

ALGEBRA

बीजगणित

SHEET-01

COMPLETE CLASS NOTES BY ADITYA RANJAN

Type: 1

$$x + \frac{1}{x} \quad \&$$

$$x - \frac{1}{x}$$

$$\boxed{x + \frac{1}{x} = n}$$

$$\checkmark * x^2 + \frac{1}{x^2} = n^2 - 2 \quad \checkmark$$

$$\checkmark * x^3 + \frac{1}{x^3} = n^3 - 3n \quad \checkmark$$

$$\textcircled{*} x^4 + \frac{1}{x^4} = (n^2 - 2)^2 - 2$$

$$* x^5 + \frac{1}{x^5} =$$

$$\checkmark * x^6 + \frac{1}{x^6} = (n^3 - 3n)^2 - 2$$

$$\# \quad x + \frac{1}{x} = n$$

$$\underline{\text{Proof:}} \quad \left(x + \frac{1}{x}\right)^2 = n^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \cancel{x} \times \cancel{x} \frac{1}{x} = n^2$$

$$\Rightarrow \boxed{x^2 + \frac{1}{x^2} = n^2 - 2}$$

Q. If $x + \frac{1}{x} = 3$ $x^2 + \frac{1}{x^2} = ?$

Ans $x^2 + \frac{1}{x^2} = n^2 - 2$
 $x^2 = 3^2 - 2 = 4$

Q. If $x + \frac{1}{x} = 7$ $x^2 + \frac{1}{x^2} = ?$

Ans $x^2 + \frac{1}{x^2} = n^2 - 2$
 $x^2 = (7)^2 - 2$
 $= 47$

$$\# \quad x + \frac{1}{x} = n$$

Proof:

$$\left(x + \frac{1}{x}\right)^3 = n^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \cancel{x} \times \frac{1}{\cancel{x}} \left(x + \frac{1}{x}\right) = n^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3n = n^3$$

$$\therefore x^3 + \frac{1}{x^3} = n^3 - 3n$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$\textcircled{1}. \quad x + \frac{1}{x} = 3 \quad x^3 + \frac{1}{x^3} = ?$$

Ans

$$\begin{aligned} x^3 + \frac{1}{x^3} &= n^3 - 3n \\ &= 27 - 9 = \textcircled{18} \end{aligned}$$

$$\textcircled{2}. \quad x + \frac{1}{x} = \sqrt{5} \quad x^3 + \frac{1}{x^3} = ?$$

Ans

$$\begin{aligned} x^3 + \frac{1}{x^3} &= n^3 - 3n \\ &= (\sqrt{5})^3 - 3\sqrt{5} \\ &= 5\sqrt{5} - 3\sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} (\sqrt{5})^3 &= \underbrace{\sqrt{5} \times \sqrt{5} \times \sqrt{5}}_{= 5\sqrt{5}} \end{aligned}$$

- Case of Double Power

$$* \quad x + \frac{1}{x} = n$$

$$* \quad x^2 + \frac{1}{x^2} = n^2 - 2$$

$$* \quad x^4 + \frac{1}{x^4} =$$

$$* \quad x^8 + \frac{1}{x^8} =$$

$$* \quad x^{16} + \frac{1}{x^{16}} =$$

Ex:- * $x + \frac{1}{x} = 3$

* $x^2 + \frac{1}{x^2} = (3)^2 - 2 = 7$

* $x^4 + \frac{1}{x^4} = (7)^2 - 2 = 47$

* $x^8 + \frac{1}{x^8} = (47)^2 - 2$
 $= 2209 - 2$
 $= \underline{\underline{2207}}$

* $x + \frac{1}{x} = 1$

$x^2 + \frac{1}{x^2} = (1)^2 - 2 = 1 - 2 = \textcircled{-1}$

$x^4 + \frac{1}{x^4} = (-1)^2 - 2 = 1 - 2 = \textcircled{-1}$

$x^8 + \frac{1}{x^8} = (-1)^2 - 2 = 1 - 2 = \textcircled{-1}$

$$* x^3 + \frac{1}{x^3} = \textcircled{18}$$

$$* x^6 + \frac{1}{x^6} = (18)^2 - 2 = 324 - 2 = 322$$

$$* x^{12} + \frac{1}{x^{12}} = (322)^2 - 2$$

$$= 103684 - 2$$

$$= \underline{103682}$$

$$\begin{array}{r} 322 \\ \times 322 \\ \hline 103684 \end{array}$$

$$\begin{array}{l}
 x + \frac{1}{x} \begin{cases} \xrightarrow{\text{Even power}} 2, 4, 6, 8, \dots \\ \xrightarrow{\text{Odd powers}} 3, 5, 7, 9, \dots \end{cases} \\
 \begin{aligned}
 & \xrightarrow{\text{Even power}} (x^2 + \frac{1}{x^2}) (x^4 + \frac{1}{x^4}) (x^6 + \frac{1}{x^6}) (x^8 + \frac{1}{x^8}) \dots \\
 & \quad (n^2 - 2) \\
 & \xrightarrow{\text{Odd powers}} (x^3 + \frac{1}{x^3})^{**} (x^5 + \frac{1}{x^5})^{**} (x^7 + \frac{1}{x^7}) \\
 & \quad (n^3 - 3n)
 \end{aligned}
 \end{array}$$

odd powers

$$\begin{aligned} * x^5 + \frac{1}{x^5} &= \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right) \\ &= x^5 + \frac{x^2}{x^3} + \frac{x^3}{x^2} + \frac{1}{x^5} - x - \frac{1}{x} \\ &= x^5 + \cancel{\frac{1}{x}} + \cancel{x} + \frac{1}{x^5} - \cancel{x} - \cancel{\frac{1}{x}} \\ &= x^5 + \frac{1}{x^5} \end{aligned}$$

$$x^5 + \frac{1}{x^5} = (n^2 - 2)(n^3 - 3n) - n$$

$$* x + \frac{1}{x} = n$$

$$* x^2 + \frac{1}{x^2} = n^2 - 2$$

$$* x^3 + \frac{1}{x^3} = n^3 - 3n$$

$$* x^4 + \frac{1}{x^4} = (n^2 - 2)^2 - 2$$

$$* x^5 + \frac{1}{x^5} = (n^2 - 2)(n^3 - 3n) - n$$

$$* x^6 + \frac{1}{x^6} = (n^3 - 3n)^2 - 2$$

$$* x^7 + \frac{1}{x^7} = \left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)$$

$$* x + \frac{1}{x} = 4$$

$$* x^2 + \frac{1}{x^2} = (4)^2 - 2 = \textcircled{14}$$

$$* x^3 + \frac{1}{x^3} = (4)^3 - 3 \times 4$$

$$= 64 - 12 = \textcircled{52}$$

$$* x^4 + \frac{1}{x^4} = (14)^2 - 2$$

$$= 196 - 2$$

$$= \textcircled{194}$$

$$* x^5 + \frac{1}{x^5} = 14 \times 52 - 4$$

$$= 728 - 4 = \textcircled{724}$$

$$* x^6 + \frac{1}{x^6} = (52)^2 - 2$$

$$= 2704 - 2 = \textcircled{2702}$$

$$* x^7 + \frac{1}{x^7} = 52 \times 194 - 4$$

$$= 10088 - 4$$

$$= \underline{10084}$$

$$x^2 + \frac{1}{x^2} = (3)^2 - 2 = 7$$

$$x^3 + \frac{1}{x^3} = (3)^3 - 3 \times 3 = 18$$

$$x^4 + \frac{1}{x^4} = (7)^2 - 2 = 47$$

$$x^5 + \frac{1}{x^5} = 7 \times 18 - 3 = 123$$

$$x^6 + \frac{1}{x^6} = (18)^2 - 2 = 322$$

$$x^7 + \frac{1}{x^7} = 18 \times 47 - 3 = 846 - 3 = 843$$

1.**If $x + \frac{1}{x} = 3$ Find**

(i) $x^2 + \frac{1}{x^2}$

(iii) $x^4 + \frac{1}{x^4}$

(v) $x^6 + \frac{1}{x^6}$

(ii) $x^3 + \frac{1}{x^3}$

(iv) $x^5 + \frac{1}{x^5}$

(vi) $x^7 + \frac{1}{x^7}$

$$\text{If } x + \frac{1}{x} = n$$

$$* x^2 + \frac{1}{x^2} = n^2 - 2 + 2$$

$$* x^3 + \frac{1}{x^3} = n^3 - 3n$$

$$* x^4 + \frac{1}{x^4} = (n^2 - 2)^2 - 2 + 2$$

$$* x^5 + \frac{1}{x^5} = (n^2 - 2)(n^3 - 3n) - n$$

$$* x^6 + \frac{1}{x^6} = (n^3 - 3n)^2 - 2$$

$$\text{If } x - \frac{1}{x} = n$$

$$* x^2 + \frac{1}{x^2} = n^2 + 2$$

$$* x^3 - \frac{1}{x^3} = n^3 + 3n$$

$$* x^4 + \frac{1}{x^4} = (n^2 + 2)^2 - 2$$

$$* x^5 - \frac{1}{x^5} = (n^2 + 2)(n^3 + 3n) - n$$

$$***$$
$$* x^6 + \frac{1}{x^6} = (n^2 + 3n)^2 + 2$$

$$* \quad x - \frac{1}{x} = n$$

$$\left(x - \frac{1}{x}\right)^2 = (n)^2$$

$$\Rightarrow x^2 + \left(\frac{1}{x}\right)^2 - 2 \times \cancel{x} \times \frac{1}{\cancel{x}} = n^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = n^2$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2} = n^2 + 2\right)$$

$$* \quad x^3 - \frac{1}{x^3} = (n^3 + 3n)$$

$$\Rightarrow \left[x^3 - \frac{1}{x^3} \right]^2 = (n^3 + 3n)^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} - 2 \cancel{x^3} \times \cancel{\frac{1}{x^3}} = (n^3 + 3n)^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} = (n^3 + 3n)^2 + 2$$

$$* x - \frac{1}{x} = 2$$

$$* x^2 + \frac{1}{x^2} = n^2 + 2 = 6$$

$$* x^3 - \frac{1}{x^3} = n^3 + 3n = 14$$

$$* x^4 + \frac{1}{x^4} = (6)^2 - 2 = 34$$

$$* x^5 - \frac{1}{x^5} = 6 \times 14 - 2 = 82$$

$$* x^6 + \frac{1}{x^6} = (14)^2 + 2 = \underline{198}$$

$$x - \frac{1}{x} = 4$$

$$* x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$$

$$* x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76$$

$$* x^4 + \frac{1}{x^4} = 18^2 - 2 = 322$$

$$* x^5 - \frac{1}{x^5} = 18 \times 76 - 4 = 1368 - 4 = 1364$$

$$* x^6 + \frac{1}{x^6} = (76)^2 + 2 = 5776 + 2 = 5778$$

2. If $x - \frac{1}{x} = 4$, Find

(i) $x^2 + \frac{1}{x^2}$

(iii) $x^3 - \frac{1}{x^3}$

(v) $x^5 - \frac{1}{x^5}$

~~(ii)~~ $x^2 - \frac{1}{x^2}$

(iv) $x^4 + \frac{1}{x^4}$

(vi) $x^6 + \frac{1}{x^6}$

$$a + \frac{1}{a} = 3$$

$$a^2 + \frac{1}{a^2} = 3^2 - 2 = 7$$

$$a^4 + \frac{1}{a^4} = 7^2 - 2 = 47$$

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

(a) 27

(b) 81

(c) 48

(d) 47

$$\begin{aligned}a^4 + \frac{1}{a^4} &= (n^2 - 2)^2 - 2 \\&= (7)^2 - 2 \\&= 47\end{aligned}$$

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

- (a) 27
- (c) 48

- (b) 81
- (d) 47

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

(a) $27+2=29$ x

(b) $81+2=83$ x

(c) $48+2=50$ x

✓ (d) $47+2=49$

4. If $a + \frac{1}{a} = 7$, then $a^5 + \frac{1}{a^5}$ is equal to:

यदि $a + \frac{1}{a} = 7$ है, तो $a^5 + \frac{1}{a^5}$ निम्न में से किसके बराबर है?

