

1089

$7 \times 11 \times 13 = \boxed{1001}$

35|87352u|351

759

1089
−759
‾‾‾‾
330

351
735
3
‾‾‾‾
1089

$\dfrac{330}{7}$ ✗ $\dfrac{330}{11}$ ✓ $\dfrac{330}{13}$ ✗

15

176

$$5\overline{\smash{\big)}342171}$$

342

342
- 176
──────
166

1

×7, ×11, ×13

136. From the following numbers, find the numbers which exactly divisible by 42. = 7×6

निम्नलिखित संख्याओं में से वे संख्याएँ ज्ञात कीजिए जो 42 से पूर्णतः विभाज्य हैं।

a) 25232   b) 25242

c) 25244   d) 25212

7×3×2

$156 = 12 \times 13$

$\rightarrow 4 \times 3 \times 13$

137. How many of the following numbers are divisible by 156?

निम्नलिखित में से कितनी संख्याएँ 156 से विभाज्य है?

312, 620, 939, 1402, 1872, 3216, 7176, 8108

a) 5   b) 3   c) 4   d) 2

$4\overline{)2}$

138. If a nine-digit number $7698x138y$ is divisible by 72, then the value of $\sqrt{4x + y}$ is: $\sqrt{36} = 6$

4 = y   9×8

यदि नौ अंक वाली संख्या $7698x138y$, 72 से विभाज्य है, तो $\sqrt{4x + y}$ का मान ज्ञात करें।

a) 8   b) 6   c) 9   d) 5

last 3 digit

$7698x1384$

$x + 1$
↓
8

18 + 1

9⟌19(2
   18
   ①

8⟌384(48
   32
   64
   64

19

39. If a nine-digit number $389x6378y$ is divisible by 72, then the value of $\sqrt{6x + 7y}$ will be:

यदि नौ अंको की संख्या $389x6378y$, 72 से विभाजित है, तो $\sqrt{6x + 7y}$ का मान होगा :

a) 6

b) $\sqrt{13}$

c) $\sqrt{46}$

d) 8

9×8

$3 8 9 \ x \ 6 3 7 8 4$

$\sqrt{36 + 28}$

$= \sqrt{64} = 8$

$\begin{array}{r} 64 \\ 78\_ \\ \hline 8 \end{array}$ y

Handwritten work

$$\frac{\overset{2\ 4\ 8}{\cdots 8\ \overset{1}{n}\cdot 6}}{8} = \frac{800 + n6}{8}$$

1, 5, 9
n

$\not{5}\, p\, \not{4}\not{2}\not{9}\not{7}\not{8}\not{5}\not{6}$
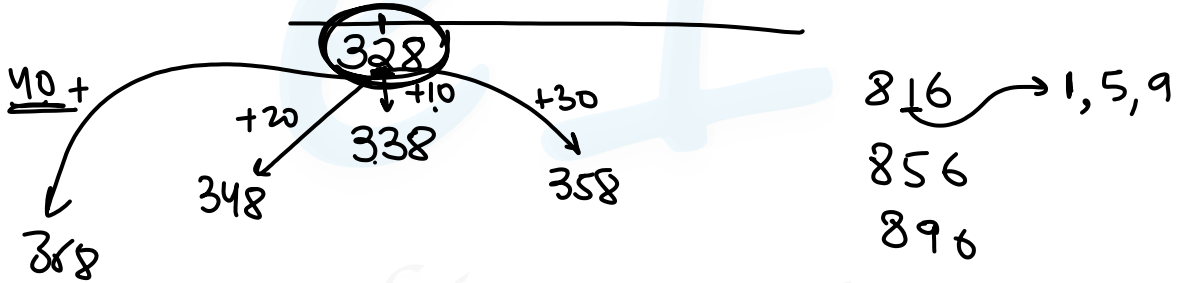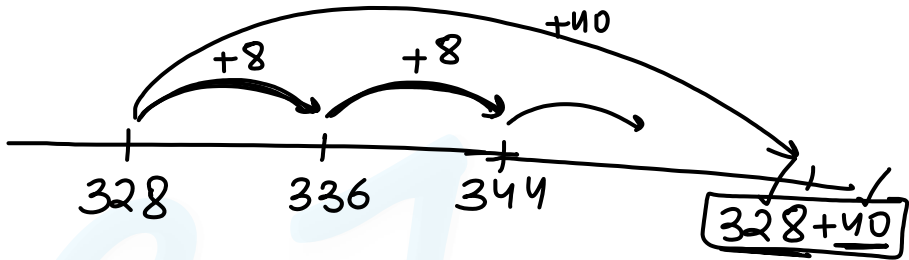8]

$2 \times 8 - 1 = 15$

9×8

Problem 140

140. If the 9-digit number 5p42978n6, is divisible by 72, what is the value of $(2p-1)$, where n is the second largest of all the possible value of n? Given that p and n are natural numbers.
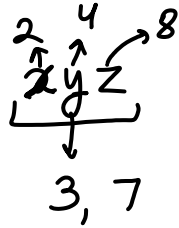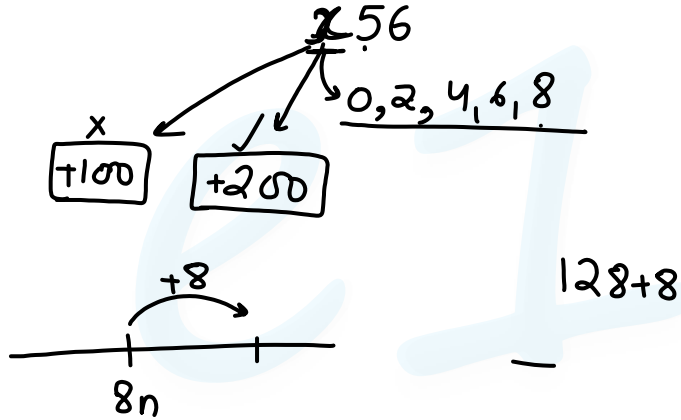
यदि 9 अंको की एक संख्या 5p42978n6 , 72 से विभाज्य है, तो $(2p-1)$, का मान क्या होगा, जहाँ n, n के सभी संभावित मानो मे से दूसरा सबसे बड़ा है? दिया गया है कि p और n प्राकृतिक संख्याएँ हैं।

a) 15
b) 21
c) 11
d) 17

328   +8   336   +8   344   +40

$\boxed{328 + 40}$

40 +

368

+20

348

328

+10

338

+30

358

8 + 6 → 1, 5, 9

856

896

$x.56$

$0, 2, 4, 6, 8$

x
| +100 |

| +200 |

$2 \quad 4 \quad 8$
$x \; y \; z$

$3, 7$

+8

128+8

8n