SSC CGL 17/07/2023 (Shift-02)

☒ (a) 15127

(b) 13127

(c) 14527

(d) 11512

$$\begin{aligned} a^5 + \frac{1}{a^5} &= (a^2 - 2)(a^3 - 3a) - a \\ &= 47 \times 322 - 7 \\ &= 15134 - 7 \\ &= 15127 \end{aligned}$$

5. If $x + \frac{1}{x} = -6$, what will be the value of $x^5 + \frac{1}{x^5}$

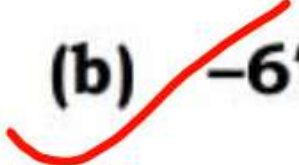
$$\begin{aligned}x^5 + \frac{1}{x^5} &= (n^2 - 2)(n^3 - 3n) - n \\&= (36 - 2)(-216 + 18) - (-6) \\&= 34(-198) + 6 \\&= -6732 + 6 \\&= -6726\end{aligned}$$

यदि $x + \frac{1}{x} = -6$, है तो $x^5 + \frac{1}{x^5}$ का मान क्या होगा?

SSC CGL 17/07/2023 (Shift-03)

(a) -7776

(c) -6730

(b)  -6726

(d) -6732

$$a + \frac{1}{a} = 3$$

6. If $a + \frac{1}{a} = 3$, then $a^6 + \frac{1}{a^6}$ is equal to :

$$\begin{aligned} a^6 + \frac{1}{a^6} &= (a^3 - \frac{1}{a^3})^2 - 2 \\ &= (27 - \frac{1}{27})^2 - 2 \\ &= 324 - 2 \\ &= 322 \end{aligned}$$

SSC CHSL 10/07/2019 (Shift- 02)

(a) 319

(c) 780

(b) 322

(d) 730

m-1

$$x - \frac{1}{x} = 10$$

$$x^4 + \frac{1}{x^4} = (x^2 + \frac{1}{x^2})^2 - 2$$

$$= (102)^2 - 2$$

$$= 10404 - 2$$

$$= 10402$$

7. If $\left(x - \frac{1}{x}\right) = 10$, what is the value of $x^4 + \frac{1}{x^4}$?

यदि $\left(x - \frac{1}{x}\right) = 10$ है, तो $x^4 + \frac{1}{x^4}$ का मान क्या है?

SSC CGL 26/07/2023 (Shift-03)

(a) 10404

(c) 10406

(b) 10402

(d) 10400

$$\begin{aligned}\text{ans} &= n^3 + 3n \\ &= (10)^3 + 3 \times 10 \\ &= 1000 + 30 \\ &= \underline{1030}\end{aligned}$$

8. If $x - \frac{1}{x} = 10$, then $x^3 - \frac{1}{x^3}$ is equal to :

SSC CHSL 4/07/2019 (Shift-02)

- (a) 970
✓ (c) 1030

- (b) 1000
(d) 1100

$$\begin{aligned}x - \frac{1}{x} &= n \\ x^3 - \frac{1}{x^3} &= n^3 + 3n\end{aligned}$$

$$3\left(y - \frac{1}{y}\right) = 5$$

$$\Rightarrow y - \frac{1}{y} = \frac{5}{3}$$

$$\begin{aligned}\therefore y^2 + \frac{1}{y^2} &= \left(\frac{5}{3}\right)^2 + 2 \\ &= \frac{25}{9} + 2 \\ &= \frac{43}{9}\end{aligned}$$

9. If $\left(3y - \frac{3}{y}\right) = 5$, find the value of $\left(y^2 + \frac{1}{y^2}\right)$.

यदि $\left(3y - \frac{3}{y}\right) = 5$ है, तो $\left(y^2 + \frac{1}{y^2}\right)$ का मान ज्ञात करें।

SSC CGL 19/07/2023 (Shift-02)

(a) $\frac{47}{9}$

(c) $\frac{41}{9}$

(b) $\frac{49}{9}$

✓ (d) $\frac{43}{9}$

$$\begin{aligned}\text{ans} &= (n^2+2)^2-2 \\ &= (13^2+2)^2-2 \\ &= (171)^2-2 \\ &= 29241-2 \\ &= \underline{29239}\end{aligned}$$

10. If $x - \frac{1}{x} = 13$, what will be the value of $x^4 + \frac{1}{x^4}$?

यदि $x - \frac{1}{x} = 13$ है, तो $x^4 + \frac{1}{x^4}$ का मान क्या होगा?

SSC CGL 03/12/2022 (Shift-01)

(a) 28561

(b) 29243

(c) 27887

✓ (d) 29239

$$x - \frac{1}{x} = n$$

$$x^4 + \frac{1}{x^4} = (n^2+2)^2-2$$

$$\begin{aligned} \text{ans} &= 24 \times 140 - 5 \\ &= 3360 - 5 \\ &= \boxed{3355} \end{aligned}$$

11. If $\left(x - \frac{1}{x}\right) = 5$, then the value of $\left(x^5 - \frac{1}{x^5}\right)$ is :

यदि $\left(x - \frac{1}{x}\right) = 5$, तो $\left(x^5 - \frac{1}{x^5}\right)$ का मान है:

SSC CHSL 07/08/2023 Shift-02

(a) ~~2535~~
(c) ~~2515~~

(b) ~~2540~~
(d) ~~2525~~

$$x - \frac{1}{x} = n$$

$$x^5 - \frac{1}{x^5} = (n^2 + 2)(n^3 + 3n) - n$$

$$\begin{aligned}
 \text{Ans} &= (36+2)(-216-18) - (-6) \\
 &= 38(-234) + 6 \\
 &= -8892 + 6 \\
 &= -\underline{8886}
 \end{aligned}$$

12. If $x - \frac{1}{x} = -6$ what will be the value of $x^5 - \frac{1}{x^5}$?

यदि $x - \frac{1}{x} = -6$ है, तो $x^5 - \frac{1}{x^5}$ का मान क्या होगा?

SSC CGL 18/07/2023 (Shift-02)

(a) - 8898

(b) - 8896

(c) - 8886

(d) - 8892

$$\begin{aligned}
 x - \frac{1}{x} &= n \\
 x^5 - \frac{1}{x^5} &= (n^2+2)(n^3+3n) - n
 \end{aligned}$$

$$\begin{aligned}\text{Ans} &= (64+12)^2 + 2 \\ &= (76)^2 + 2 \\ &= 5776 + 2 \\ &= \underline{5778}\end{aligned}$$

13. If $\left(y - \frac{1}{y}\right) = 4$, find the value of $\left(y^6 + \frac{1}{y^6}\right)$.

यदि $\left(y - \frac{1}{y}\right) = 4$, है तो $\left(y^6 + \frac{1}{y^6}\right)$ का मान ज्ञात करें

✓ SSC CGL 25/07/2023 (Shift-01)

(a) 5774

(b) 4096

(c) 5776

✓ (d) 5778

$$\begin{aligned}y - \frac{1}{y} &= n \\ y^6 + \frac{1}{y^6} &= (n^3 + 3n)^2 + 2\end{aligned}$$

Questions related to root

$$\left. \begin{aligned} (\sqrt{x})^2 &= \sqrt{x} \times \sqrt{x} = x \\ (\sqrt{x})^3 &= \underbrace{\sqrt{x} \times \sqrt{x}} \times \sqrt{x} \\ &= x\sqrt{x} \end{aligned} \right\} \begin{aligned} (\sqrt{5})^2 &= 5 \\ (\sqrt{5})^3 &= 5\sqrt{5} \end{aligned}$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 4^2$$

14. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

$$\Rightarrow (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2 \times \cancel{\sqrt{x}} \times \cancel{\frac{1}{\sqrt{x}}} = 16$$

(a) 192

(b) 326

(c) 322

(d) 256

$$\Rightarrow x + \frac{1}{x} - 2 = 16$$

$$\Rightarrow x + \frac{1}{x} = 18$$

$$\begin{aligned} x^2 + \frac{1}{x^2} &= (18)^2 - 2 \\ &= 324 - 2 \\ &= 322 \end{aligned}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$x + \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 - 2$$

$$x - \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 + 2$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 4$$

$$x + \frac{1}{x} = (4)^2 + 2 = 18$$

$$x^2 + \frac{1}{x^2} = (18)^2 - 2 = 322$$

14. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

(a) 192

(b) 326

(c) 322

(d) 256

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$x + \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 - 2$$

$$x - \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 + 2$$

14. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

- ~~(a) 192~~ +2
(c) 322 +2

- ~~(b) 326~~ +2
~~(d) 256~~ +2

15. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6}$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 11/06/2019 (Shift-02)

☒ (a) 62 + 2 = 64
(c) 54

(b) 40
(d) 66

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{6})^2$$

$$\Rightarrow x + \frac{1}{x} - 2 \times \cancel{\sqrt{x}} \times \frac{1}{\cancel{\sqrt{x}}} = 6$$

$$\Rightarrow x + \frac{1}{x} = 8$$

$$\begin{aligned} \rightarrow x^2 + \frac{1}{x^2} &= x^2 - 2 \\ &= 8^2 - 2 \\ &= \underline{\underline{62}} \end{aligned}$$

15. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6}$, then $x^2 + \frac{1}{x^2}$ is equal to :

- (a) 62
(c) 54

- (b) 40
(d) 66

SSC CGL 11/06/2019 (Shift-02)

$$\textcircled{Q}. \quad x + \frac{1}{x} = 3\sqrt{3}$$

$$x^2 + \frac{1}{x^2} = ?$$

$$\begin{aligned} \underline{Ans} &= n^2 - 2 \\ &= (3\sqrt{3})^2 - 2 \\ &= 9 \times 3 - 2 \\ &= \textcircled{25} \end{aligned}$$

$$\textcircled{a} \quad 24$$

$$\textcircled{b} \quad 23$$

$$\textcircled{c} \quad 25$$

$$\textcircled{d} \quad 27$$

Concept of Bridging

$$\begin{cases} \# (a+b)^2 = (a-b)^2 + 4ab \\ \# (a-b)^2 = (a+b)^2 - 4ab \end{cases}$$


$$\begin{aligned} \# \text{ If } x - \frac{1}{x} = n & \quad \text{then } x + \frac{1}{x} = \sqrt{n^2 + 4} \\ \# \text{ If } x + \frac{1}{x} = n & \quad \text{then } x - \frac{1}{x} = \sqrt{n^2 - 4} \end{aligned}$$

$$\textcircled{1}. \text{ If } x + \frac{1}{x} = 3 \quad \text{then } x - \frac{1}{x} = \frac{\sqrt{n^2 - 4}}{\sqrt{9 - 4}} = \sqrt{5}$$

$$\textcircled{1}. \text{ If } x + \frac{1}{x} = 7 \quad \text{then } x - \frac{1}{x} = \frac{\sqrt{n^2 - 4}}{\sqrt{49 - 4}} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

$$\textcircled{1}. \text{ If } x - \frac{1}{x} = 4 \quad \text{then } x + \frac{1}{x} = \frac{\sqrt{n^2 + 4}}{\sqrt{16 + 4}} = \sqrt{20} = 2\sqrt{5}$$

$$\textcircled{1}. \text{ If } x - \frac{1}{x} = 3 \quad \text{then } x + \frac{1}{x} = \frac{\sqrt{n^2 + 4}}{\sqrt{9 + 4}} = \sqrt{13}$$

$$\begin{aligned} a + \frac{1}{a} &= \sqrt{n^2 + 4} \\ &= \sqrt{16 + 4} = \sqrt{20} \\ &= \sqrt{4 \times 5} \\ &= \underline{2\sqrt{5}} \end{aligned}$$

16. If $a - \frac{1}{a} = 4$, then the value of $a + \frac{1}{a}$ is:

यदि $a - \frac{1}{a} = 4$ है, तो $a + \frac{1}{a}$ का मान है:

✓ **SSC CPO 05.10.2023 (Shift-2)**

(a) $5\sqrt{5}$

(b) $4\sqrt{5}$

✓ (c) $2\sqrt{5}$

(d) $3\sqrt{5}$

Q. $a + \frac{1}{a} = 3$ then $a^2 - \frac{1}{a^2} = ?$

$$\downarrow$$
$$\left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$$

$$a^2 - b^2 = (a - b)(a + b)$$

Solⁿ $a + \frac{1}{a} = 3$

$$a - \frac{1}{a} = \sqrt{(3)^2 - 4} = \sqrt{5}$$

$$\therefore a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$$

$$= \sqrt{5} \times 3$$

$$= 3\sqrt{5} \checkmark$$

$$\begin{aligned}
 x^2 - \frac{1}{x^2} &= \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \\
 &= \frac{2}{5} \sqrt{\left(\frac{2}{5}\right)^2 + 4} \\
 &= \frac{2}{5} \sqrt{\frac{4}{25} + 4} \\
 &= \frac{2}{5} \sqrt{\frac{104}{25}} \\
 &= \frac{2}{5} \times \frac{2}{5} \sqrt{26} \\
 &= \frac{4}{25} \sqrt{26}
 \end{aligned}$$

17. If $\left(x - \frac{1}{x}\right) = 0.4$, and $x > 0$, what is the value of $\left(x^2 - \frac{1}{x^2}\right)$?

यदि $\left(x - \frac{1}{x}\right) = 0.4$, और $x > 0$ तो $\left(x^2 - \frac{1}{x^2}\right)$ का मान क्या है?

SSC CHSL 07/08/2023 Shift-02

- (a) 4.16
- (b) 3.32
- (c) 1.84
- (d) 2.16

$$* \quad x + \frac{1}{x} = 3$$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$* \quad x^{10} + \frac{1}{x^{10}} = 3$$

$$x^{20} + \frac{1}{x^{20}} = 3^2 - 2 = \textcircled{7}$$

$$* \quad x^3 + \frac{1}{x^3} = 5$$

$$x^6 + \frac{1}{x^6} = (5)^2 - 2 = \underline{\underline{23}}$$

let $x^2 = a$

$$a + \frac{1}{a} = 6, \quad a^2 - \frac{1}{a^2} = ?$$

$$\begin{aligned} a^2 - \frac{1}{a^2} &= \left(a + \frac{1}{a}\right) \left(a - \frac{1}{a}\right) \\ &= 6 \sqrt{36 - 4} \\ &= 6 \times 4\sqrt{2} \\ &= 24\sqrt{2} \end{aligned}$$

18. If $\left(x^2 + \frac{1}{x^2}\right) = 6$ and $0 < x < 1$, what is the value of $x^4 - \frac{1}{x^4}$?

यदि $\left(x^2 + \frac{1}{x^2}\right) = 6$ और $0 < x < 1$ है, तो $x^4 - \frac{1}{x^4}$ का मान क्या होगा?

SSC CGL 26/07/2023 (Shift-01)

✓ (a) $24\sqrt{2}$

(b) $-24\sqrt{2}$

(c) $-12\sqrt{10}$

(d) $12\sqrt{10}$

$$a - \frac{1}{a} = 4\sqrt{2} \quad ; \quad a^2 - \frac{1}{a^2} = ?$$

$$\begin{aligned} a^2 - \frac{1}{a^2} &= \left(a - \frac{1}{a}\right) \left(a + \frac{1}{a}\right) \\ &= 4\sqrt{2} \sqrt{(4\sqrt{2})^2 + 4} \\ &= 4\sqrt{2} \sqrt{16 \times 2 + 4} \\ &= 4\sqrt{2} \times 6 \\ &= \underline{24\sqrt{2}} \end{aligned}$$

19. If $x^2 - \frac{1}{x^2} = 4\sqrt{2}$, what is the value of

$$x^4 - \frac{1}{x^4} ?$$

यदि $x^2 - \frac{1}{x^2} = 4\sqrt{2}$ है, तो $x^4 - \frac{1}{x^4}$ का मान क्या है?

✓ SSC CGL 27/07/2023 (Shift-02)

(a) $16\sqrt{2}$

(b) $8\sqrt{2}$

✓ (c) $24\sqrt{2}$

(d) $32\sqrt{2}$

Expansion

$$\checkmark * x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$\checkmark * x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$\checkmark * x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$* x^6 - \frac{1}{x^6} = (x^3)^2 - \frac{1}{(x^3)^2} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

Expansion

$$* x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$= \left(x^2 + \frac{1}{x^2}\right) \left(x^2 - \frac{1}{x^2}\right)$$

$$= \left((x^2)^2 - \frac{1}{(x^2)^2}\right)$$

$$= x^4 - \frac{1}{x^4}$$

$$* \text{ If } x + \frac{1}{x} = 3$$

$$\begin{aligned} \textcircled{a} \quad x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ &= 3 \times (\sqrt{3^2 - 4}) \\ &= 3 \times \sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

$$x + \frac{1}{x} = n \quad x - \frac{1}{x} = \sqrt{n^2 - 4}$$

$$* \text{ If } x + \frac{1}{x} = 3$$

$$\begin{aligned} \textcircled{b} \quad x^4 - \frac{1}{x^4} &= \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ &= (3^2 - 2) \times 3 \times (\sqrt{3^2 - 4}) \\ &= 7 \times 3 \times \sqrt{5} \\ &= 21\sqrt{5} \end{aligned}$$

* If $x - \frac{1}{x} = 3$

$$\begin{aligned}
 \textcircled{d} \quad x^8 - \frac{1}{x^8} &= \left(x^4 + \frac{1}{x^4} \right) \left(x^2 + \frac{1}{x^2} \right) \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right) \\
 &= \overset{11^2-2}{119} \times \overset{(\sqrt{13})^2-2}{11} \times \overset{\sqrt{3^2+4}}{\sqrt{13}} \times 3 \\
 &= 357 \times 11 \times \sqrt{13} \\
 &= 3927\sqrt{13} \quad \checkmark
 \end{aligned}$$

* If $x - \frac{1}{x} = 2$

$$\text{find } x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4} \right) \left(x^2 + \frac{1}{x^2} \right) \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)$$

$$= 34 \times 6 \times \sqrt{8} \times 2$$

$$= 34 \times 12 \times 2\sqrt{2}$$

$$= \underline{816\sqrt{2}}$$

$$* \quad x - \frac{1}{x} = 4$$

$$x^8 - \frac{1}{x^8} = \frac{(18)^2 - 2}{322} \times \frac{(\sqrt{20})^2 - 2}{18} \times \frac{\sqrt{4^2 + 4}}{\sqrt{20}} \times 4$$

$$= 322 \times 72 \times 2\sqrt{5}$$

$$= 644 \times 72 \sqrt{5}$$

$$= 46368 \sqrt{5}$$

$$\textcircled{1}. \quad x + \frac{1}{x} = 3$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$= 18 \times 8\sqrt{5}$$

$$= \underline{\underline{144\sqrt{5}}}$$

$$\rightarrow x + \frac{1}{x} = 3$$

$$x^3 + \frac{1}{x^3} = (3)^3 - 3 \times 3 = 18$$

$$\rightarrow x - \frac{1}{x} = \sqrt{3^2 - 4} = \sqrt{5}$$

$$x^3 - \frac{1}{x^3} = (\sqrt{5})^3 + 3\sqrt{5}$$

$$= 5\sqrt{5} + 3\sqrt{5}$$

$$= 8\sqrt{5}$$

$$\textcircled{1}. \quad x + \frac{1}{x} = 3$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$= 18 \times \sqrt{18^2 - 4}$$

$$= 18 \times \sqrt{320}$$

$$= 18 \times 8\sqrt{5}$$

$$= 144\sqrt{5}$$

$$x + \frac{1}{x} = 3$$

$$x^3 + \frac{1}{x^3} = x^3 - 3x$$

$$= 27 - 9$$

$$= \textcircled{18}$$

$$\textcircled{1}. x + \frac{1}{x} = 4$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$= 52 \times 30\sqrt{3}$$

$$= \underline{1560\sqrt{3}}$$

$$x + \frac{1}{x} = 4$$

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$$

$$= \textcircled{52}$$

$$x - \frac{1}{x} = \sqrt{4^2 - 4} = \sqrt{12}$$

$$= 2\sqrt{3}$$

$$x^3 - \frac{1}{x^3} = (2\sqrt{3})^3 + 3 \times 2\sqrt{3}$$

$$= 8 \times 3\sqrt{3} + 6\sqrt{3}$$

$$= 30\sqrt{3}$$

$$\textcircled{1}. \quad x + \frac{1}{x} = 4$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$= 52 \times \sqrt{(52)^2 - 4}$$

$$= 52 \times \sqrt{2704 - 4}$$

$$= 52 \times \sqrt{2700}$$

$$= 52 \times \sqrt{900 \times 3}$$

$$= 52 \times 30\sqrt{3}$$

$$= 1560\sqrt{3}$$

$$x + \frac{1}{x} = 4$$

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$$

$$= \textcircled{52}$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right) \quad 20.$$

$$\begin{aligned} &= 235\sqrt{2} \left(\sqrt{(235\sqrt{2})^2 - 4}\right) \\ &= 235\sqrt{2} \sqrt{55225 \times 2 - 4} \\ &= 235\sqrt{2} \sqrt{110446} \\ &= 235\sqrt{2} \sqrt{46 \times 2401} \\ &= 235\sqrt{2} \times \sqrt{2} \times \sqrt{23} \times 49 \\ &= 470 \times 49 \times \sqrt{23} \\ &= 23030\sqrt{23} \end{aligned}$$

If $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$, and $x > 1$, what is the value

of $\left(x^6 - \frac{1}{x^6}\right)$?

$$\begin{aligned} x + \frac{1}{x} &= 5\sqrt{2} \\ x^3 + \frac{1}{x^3} &= (5\sqrt{2})^3 - 3 \times 5\sqrt{2} \\ &= 125 \times 2\sqrt{2} - 15\sqrt{2} \\ &= 235\sqrt{2} \end{aligned}$$

यदि $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$, और $x > 1$, तो $\left(x^6 - \frac{1}{x^6}\right)$

का मान क्या है?

SSC CHSL 02/08/2023 Shift-02

- (a) $22970\sqrt{23}$
- (c) $23060\sqrt{23}$

- (b) $23030\sqrt{23}$
- (d) $22960\sqrt{23}$

$$\begin{aligned}
 x^6 - \frac{1}{x^6} &= \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right) \\
 &= 235\sqrt{2} \times 49\sqrt{46} \\
 &= 235 \times 49 \times \sqrt{2} \times \sqrt{2} \times \sqrt{23} \\
 &= 470 \times 49 \times \sqrt{23} \\
 &= 23030\sqrt{23}
 \end{aligned}$$

20. If $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$, and $x > 1$, what is the value

of $\left(x^6 - \frac{1}{x^6}\right)$?

$$x + \frac{1}{x} = 5\sqrt{2}$$

$$x^3 + \frac{1}{x^3} = (5\sqrt{2})^3 - 3x \cdot 5\sqrt{2}$$

$$x^3 + \frac{1}{x^3} = 250\sqrt{2} - 15\sqrt{2}$$

$$= 235\sqrt{2}$$

यदि $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$, और $x > 1$, तो $\left(x^6 - \frac{1}{x^6}\right)$

का मान क्या है?

SSC CHSL 02/08/2023 Shift-02

(a) $22970\sqrt{23}$

(b) $23030\sqrt{23}$

(c) $23060\sqrt{23}$

(d) $22960\sqrt{23}$

$$\begin{aligned}
 x - \frac{1}{x} &= \sqrt{(5\sqrt{2})^2 - 4} \\
 &= \sqrt{46}
 \end{aligned}$$

$$\begin{aligned}
 x^3 - \frac{1}{x^3} &= (\sqrt{46})^3 + 3\sqrt{46} \\
 &= 46\sqrt{46} + 3\sqrt{46} \\
 &= 49\sqrt{46}
 \end{aligned}$$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$= \sqrt{(22\sqrt{2})^2 + 4} \times 22\sqrt{2}$$

$$= \sqrt{484 \times 2 + 4} \times 22\sqrt{2}$$

$$= \sqrt{972} \times 22\sqrt{2}$$

$$= \sqrt{3 \times 324} \times 22\sqrt{2}$$

$$= 18 \times \sqrt{3} \times 22\sqrt{2}$$

$$= \underline{396\sqrt{6}}$$

21. If $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$, and $x > 1$, what is the value of $\left(x^6 - \frac{1}{x^6}\right)$.

$$x - \frac{1}{x} = 2\sqrt{2}$$

$$x^3 - \frac{1}{x^3} = (2\sqrt{2})^3 + 3 \times 2\sqrt{2}$$

$$= 16\sqrt{2} + 6\sqrt{2}$$

$$= 22\sqrt{2}$$

यदि $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$ और $x > 1$ है, तो $\left(x^6 - \frac{1}{x^6}\right)$

का मान क्या होगा?

SSC CGL 20/07/2023 (Shift-03)

(a) $372\sqrt{6}$

(b) $384\sqrt{6}$

(c) $396\sqrt{6}$

(d) $420\sqrt{6}$

$$\begin{aligned}
 x^8 - \frac{1}{x^8} &= \left(x^4 + \frac{1}{x^4}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\
 &= 527 \times 23 \times 5 \times \sqrt{25-4} \\
 &= 527 \times 115 \times \sqrt{21} \\
 &= 60605\sqrt{21}
 \end{aligned}$$

22. If $\left(x + \frac{1}{x}\right) = 5$, and $x > 1$, what is the value of $\left(x^8 - \frac{1}{x^8}\right)$?

यदि $\left(x + \frac{1}{x}\right) = 5$ और $x > 1$ है, तो $\left(x^8 - \frac{1}{x^8}\right)$ का मान क्या होगा?

SSC CGL 18/07/2023 (Shift-03)

☒ (a) $60605\sqrt{21}$

(b) $60615\sqrt{21}$

(c) $60705\sqrt{21}$

(d) $60725\sqrt{21}$

$N - Q?$



$Q \longrightarrow N?$



$Q \longrightarrow Q'$

.

.

23. If $x + \frac{1}{x} = 8$, then find the value of

$$\frac{5x}{x^2 + 1 - 6x}$$

SSC CHSL 14/10/2020 (Shift- 03)

- (a) 2.5
(c) 5

- (b) 6
(d) 6.5


$$\frac{\frac{5x}{\cancel{x}}}{\frac{x^2}{\cancel{x}} + \frac{1}{\cancel{x}} - \frac{6x}{\cancel{x}}} = \frac{5}{x + \frac{1}{x} - 6} = \frac{5}{8 - 6} = \frac{5}{2}$$

$$\frac{\frac{7x}{x}}{\frac{x^2}{x} + \frac{1}{x} - \frac{8x}{x}}$$

$$= \frac{7}{x + \frac{1}{x} - 8} = \frac{7}{10 - 8} = \frac{7}{2}$$

24. If $x + \frac{1}{x} = 10$, then find the value of

$$\left\{ \frac{7x}{x^2 + 1 - 8x} \right\}$$

SSC CHSL 03/06/2022 (Shift- 01)

- ☒ (a) 3.5
- (b) 4.5
- (c) 2.5
- (d) 5.5

$$\begin{aligned} & \frac{\frac{x^4}{x} + \frac{1}{x^2 \cdot x}}{\frac{x^2 - \cancel{3x}}{\cancel{x}} + \frac{1}{x}} \\ &= \frac{x^3 + \frac{1}{x^3}}{x - 3 + \frac{1}{x}} = \frac{x^3 + \frac{1}{x^3}}{x + \frac{1}{x} - 3} \\ &= \frac{5^3 - 3 \times 5}{5 - 3} \\ &= \frac{110}{2} = 55 \end{aligned}$$

25. If $x + \frac{1}{x} = 5$, $x \neq 0$ then the value of

$$\frac{x^4 + \frac{1}{x^2}}{x^2 - \underline{3x} + 1} \text{ is equal to :}$$

SSC CHSL 20/10/2020 (Shift- 01)

- ☒ (a) 55
(c) 65

- (b) 60
(d) 50

26. If $x - \frac{1}{x} = 5$, $x \neq 0$, then what is the value

of $\frac{x^6 + 3x^3 - 1}{x^6 - 8x^3 - 1}$?

SSC CGL TIER II 16/11/2020

$$\frac{\frac{x^6}{x^3} + \frac{3x^3}{x^3} - \frac{1}{x^3}}{\frac{x^6}{x^3} - \frac{8x^3}{x^3} - \frac{1}{x^3}}$$

$$= \frac{x^3 + 3 - \frac{1}{x^3}}{x^3 - 8 - \frac{1}{x^3}} = \frac{x^3 - \frac{1}{x^3} + 3}{x^3 - \frac{1}{x^3} - 8} \quad (a) \quad \frac{13}{12}$$

$$= \frac{\cancel{143} \frac{13}{12}}{\cancel{132}} \quad (c) \quad \frac{3}{8}$$

$$(b) \quad \frac{11}{13}$$

$$(d) \quad \frac{4}{9}$$

$$\begin{aligned} x - \frac{1}{x} &= 5 \\ x^3 - \frac{1}{x^3} &= 5^3 + 3 \times 5 = 140 \end{aligned}$$

$$\frac{x^2}{x} + \frac{3x}{x} + \frac{1}{x} = 0$$

$$\Rightarrow x + 3 + \frac{1}{x} = 0$$

$$\Rightarrow \left(x + \frac{1}{x} = -3 \right)$$

$$\begin{aligned} x^6 + \frac{1}{x^6} &= [-24 - 3(-3)]^2 - 2 \\ &= [-24 + 9]^2 - 2 \\ &= 324 - 2 \\ &= \underline{322} \end{aligned}$$

27. If $x^2 + 3x + 1 = 0$, then what is the value of $x^6 + \frac{1}{x^6}$?

- (a) 324
(c) 318

SSC CGL 3/03/2020 (Shift- 02)

- (b) 322 ✓
(d) 327

$$x + \frac{1}{x} = n$$

$$x^6 + \frac{1}{x^6} = (n^3 - 3n)^2 - 2$$

28. If $x^2 - 2\sqrt{5}x + 1 = 0$, then what is the value of $x^5 + \frac{1}{x^5}$?

SSC CGL 5/03/2020 (Shift- 01)

$$\frac{x^2 - 2\sqrt{5}x + 1}{x} = 0$$

$$\Rightarrow x - 2\sqrt{5} + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 2\sqrt{5}$$

- (a) $610\sqrt{5}$
(c) $612\sqrt{5}$

- (b) $408\sqrt{5}$
(d) $406\sqrt{5}$

$$\begin{aligned} x^5 + \frac{1}{x^5} &= (20-2)(40\sqrt{5}-6\sqrt{5}) - 2\sqrt{5} \\ &= 18 \times 34\sqrt{5} - 2\sqrt{5} \\ &= 612\sqrt{5} - 2\sqrt{5} \\ &= 610\sqrt{5} \end{aligned}$$

$$\begin{aligned} x + \frac{1}{x} &= n \\ x^5 + \frac{1}{x^5} &= (n^2-2)(n^3-3n) - n \end{aligned}$$

$$6 = 2x + 5 - \frac{2}{x}$$

$$\Rightarrow 1 = 2\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \boxed{\frac{1}{2} = x - \frac{1}{x}}$$

$$\therefore x + \frac{1}{x} = \sqrt{\left(\frac{1}{2}\right)^2 + 4} = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$$

$$x^3 + \frac{1}{x^3} = \left(\frac{\sqrt{17}}{2}\right)^3 - 3\left(\frac{\sqrt{17}}{2}\right)$$

$$= \frac{17\sqrt{17}}{8} - \frac{4 \times 3\sqrt{17}}{4 \times 2}$$

$$= \frac{5\sqrt{17}}{8}$$

29. If $\frac{6x}{2x^2 + 5x - 2} = 1, x > 0$, then the value of $x^3 + \frac{1}{x^3}$ is :

$$\frac{6}{2x + 5 - \frac{2}{x}} = 1$$

SSC CGL 07/06/2019 (Shift- 03)

(a) $\frac{3}{8}\sqrt{17}$

(b) $\frac{5}{8}\sqrt{17}$

(c) $\frac{5}{16}\sqrt{17}$

(d) $\frac{3}{4}\sqrt{17}$

$$\frac{x^2}{x} - \frac{5x}{x} + \frac{1}{x} = 0$$

$$\Rightarrow x - 5 + \frac{1}{x} = 0$$

$$\Rightarrow \left(x + \frac{1}{x} \right) = 5$$

$$\frac{x^3 + x + \frac{1}{x} + \frac{1}{x^3}}{5} = \frac{\left(x + \frac{1}{x} \right) + \left(x^3 + \frac{1}{x^3} \right)}{5} = \frac{5 + 10}{5} = \frac{15}{5} = 3$$

30. If $x^2 - 5x + 1 = 0$, then the value of

$$\left\{ \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3} \right\} = ?$$

SSC CGL 01/12/2022 (Shift-01)

- (a) 30
~~(c) 23~~

- (b) 25
 (d) 28

$$x - 3 + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 3$$

$$\frac{x^3 + \frac{1}{x^3}}{x + 5 + \frac{1}{x}} = \frac{(3)^3 - 3 \times 3}{3 + 5} = \frac{9}{8} \quad \text{4}$$

31. If $\frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x} = 0$, then the value of

$$\frac{\frac{x^4}{x} + \frac{1}{x^2} x}{\frac{x^2}{x} + 5\cancel{x} + \frac{1}{x}} \text{ is:}$$

SSC CGL MAINS 03/02/2022

☒ (a) $\frac{9}{4}$

(b) $\frac{27}{8}$

(c) $\frac{5}{2}$

(d) 2

$$\frac{x^2}{x} - \frac{3x}{x} - \frac{1}{x} = 0$$

$$\Rightarrow x - 3 - \frac{1}{x} = 0 \quad \left\} \Rightarrow x - \frac{1}{x} = 3 \quad \checkmark$$

$$\left(\frac{x^2}{x} + \frac{8x}{x} - \frac{1}{x} \right) \times \frac{1}{\left(\frac{x^3}{x} + \frac{1}{x \cdot x} \right)^1}$$

$$= \frac{\left(x + 8 - \frac{1}{x} \right)}{\left(x^2 + \frac{1}{x^2} \right)} = \frac{3 + 8}{3^2 + 2} = \frac{11}{11} = 1$$

32. If $x^2 - 3x - 1 = 0$, then the value of $(x^2 + 8x - 1)(x^3 + x^{-1})^{-1}$ is :

SSC CGL 07/06/2019 (Shift- 02)

(a) $\frac{3}{8}$

(b) 8

☒ (c) 1

(d) 3

$$\frac{a^{-1}}{1} = \frac{1}{a^1}$$

$$2^{-1} = \frac{1}{2}$$

$$x - \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 + 2$$

Equal Coefficient

$$* \quad x + \frac{1}{x} = 3$$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$* \quad 2x + \frac{1}{2x} = 3$$

$$\left(2x + \frac{1}{2x}\right)^2 = (3)^2$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} + 2 \times \cancel{2x} \times \frac{1}{\cancel{2x}} = (3)^2$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} = (3)^2 - 2 \times \cancel{2x} \times \frac{1}{\cancel{2x}}$$

Unequal Coefficient

$$* \quad 3x + \frac{1}{2x} = 4$$

$$\underline{\text{Soln}} \quad \left(3x + \frac{1}{2x}\right)^2 = (4)^2$$

$$\Rightarrow 9x^2 + \frac{1}{4x^2} + 2 \times \cancel{3x} \times \frac{1}{\cancel{2x}} = (4)^2$$

$$\Rightarrow 9x^2 + \frac{1}{4x^2} = (4)^2 - \cancel{2 \times 3x} \times \frac{1}{\cancel{2x}} = \textcircled{13}$$

$$\textcircled{1}. 4x + \frac{1}{2x} = 2$$

$$16x^2 + \frac{1}{4x^2} = ?$$

Ans

$$\begin{aligned} & (2)^2 - \cancel{2} \times 4x \times \frac{1}{\cancel{2}} \\ & = 4 - 4 \\ & = \underline{\underline{0}} \end{aligned}$$

$$\textcircled{1}. 3x + \frac{1}{2x} = 4$$

$$27x^3 + \frac{1}{8x^3} = ?$$

Ans

$$\begin{aligned} & (4)^3 - 3 \times \cancel{4}^2 \times 3x \times \frac{1}{\cancel{2}} \\ & = 64 - 18 \\ & = \underline{\underline{46}} \end{aligned}$$

$$\textcircled{1}. 3x + \frac{1}{2x} = 4$$

$$\begin{aligned} 3x - \frac{1}{2x} &= \sqrt{(4)^2 - \cancel{4}^2 \times 3 \times \frac{1}{\cancel{2}}} \\ &= \sqrt{16-6} \\ &= \underline{\underline{\sqrt{10}}} \end{aligned}$$

$$\textcircled{1}. 4x + \frac{1}{2x} = 5$$

$$\begin{aligned} 4x - \frac{1}{2x} &= \sqrt{(5)^2 - 4 \times \cancel{4}^2 \times \frac{1}{\cancel{2}}} \\ &= \sqrt{25-8} = \sqrt{17} \end{aligned}$$

$$\textcircled{1} \quad 3x + \frac{1}{4x} = 4$$

$$9x^2 + \frac{1}{16x^2} = ?$$

Ans

$$\begin{aligned} 9x^2 + \frac{1}{16x^2} &= (4)^2 - \cancel{2} \times 3x \times \frac{1}{\cancel{4}2} \\ &= 16 - \frac{3}{2} \\ &= \frac{29}{2} \end{aligned}$$

$$\textcircled{1}. \quad 7x + \frac{1}{2x} = 11$$

$$49x^2 + \frac{1}{4x^2} = ?$$

Ans

$$\begin{aligned} 49x^2 + \frac{1}{4x^2} &= (11)^2 - \cancel{2} \times 7x \times \frac{1}{\cancel{2}} \\ &= 121 - 7 \\ &= \textcircled{114} \end{aligned}$$

$$\textcircled{Q}. 2x + \frac{1}{3x} = 4$$

$$9x^2 + \frac{1}{4x^2} = ?$$

$$3x + \frac{1}{2x}$$

$$\text{Sol}^n \quad 2x + \frac{1}{3x} = \textcircled{4}$$

$$3x + \frac{1}{2x} = \frac{2}{\cancel{4}} \times 3 \times \frac{1}{\cancel{2}} = 6$$

$$9x^2 + \frac{1}{4x^2} = \frac{(6)^2 - \cancel{2} \times 3 \times \frac{1}{\cancel{2}}}{\underline{\underline{33}}}$$

$$\left. \begin{array}{l} \times \frac{3}{2} \left(\begin{array}{l} 2x + \frac{1}{3x} = 4 \\ 3x + \frac{1}{2x} = 4 \times \frac{3}{2} \end{array} \right) \times \frac{3}{2} \end{array} \right\}$$

$$\textcircled{1}. \quad 4x + \frac{1}{3x} = 16$$

$$3x + \frac{1}{4x} = ?$$

$$\underline{\underline{\text{Ans}}} \quad 3x + \frac{1}{4x} = \overset{4}{16} \times 3 \times \frac{1}{\cancel{4}} = \underline{\underline{12}}$$

33. If $5x + \frac{1}{3x} = 5$, find the value of $9x^2 + \frac{1}{25x^2}$

(a) 0

(b) 5

(c) 4

(d) 7.8

$$5x + \frac{1}{3x} = 5$$

$$\Rightarrow 3x + \frac{1}{5x} = \cancel{5} \times 3 \times \frac{1}{\cancel{5}}$$

$$9x^2 + \frac{1}{25x^2} = (3)^2 - 2 \times 3 \times \frac{1}{5}$$

$$= 9 - 1.2$$

$$= \underline{\underline{7.8}}$$

34. If $3x + \frac{1}{2x} = 5$ then find $8x^3 + \frac{1}{27x^3}$

$$3x + \frac{1}{2x} = 5$$

$$\Rightarrow 2x + \frac{1}{3x} = 5 \times 2 \times \frac{1}{3}$$

$$\Rightarrow 8x^3 + \frac{1}{27x^3} = \left(\frac{10}{3}\right)^3 - \cancel{3} \times \frac{10}{\cancel{3}} \times 2 \times \frac{1}{3}$$

$$= \frac{1000}{27} - \frac{20}{3}$$

$$= \frac{1000 - 180}{27} = \frac{820}{27} = 30\frac{10}{27}$$

(a) $118\frac{1}{2}$

(c) 0

(b) $30\frac{10}{27}$

(d) 1

$$x + \frac{1}{16x} = 3$$

$$\Rightarrow x^3 + \frac{1}{(16)^3 x^3} = (3)^3 - 3 \times 3 \times 1 \times \frac{1}{16}$$

$$\Rightarrow x^3 + \frac{1}{(16)^3 x^3} = 27 - \frac{9}{16}$$

$$\Rightarrow 16x^3 + 16x \frac{1}{16 \times 16 \times 16 x^3} = 27 \times 16 - \frac{9}{16} \times 16$$

$$\Rightarrow 16x^3 + \frac{1}{256x^3} = 432 - 9 = \underline{423}$$

35. If $x + \frac{1}{16x} = 3$, then the value of

$$16x^3 + \frac{1}{256x^3} \text{ is :}$$

- (a) 423
(c) 432

- (b) 441
(d) 414

✓✓ SSC CGL TIER II 12/09/2019

✓
H.W 36. If $5x + \frac{1}{3x} = 4$, then the value of $9x^2 + \frac{1}{25x^2}$ is :

SSC CGL 04/03/2020(Shift-01)

(a) $\frac{174}{125}$

(b) $\frac{144}{125}$

(c) $\frac{114}{25}$

(d) $\frac{119}{25}$

$$20x - 30 + \frac{1}{x} = 0$$
$$\Rightarrow \frac{\overset{5}{\cancel{20}x} + \frac{1}{\cancel{4}x} = \frac{\cancel{30}}{\cancel{4}} \frac{15}{2}$$

$$5x + \frac{1}{4x} = \frac{15}{2}$$
$$\rightarrow 25x^2 + \frac{1}{16x^2} = \left(\frac{15}{2}\right)^2 - 2 \times 5 \times \frac{1}{4}$$
$$= \frac{225}{4} - \frac{10}{4} = \frac{215}{4} = 53\frac{3}{4}$$

37. If $\frac{20x^2}{x} - \frac{30x}{x} + \frac{1}{x} = 0$, then what is the value of $25x^2 + \frac{1}{16x^2}$ is :

SSC CGL 05/03/2020 (Shift-02)

(a) $53\frac{1}{2}$

✓ (c) $53\frac{3}{4}$

(b) $58\frac{1}{2}$

(d) $58\frac{3}{4}$

38. H.W If $4a + \frac{1}{5a} = 4$, then the value of $25a^2 + \frac{1}{16a^2}$ is :

SSC CHSL 21/10/2020 (Shift-03)

(a) $\frac{45}{2}$

(b) $\frac{55}{2}$

(c) $\frac{43}{2}$

(d) $\frac{45}{4}$

$$2x + 5 + \frac{1}{x} = 0$$

$$\Rightarrow 2x + \frac{1}{x} = -5$$

$$\Rightarrow x + \frac{1}{2x} = -5 \times 1 \times \frac{1}{2} = \left(-\frac{5}{2}\right)$$

$$x - \frac{1}{2x} = \sqrt{\left(-\frac{5}{2}\right)^2 - 4 \times 1 \times \frac{1}{2}}$$

$$= \sqrt{\frac{25}{4} - 2}$$

$$= \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$$

39. If $\frac{2x^2}{x} + \frac{5x}{x} + \frac{1}{x} = 0$, then one of the values of $x - \frac{1}{2x}$?

SSC CGL MAINS 29/01/2022

✓ (a) $\frac{\sqrt{17}}{2}$

(b) $\frac{13}{2}$

(c) $\frac{5}{2}$

(d) $\frac{\sqrt{13}}{2}$

$$a + \frac{1}{a} = n$$

$$a - \frac{1}{a} = \sqrt{n^2 - 4}$$

$$\begin{aligned} & 25x^2 - \frac{1}{16x^2} \\ &= (5x)^2 - \frac{1}{(4x)^2} \\ &= \left(5x - \frac{1}{4x}\right) \left(5x + \frac{1}{4x}\right) \\ &= 6 \sqrt{(6)^2 + 4 \times 5x \times \frac{1}{4}} \\ &= 6 \sqrt{36 + 5} \\ &= 6 \sqrt{41} \end{aligned}$$

40. If $5x - \frac{1}{4x} = 6$, $x > 0$, then find the value

of $25x^2 - \frac{1}{16x^2}$.

SSC CGL 21/04/2022 (Shift 01)

✓ (a) $6\sqrt{41}$

(b) 36

(c) $\sqrt{246}$

(d) $6\sqrt{31}$

$$(2x)^2 + \frac{1}{(2x)^2} = 2$$

$$t^2 + \frac{1}{t^2} = 2$$

$$t^3 + \frac{1}{t^3} = ?$$

41. If $\left(4x^2 + \frac{1}{4x^2}\right) = 2$, then what is the value of

$$\left(8x^3 + \frac{1}{8x^3}\right)? \quad (2x)^3 + \frac{1}{(2x)^3}$$

☒ (a) +2

(b) -2

(c) +4

(d) -4

let $2x = t$

$$2x = 1$$

$$x = \frac{1}{2}$$

~~$4x \cdot \frac{1}{4} + \frac{1}{4x \cdot \frac{1}{4}} = 2$~~ 41. If $\left(4x^2 + \frac{1}{4x^2}\right) = 2$, then what is the value of

$8x \cdot \frac{1}{8} + \frac{1}{8x \cdot \frac{1}{8}} = 2$

$\left(8x^3 + \frac{1}{8x^3}\right) ? \quad (2x)^3 + \frac{1}{(2x)^3}$

☒ (a) +2

(b) -2

(c) +4

(d) -4

let $2x = t$

$2x = 1$

$x = \frac{1}{2}$

42. If $\left(\underline{0.4x} + \frac{1}{x}\right) = 5$, what is the value of

$$\left(0.064x^3 + \frac{1}{x^3}\right) = (5)^3 - 3 \times 5 \times \frac{0.4^2 \times 1}{10} = 125 - 6 = 119$$

SSC CGL 20/04/2022 (Shift 01)

☒ (a) 119

(b) 125

(c) 110

(d) 105

$$\begin{cases} (0.4)^2 = 0.16 \\ (0.4)^3 = \underline{0.064} \end{cases}$$

$$\textcircled{Q}. \quad x^2 + \frac{1}{x^2} = 119$$

$$x + \frac{1}{x} = ?$$

Ans

$$x^2 + \frac{1}{x^2} = 119$$

$$x + \frac{1}{x} = \sqrt{119+2} = 11$$

$$\textcircled{Q}. \quad x^4 + \frac{1}{x^4} = 119$$

$$x + \frac{1}{x} = ?$$

Ans

$$x^4 + \frac{1}{x^4} = 119$$

$$\rightarrow x^2 + \frac{1}{x^2} = \sqrt{119+2} = 11$$

$$\rightarrow x + \frac{1}{x} = \sqrt{11+2} = \sqrt{13}$$

$$\rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{\sqrt{13}+2} \quad \checkmark$$

$$\textcircled{1}. x^4 + \frac{1}{x^4} = 119$$

$$\text{find } x + \frac{1}{x}$$

$$x - \frac{1}{x}$$

Soln $x^4 + \frac{1}{x^4} = 119$

$$x^2 + \frac{1}{x^2} = \sqrt{119+2} = 11$$

$$x + \frac{1}{x} = \sqrt{11+2} \\ = \sqrt{13} .$$

$$x - \frac{1}{x} = \sqrt{11-2} = 3$$

$$\begin{array}{l} x^2 + \frac{1}{x^2} = 27 \\ \swarrow \quad \searrow \\ x + \frac{1}{x} = \sqrt{27+2} \quad x - \frac{1}{x} = \sqrt{27-2} \\ = \sqrt{29} \quad = \sqrt{25} = 5 \end{array}$$

43. If $x^2 + \frac{1}{x^2} = 27$, then find

(i) $x + \frac{1}{x}$

(ii) $x - \frac{1}{x}$

$$x^4 + \frac{1}{x^4} = 23$$

$$x^2 + \frac{1}{x^2} = \sqrt{23+2} = 5$$

$$* \quad x + \frac{1}{x} = \sqrt{5+2} = \sqrt{7}$$

$$\begin{aligned} * \quad x^3 + \frac{1}{x^3} &= (\sqrt{7})^3 - 3\sqrt{7} \\ &= 7\sqrt{7} - 3\sqrt{7} = 4\sqrt{7} \end{aligned}$$

44. If $x^4 + \frac{1}{x^4} = 23$, find

(i) $x + \frac{1}{x}$

(ii) $x^3 + \frac{1}{x^3}$

$$x^4 - 1442 + \frac{1}{x^4} = 0$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 1442$$

$$x^2 + \frac{1}{x^2} = \sqrt{1442 + 2} = 38$$

$$x - \frac{1}{x} = \sqrt{38 - 2} = 6$$

45. If $\frac{x^8}{x^4} - 1442\frac{x^4}{x^4} + \frac{1}{x^4} = 0$, then a possible value of $x - \frac{1}{x}$ is :

- (a) 5
- (c) 4

SSC CGL TIER-II (11/09/2019)

- (b) 8
- (d) 6

$$x^4 + \frac{1}{x^4} = 142$$

$$x^2 + \frac{1}{x^2} = \sqrt{142+2} = 12$$

$$x + \frac{1}{x} = \sqrt{12+2} = \sqrt{14}$$

$$\begin{aligned} x^7 + \frac{1}{x^7} &= (n^3 - 3n) \times 142 - \sqrt{14} \\ &= (14\sqrt{14} - 3\sqrt{14}) \times 142 - \sqrt{14} \\ &= 11\sqrt{14} \times 142 - \sqrt{14} \\ &= 1562\sqrt{14} - \sqrt{14} \\ &= 1561\sqrt{14} \end{aligned}$$

46. If $x > 0$ and $x^4 + \frac{1}{x^4} = 142$, what is the value of $x^7 + \frac{1}{x^7}$?

SSC CGL 20/07/2023 (Shift-02)

✓ (a) $1561\sqrt{14}$

(b) $1563\sqrt{14}$

(c) $1560\sqrt{14}$

(d) $1562\sqrt{14}$

$$x^7 + \frac{1}{x^7} = \left(x^3 + \frac{1}{x^3}\right) \left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)$$

47. H.W If $x > 0$, and $x^4 + \frac{1}{x^4} = 2207$, what is the value of $x^7 + \frac{1}{x^7}$?

SSC CGL 02/12/2022 (Shift-01)

(a) 710649

(b) 710647

(c) 710654

(d) 710661

$$\begin{aligned}
 & x^4 + \frac{1}{x^4} = 47 \\
 & \rightarrow x^2 + \frac{1}{x^2} = \sqrt{47+2} = 7 \\
 & \rightarrow x + \frac{1}{x} = \sqrt{7+2} = 3
 \end{aligned}$$

$$\Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow \boxed{x^2 - 3x = -1}$$

48. If $x^4 + x^{-4} = 47$, ($x > 0$), then the value of $(2x - 3)^2$ is :

SSC CHSL 8/07/2019 (Shift-03)

- (a) 2
(c) 5

- (b) 3
(d) 4

$$\begin{aligned}
 & (2x - 3)^2 \\
 &= 4x^2 + 9 - 2 \times 2x \times 3 \\
 &= 4x^2 + 9 - 12x \\
 &= 4x^2 - 12x + 9 \\
 &= 4(x^2 - 3x) + 9 \\
 &= 4(-1) + 9 \\
 &= -4 + 9 = 5
 \end{aligned}$$

$$x^4 + \frac{1}{x^4} = 194$$

$$x^2 + \frac{1}{x^2} = \sqrt{194+2} = 14$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{14+2} = 4$$

$$\Rightarrow x^2 + 1 = 4x$$

$$\Rightarrow x^2 - 4x = -1$$

49. If $x^4 + x^{-4} = 194$, ($x > 0$), then the value of $(2x - 4)^2$ is :

SSC CHSL 09/07/2019 (Shift-01)

(a) 15

(b) 20

(c) 12

(d) 16

$$\begin{aligned} & \rightarrow 4x^2 + 16 - 2 \times 2x \times 4 \\ & = 4x^2 - 16x + 16 \\ & = 4(x^2 - 4x) + 16 \\ & = 4(-1) + 16 \\ & = 12 \end{aligned}$$

$$x^4 + \frac{1}{x^4} = 194$$

$$x^2 + \frac{1}{x^2} = \sqrt{194+2} = 14$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{14+2} = 4$$

$$x - \frac{1}{x} = \sqrt{4^2 - 4} = 2\sqrt{3}$$

$$\begin{array}{r} x + \frac{1}{x} = 4 \\ (+) \quad x - \frac{1}{x} = 2\sqrt{3} \\ \hline 2x = (4 + 2\sqrt{3}) \end{array}$$

49. If $x^4 + x^{-4} = 194$, ($x > 0$), then the value of $(2x - 4)^2$ is :

- (a) 15
(c) 12

- (b) 20
(d) 16

SSC CHSL 09/07/2019 (Shift-01)

$$\begin{array}{l} (2x - 4)^2 = (2\sqrt{3})^2 \\ \Rightarrow (2x - 4)^2 = \underline{\underline{12}} \end{array}$$

$$x = \sqrt{9}$$

$$\boxed{x = 3}$$

$$x^2 = 9$$

$$\Rightarrow x = \pm \sqrt{9}$$

$$\Rightarrow \boxed{x = \pm 3}$$

$$x^4 + \frac{1}{x^4} = 322$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{322+2} = 18$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 18 - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16$$

$$\Rightarrow x - \frac{1}{x} = \pm \sqrt{16}$$

$$\Rightarrow \boxed{x - \frac{1}{x} = \pm 4}$$

50. If $x^4 + \frac{1}{x^4} = 322$, find $x^3 - \frac{1}{x^3}$

(a) ± 76
(c) -76

(b) 76
(d) 95

$$\left. \begin{aligned} x - \frac{1}{x} &= 4 \\ x^3 - \frac{1}{x^3} &= 4^3 + 3 \times 4 \\ &= 76 \end{aligned} \right\}$$

$$\left. \begin{aligned} x - \frac{1}{x} &= -4 \\ x^3 - \frac{1}{x^3} &= (-4)^3 + 3 \times (-4) \\ &= -64 - 12 \\ &= -76 \end{aligned} \right\}$$

Special Type

$$x + \frac{1}{x} = 2$$

$$\therefore x = 1$$

$$x + \frac{1}{x} = -2$$

$$\therefore x = -1$$

Q. $x + \frac{1}{x} = 2$ find $x^{99} + \frac{1}{x^{100}} = ?$

Ans

$x = 1$

$$\therefore (1)^{99} + \frac{1}{(1)^{100}}$$

$$= 1 + \frac{1}{1}$$

$$= \underline{\underline{2}}$$

51. If $x + \frac{1}{x} = 2$, find

(i) $x^{20} + x^{19} + x^{18} + \dots + x^2 + x + 1$

(ii) $x^{49} - x^{48} + x^{47} - x^{46} + \dots + x^3 - x^2 + x + 1$

(iii) $x^{12} + \frac{1}{x^{12}}$

(iv) $x^{17} + \frac{1}{x^9}$

(v) $x^{15} - \frac{1}{x^{17}}$

51. If $x + \frac{1}{x} = 2$, find

(i) $x^{20} + x^{19} + x^{18} + \dots + x^2 + x + 1$

$$= 1 + 1 + 1 + \dots + 1 + 1 + 1$$

$$= 20 + 1$$

$$= 21$$

51. If $x + \frac{1}{x} = 2$, find

$$\begin{aligned}
 \text{(ii)} \quad & \cancel{x^{49}} - \cancel{x^{48}} + \cancel{x^{47}} - \cancel{x^{46}} + \dots + \cancel{x^3} - \cancel{x^2} + \underline{x + 1} \\
 &= \cancel{+1} - \cancel{1} + \cancel{1} - 1 + \dots - \dots + \cancel{1} - \cancel{1} + \underline{1 + 1} \\
 &= \underline{\underline{2}}
 \end{aligned}$$

51. If $x + \frac{1}{x} = 2$, find

(iii) $x^{12} + \frac{1}{x^{12}}$

$$= (1)^{12} + \frac{1}{(1)^{12}}$$

$$= 1 + 1$$

$$= 2$$

51. If $x + \frac{1}{x} = 2$, find

(iv) $x^{17} + \frac{1}{x^9}$

$$= (1)^{17} + (1)^9$$

$$= \underline{\underline{2}}$$

51. If $x + \frac{1}{x} = 2$, find

(v) $x^{15} - \frac{1}{x^{17}}$

$= 1 - 1$

$= 0$

52. If $p + \left(\frac{1}{p}\right) = 2$, then find the value of $p \times p \times p$. ^{$1 \times 1 \times 1$}

✓ SSC CHSL 18 /03/2020 (Shift-02)

(a) 4

(b) 8

✓ (c) 1

(d) 2

$$\textcircled{1} \quad x + \frac{1}{x} = -2 \quad \therefore x = -1$$

$$\textcircled{2} \quad x^{10} + \frac{1}{x^{10}} = 1 + 1 = 2$$

$$\textcircled{3} \quad x^9 + \frac{1}{x^9} = -1 - 1 = -2$$

$$\textcircled{4} \quad x^9 + \frac{1}{x^{10}} = -1 + 1 = 0$$

$$(-1)^{\text{even}} = 1$$

$$(-1)^{\text{odd}} = -1$$

53. If $x + \frac{1}{x} = -2$

(i) $x^{39} + x^{38} + \dots + x^2 + x + 1$

(ii) $x^{100} - x^{99} + x^{98} - x^{97} + \dots + x^2 - x + 1$

(iii) $x^5 - \frac{1}{x^5}$

(iv) $x^{18} + \frac{1}{x^{18}}$

(v) $x^7 - \frac{1}{x^9}$

53. If $x + \frac{1}{x} = -2$

(i) $\cancel{x^{39}} + \cancel{x^{38}} + \dots \dots \dots \cancel{x^3} + \cancel{x^2} + x + 1$

$= \cancel{x} + \cancel{x} - \cancel{x} + \cancel{x} - \dots - \cancel{x} + \cancel{x} - \underbrace{1 + 1}$

$= 0$

53. If $x + \frac{1}{x} = -2$

(ii) $x^{100} - x^{99} + x^{98} - x^{97} + \dots + x^2 - x + 1$

$$= 1 - (-1) + 1 - (-1) + \dots + 1 - (-1) + 1$$

$$= 1 + 1 + 1 + 1 + \dots + 1 + 1 + 1$$

$$= 100 + 1$$

$$= \underline{\underline{101}}$$

$$x = -1$$

53. If $x + \frac{1}{x} = -2$

(iii) $x^5 - \frac{1}{x^5}$

$$\begin{aligned} & (-1)^5 - \frac{1}{(-1)^5} \\ &= -1 - \frac{1}{(-1)} \\ &= -1 + 1 \\ &= 0 \end{aligned}$$

53. If $x + \frac{1}{x} = -2$

$$\text{(iv) } x^{18} + \frac{1}{x^{18}}$$

$$= (-1)^{18} + \frac{1}{(-1)^{18}}$$

$$= 1 + 1$$

$$= 2$$

53. If $x + \frac{1}{x} = -2$

(v) $x^7 - \frac{1}{x^9}$

$$\begin{aligned} &= (-1)^7 - \frac{1}{(-1)^9} \\ &= -1 - \frac{1}{(-1)} \\ &= -1 + 1 \\ &= 0 \end{aligned}$$

$$* \quad x + \frac{1}{x} = 2, \quad (x = 1)$$

$$* \quad x + \frac{1}{x} = -2, \quad (x = -1)$$

$$* \quad \boxed{x + \frac{1}{x} = 1} \quad \therefore (x^3 = -1)$$

$$* \quad x + \frac{1}{x} = -1 \quad x^3 = 1$$

$$* \quad x + \frac{1}{x} = 1$$

$$\Rightarrow x^2 + 1 = x$$

$$\Rightarrow (x^2 + 1 - x) = 0$$

$$\Rightarrow (x+1)(x^2+1-x)=0$$

$$\Rightarrow (x^3+1)=0$$

$$\Rightarrow x^3 = -1$$

$$(a^3+b^3) = (a+b)(a^2+b^2-ab)$$

Q. $x + \frac{1}{x} = 1$ find $x^{99} + \frac{1}{x^{33}}$

Ans $x^3 = -1$

$$\begin{aligned} & (x^3)^{33} + \frac{1}{(x^3)^{11}} \\ &= (-1)^{33} + \frac{1}{(-1)^{11}} \\ &= -1 - 1 \\ &= \underline{\underline{-2}} \end{aligned}$$

54. If $x + \frac{1}{x} = 1$, find

(i) $x^{18} + x^{12} + x^6 + 1$

(ii) $x^{99} + x^{96} + x^{93} + x^{90} + \dots + x^6 + x^3 + 1$

(iii) $x^{45} - x^{42} + x^{39} - x^{36} + \dots + x^9 - x^6 + x^3 + 1$

(iv) $x^{32} + x^{46} + x^{90} + 2$

✓ ✓

54. If $x + \frac{1}{x} = 1$, find

$$\therefore x + \frac{1}{x} = 1$$

$$\therefore x^3 = -1$$

(i) $x^{18} + x^{12} + x^6 + 1$

$$= (x^3)^6 + (x^3)^4 + (x^3)^2 + 1$$

$$= (-1)^6 + (-1)^4 + (-1)^2 + 1$$

$$= 1 + 1 + 1 + 1$$

$$= \textcircled{4}$$

$$* \quad x + \frac{1}{x} = 1$$

$$\text{find } x^{100} + x^{103} + x^{201} + x^{204}$$

Ans $\boxed{x^3 = -1}$

$$\begin{aligned} \therefore & x^{100} + x^{103} + x^{201} + x^{204} \\ &= x^{100}(1 + x^3) + x^{201}(1 + x^3) \\ &= x^{100}(1 - 1) + x^{201}(1 - 1) \\ &= \underline{0 + 0} \end{aligned}$$

$$* \quad x + \frac{1}{x} = 1$$

Ans $\boxed{x^3 = -1}$

find $x^{100} + x^{103} + x^{201} + x^{204}$
0

54. If $x + \frac{1}{x} = 1$, find

(ii) ~~x^{99}~~ + ~~x^{96}~~ + ~~x^{93}~~ + ~~x^{90}~~ + ~~x^9~~ + ~~x^6~~ + $x^3 + 1$

Ans $x^3 = -1$

$\therefore$

$= -1 + 1$
 $= 0$

54. If $x + \frac{1}{x} = 1$, find

$$(iii) x^{45} - x^{42} + x^{39} - x^{36} + \dots + x^9 - x^6 + x^3 + 1$$

$$= \frac{(x^3)^{15} - (x^3)^{14} + (x^3)^{13} - (x^3)^{12} + \dots + (x^3)^3 - (x^3)^2 + x^3 + 1}{}$$

$$= (-1)^{15} - (-1)^{14} + (-1)^{13} - (-1)^{12} + \dots + (-1)^3 - (-1)^2 + (-1) + 1$$

$$= -1 - 1 - 1 - 1 - 1 - 1 - \dots - 1 + 1$$

$$= -15 + 1$$

$$= -14$$

$$x^3 = -1$$

$$\begin{aligned} &= -\left(x + \frac{1}{x}\right) + 3 \\ &= -1 + 3 \\ &= 2 \end{aligned}$$

54. If $x + \frac{1}{x} = 1$, find

(iv) $x^{32} + x^{46} + x^{90} + 2$

$$\begin{aligned} &\rightarrow x^{32} \cdot \frac{x^1}{x^1} + x^1 \cdot x^{45} + x^{90} + 2 \\ &= \frac{x^{33}}{x} + x \cdot x^{45} + x^{90} + 2 \\ &= \frac{(x^3)^{11}}{x} + x \cdot (x^3)^{15} + (x^3)^{30} + 2 \\ &= \frac{-1}{x} - x + 1 + 2 \end{aligned}$$

$$* \quad x + \frac{1}{x} = 2 \quad \therefore x = 1$$

$$* \quad x + \frac{1}{x} = -2 \quad \therefore x = -1$$

$$* \quad x + \frac{1}{x} = 1 \quad x^3 = -1$$

$$* \quad x + \frac{1}{x} = -1 \quad x^3 = 1$$

Special case

$$x + \frac{1}{x} = \sqrt{3}$$

Proof:

$$x + \frac{1}{x} = \sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = (\sqrt{3})^3 - 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0 \Rightarrow \frac{x^3 \cdot x^3 + 1}{x^3} = 0$$

$$\Rightarrow \frac{x^6 + 1}{x^3} = 0$$

$$\Rightarrow x^6 + 1 = 0$$

$$\Rightarrow \boxed{x^6 = -1}$$

$$* \quad x + \frac{1}{x} = \sqrt{3} \quad \therefore x^6 = -1$$

$$* \quad x + \frac{1}{x} = -\sqrt{3} \quad (x^6 = -1)$$

proof: $x + \frac{1}{x} = -\sqrt{3}$

$$\Rightarrow x^3 + \frac{1}{x^3} = (-\sqrt{3})^3 - 3(-\sqrt{3})$$

$$\Rightarrow x^3 + \frac{1}{x^3} = -3\cancel{\sqrt{3}} + 3\cancel{\sqrt{3}}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0$$

$$x^6 + 1 = 0$$

$$\therefore x^6 = -1$$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(i) $x^{90} + \frac{1}{x^{90}}$

(ii) $x^{92} + \frac{1}{x^{92}}$

(iii) $x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$ is

(iv) $x^6 - \frac{1}{x^6} + 2$

(v) $x^{18} + x^{12} + x^6 + 1$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(i) $x^{90} + \frac{1}{x^{90}}$

$x^6 = -1$

$$\begin{aligned} & (x^6)^{15} + \frac{1}{(x^6)^{15}} \\ &= (-1)^{15} + \frac{1}{(-1)^{15}} \\ &= -1 - 1 \\ &= -2 \end{aligned}$$

$$x^6 = -1$$

$$\begin{aligned} &= -x^2 - \frac{1}{x^2} \\ &= -\left(x^2 + \frac{1}{x^2}\right) \\ &= -\left\{(\sqrt{3})^2 - 2\right\} \\ &= -\{3-2\} \\ &= -1 \end{aligned}$$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(ii) $x^{92} + \frac{1}{x^{92}}$

$$\begin{aligned} &\rightarrow x^2 \cdot x^{90} + \frac{1}{x^2 \cdot x^{90}} \\ &= x^2 \cdot (x^6)^{15} + \frac{1}{x^2 \cdot (x^6)^{15}} \\ &= x^2 (-1)^{15} + \frac{1}{x^2 (-1)^{15}} \end{aligned}$$

$$\textcircled{1}. \quad x + \frac{1}{x} = \sqrt{3}$$

$$(a) \quad x^{63} + \frac{1}{x^{63}}$$

$$= x^3 \cdot x^{60} + \frac{1}{x^3 \cdot x^{60}}$$

$$= x^3 (x^6)^{10} + \frac{1}{x^3 (x^6)^{10}}$$

$$= x^3 (-1)^{10} + \frac{1}{x^3 (-1)^{10}}$$

$$= x^3 + \frac{1}{x^3}$$

$$= (x^3 - 3x)$$

$$= (\sqrt{3})^3 - 3\sqrt{3}$$

$$= 3\sqrt{3} - 3\sqrt{3}$$

$$= 0$$

$$\textcircled{1}. x + \frac{1}{x} = \sqrt{3}$$

Ans $x^6 = -1$

$$\text{find } x^{100} + x^{106} + x^{200} + x^{206}$$

$$= x^{100}(1 + x^6) + x^{200}(1 + x^6)$$

$$= x^{100}(1 - 1) + x^{200}(1 - 1)$$

$$= 0 + 0$$

$$= 0$$

$$\omega^6 = -1$$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(iii) ~~$x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$~~ is

$$x^6 = -1$$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(iv) $x^6 - \frac{1}{x^6} + 2$

Ans $(-1) - \frac{1}{(-1)} + 2$
 $= -\cancel{1} + \cancel{1} + 2$
 $= 2$

$$x^6 = -1$$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(v) $x^{18} + x^{12} + x^6 + 1$

$$(x^6)^3 + (x^6)^2 + x^6 + 1$$

$$= (-1)^3 + (-1)^2 - 1 + 1$$

$$= \cancel{-1} + \cancel{1} \cancel{-1} + \cancel{1}$$

$$= 0$$

56. If $\left(x + \frac{1}{x}\right)^2 = 3$, then what is the value of $x^6 + x^{-6}$?

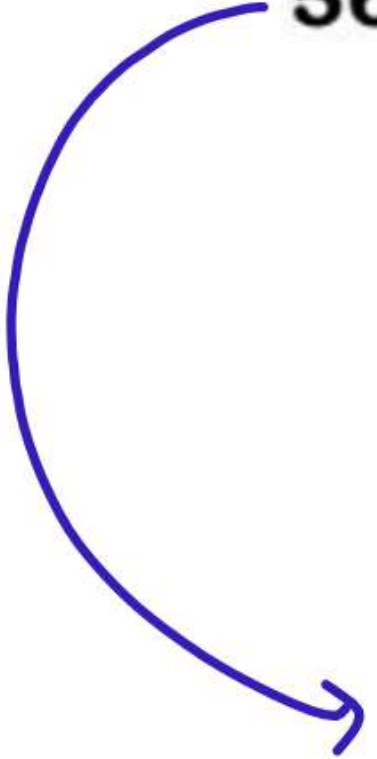
SSC CGL MAINS (08/08/2022)

(a) 6

(b) 2

☒ (c) -2

(d) -6


$$\begin{aligned}x + \frac{1}{x} &= \sqrt{3} \\&= x^6 + \frac{1}{x^6} \\&= -1 - 1 \\&= -2\end{aligned}$$

57. If $x + \frac{1}{x} = \sqrt{3}$, then the value of
 ~~$x^{18} + x^{12}$~~ + $x^6 + 1$ is :

SSC CHSL 21/10/2020 (Shift- 03)

- ~~(a) 0~~
(c) 3

- (b) 2
(d) 1

$$\begin{aligned} &= -1 + 1 \\ &= 0 \end{aligned}$$

58. If $x + \frac{1}{x} = -\sqrt{3}$ then $x^{67} + x^{53} + x^{43} + x^{29} +$

$x^{24} + x^{12} + x^6 + 3$ is

(a) $\sqrt{3}$

(b) 0

(c) $2(2 + \sqrt{3})$

(d) $2(2 - \sqrt{3})$

$$x^{67} + x^{53} + x^{43} + x^{29} + x^{24} + 3$$

$$= x \cdot x^{66} + x^{\frac{53}{x}} + x^1 \cdot x^{42} + x^{29} \cdot \frac{x}{x} + (x^6)^4 + 3$$

$$= x \cdot (x^6)^{11} + \frac{(x^6)^9}{x} + x \cdot (x^6)^7 + \frac{(x^6)^5}{x} + (x^6)^4 + 3$$

$$= x(-1) - \frac{1}{x} + x(-1) + \frac{(-1)}{x} + 1 + 3$$

$$= -x - \frac{1}{x} - x - \frac{1}{x} + 4$$

$$= -2x - \frac{2}{x} + 4$$

$$= -2\left(x + \frac{1}{x}\right) + 4$$

$$= -2(-\sqrt{3}) + 4$$

$$= 2\sqrt{3} + 4$$

$$= 2(\sqrt{3} + 2)$$

$$\text{ans} = x + \frac{1}{x}$$

$$= (2 + \sqrt{3}) + \frac{1}{(2 + \sqrt{3})}$$

$$= 2 + \sqrt{3} + \frac{1}{(2 + \sqrt{3})} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})}$$

$$= 2 + \sqrt{3} + \frac{(2 - \sqrt{3})}{4 - 3}$$

$$= 2 + \cancel{\sqrt{3}} + \frac{2 - \cancel{\sqrt{3}}}{1}$$

$$= 4$$

59. If $x = 2 + \sqrt{3}$, find $x + \frac{1}{x}$

(a) 5

(c) -4

(b) 4

(d) $2\sqrt{3}$

Note:

$$x = \overset{\beta}{\textcircled{2}} + \overset{s}{\textcircled{\sqrt{3}}}$$

$$\text{If } \beta^2 - s^2 = 1$$

$$\# x + \frac{1}{x} = 2 \times \beta = 2 \times 2 = 4$$

$$\# x - \frac{1}{x} = 2 \times s = 2 \times \sqrt{3} = 2\sqrt{3}$$

59. If $x = 2 + \sqrt{3}$, find $x + \frac{1}{x}$

(a) 5

(b) 4

(c) -4

(d) $2\sqrt{3}$

$$\therefore x + \frac{1}{x} = 2 \times 2 = 4$$

/

60. If $x = \overset{B}{\textcircled{7}} + \overset{S}{\textcircled{4\sqrt{3}}}$ then $x - \frac{1}{x}$

☒ (a) $8\sqrt{3}$

(b) 4

(c) -4

(d) $-2\sqrt{3}$

$$\therefore x + \frac{1}{x} = 2 \times 7 = 14$$

$$x - \frac{1}{x} = 2 \times 4\sqrt{3} = 8\sqrt{3}$$

61. If $x = 2 + \sqrt{5}$ then the value of $x^3 - x^{-3}$ is :

SSC CHSL 08/07/2019 (Shift-03)

(a) - 52

(b) 52

☒ (c) 76

(d) - 76

$$x = \overset{S}{\textcircled{2}} + \overset{B}{\textcircled{\sqrt{5}}}$$

$$x^3 - \frac{1}{x^3}$$

$$x - \frac{1}{x} = 2 \times 2 = 4$$

$$x^3 - \frac{1}{x^3} = (4)^3 + 3 \times 4 = \underline{\underline{76}}$$

m-1

$$x = 3 + 2\sqrt{2}$$

$$\therefore \sqrt{x} = \sqrt{3 + 2\sqrt{2}}$$

$$= \sqrt{2 + 1 + 2\sqrt{2}}$$

$$= \sqrt{(\sqrt{2})^2 + (1)^2 + 2 \times \sqrt{2} \times 1}$$

$$\sqrt{x} = \sqrt{(\sqrt{2} + 1)^2}$$

$$\therefore \sqrt{x} = \overset{B}{\sqrt{2}} + \overset{S}{1}$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 2 \times 1 = 2$$

62. If $x = 3 + 2\sqrt{2}$, then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is:

- (a) 2
(c) 0

SSC CHSL 26/10/2020 (Shift-03)

- (b) 1
(d) 3

m-1

$$x = 3 + 2\sqrt{2}$$

$$\therefore x + \frac{1}{x} = 6$$

$$\begin{array}{l} \swarrow \quad \searrow \\ \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6+2} \quad \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6-2} \\ \phantom{\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6+2}} \phantom{\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6-2}} = 2 \end{array}$$

62. If $x = 3 + 2\sqrt{2}$, then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is:

- (a) 2
(c) 0

SSC CHSL 26/10/2020 (Shift-03)

- (b) 1
(d) 3

$$x = \overset{S}{\textcircled{1}} + \overset{B}{\textcircled{\sqrt{2}}}$$

$$\therefore x + \frac{1}{x} = 2\sqrt{2}$$



$$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{2\sqrt{2} + 2} = \sqrt{2 \times 1.414 + 2} = \sqrt{4.82}$$

63. If $x = 1 + \sqrt{2}$, then the value of $\sqrt{x} + \left(\frac{1}{\sqrt{x}}\right)$ is:

SSC CHSL 17/03/2020 (Shift-03)

- (a) 2.1014
(c) 1.9876

- ☒ (b) 2.1973
(d) 1.9996

$$\begin{aligned} (21)^2 &= 441 \\ (22)^2 &= 484 \end{aligned}$$

$$p + \frac{1}{p} = 14$$

$$\left(p + \frac{1}{p}\right) + \left(p^3 + \frac{1}{p^3}\right)$$

$$= 14 + (14)^3 - 3 \times 14$$

$$= 2744 - 28$$

$$= \underline{2716}$$

64. If $p = \overset{B}{7} + \overset{S}{4\sqrt{3}}$, then what is the value of $\frac{p^6 + p^4 + p^2 + 1}{p^3}$?

SSC CGL 06/12/2022 (Shift-02)

(a) 2617

(b) 2167

(c) ~~2716~~

(d) 2176


$$p^3 + p + \frac{1}{p} + \frac{1}{p^3}$$

$$PQ = 1$$
$$P = \frac{1}{Q}$$

$$\therefore P + \frac{1}{P} = 14$$
$$P^2 + \frac{1}{P^2} = (14)^2 - 2$$
$$= 194$$

65. If $P = 7 + 4\sqrt{3}$ and $PQ = 1$, then what is

the value of $\left(\frac{1}{P^2} + \frac{1}{Q^2}\right) = ?$

- (a) 148
- (c) 194

- (b) 189
- (d) 204

$$\frac{1}{P^2} + P^2$$

$$x = \frac{(\sqrt{5} - \sqrt{4})}{\sqrt{5} + \sqrt{4}} \times \frac{(\sqrt{5} - \sqrt{4})}{\sqrt{5} - \sqrt{4}}$$

$$= \frac{(\sqrt{5} - \sqrt{4})^2}{5 - 4} = \frac{5 + 4 - 2 \times \sqrt{5} \times \sqrt{4}}{1}$$

$$\boxed{x = 9 - 4\sqrt{5}} \therefore x + \frac{1}{x} = 18$$

$$\frac{x^2 - x \times \frac{1}{x} + \frac{1}{x^2}}{x^2 + x \times \frac{1}{x} + \frac{1}{x^2}} = \frac{x^2 + \frac{1}{x^2} - 1}{x^2 + \frac{1}{x^2} + 1}$$

$$= \frac{(18)^2 - 2 - 1}{(18)^2 - 2 + 1} = \frac{321}{323}$$

$$\boxed{y = \frac{1}{x}}$$

66. If $x = \frac{\sqrt{5} - \sqrt{4}}{\sqrt{5} + \sqrt{4}}$ and $y = \frac{\sqrt{5} + \sqrt{4}}{\sqrt{5} - \sqrt{4}}$ then the

value of $\frac{x^2 - xy + y^2}{x^2 + xy + y^2} = ?$

SSC CGL 08/12/2022 (Shift-02)

(a) $\frac{361}{363}$

(b) $\frac{341}{343}$

(c) $\frac{384}{387}$

(d) $\frac{321}{323}$

67. If $m + \frac{1}{m-3} = 5$, then $\underline{(m-3)^8} + \underline{\frac{1}{(m-3)^8}} = ?$

let $(m-3) = x$

~~(a) 2~~

(b) 0

(c) -2

(d) 15

$$m + \frac{1}{(m-3)} = 5$$

$$\Rightarrow m + \frac{1}{(m-3)} - 3 = 5 - 3$$

$$\Rightarrow (m-3) + \frac{1}{(m-3)} = 2$$

$$x + \frac{1}{x} = 2$$

$$x^8 + \frac{1}{x^8} = 2$$

$$x + \frac{1}{x-9} = 11$$

$$\Rightarrow (x-11) = -\frac{1}{(x-9)}$$

$$\Rightarrow (x-9) = -\frac{1}{(x-11)}$$

$$\Rightarrow x-9-2 = -\frac{1}{(x-11)} - 2$$

$$\Rightarrow (x-11) + \frac{1}{(x-11)} = -2$$

$$(x-11)^{12} + \frac{1}{(x-11)^{13}}$$

$$= (-1)^{12} + \frac{1}{(-1)^{13}} = 1 - 1 = 0$$

68. If $x + \frac{1}{x-9} = 11$, then $(x-11)^{12} +$

$$\frac{1}{(x-11)^{13}} = ?$$

- ☒ (a) 0
(c) -2

- (b) 2
(d) 1

$$\begin{aligned} & (x-11)^{12} + \frac{1}{(x-11)^{13}} \\ & \rightarrow (x-11) + \frac{1}{(x-11)} \end{aligned}$$

$$\frac{1}{a-4} = 6-a$$

$$\Rightarrow 1 = 6a - \underline{a^2} - 24 + 4a$$

$$\Rightarrow a^2 - 10a + 25 = 0$$

$$\Rightarrow a^2 - 5a - 5a + 25 = 0$$

$$\Rightarrow a(a-5) - 5(a-5) = 0$$

$$\Rightarrow (a-5)(a-5) = 0$$

$$\underline{a=5}$$

69. If $a + \frac{1}{a-4} = 6$, then $(a-3)^7 + \frac{1}{(a-7)^3} = ?$

(a) $63\frac{7}{8}$

(b) $255\frac{1}{8}$

(c) $127\frac{7}{8}$

(d) 216

$$\begin{aligned} & 2^7 + \left(-\frac{1}{2}\right)^3 \\ &= 128 - \frac{1}{8} = 127 + 1 - \frac{1}{8} \\ &= 127\frac{7}{8} \end{aligned}$$

$\rightarrow (a-3) + \frac{1}{(a-7)^3}$

$$(a-4) + \frac{1}{(a-4)} = 2$$

$$\therefore a-4 = 1$$

$$\boxed{a=5}$$

$$2^7 + \frac{1}{(-2)^3}$$

$$= 128 - \frac{1}{8}$$

$$= 127\frac{7}{8}$$

69. If $a + \frac{1}{a-4} = 6$, then $(a-3)^7 + \frac{1}{(a-7)^3} = ?$

(a) $63\frac{7}{8}$

(b) $255\frac{1}{8}$

✓ (c) $127\frac{7}{8}$

(d) 216

$$(a-3) + \frac{1}{(a-7)^3}$$

M-1 (Basic)

$$x - 8 = t$$

$$\Rightarrow x = 8 + t$$

$$\therefore (8+t)^2 - 22(8+t) + 111 = 0$$

$$\Rightarrow 64 + t^2 + 16t - 176 - 22t + 111 = 0$$

$$\Rightarrow t^2 - 6t - 1 = 0$$

$$\Rightarrow t - 6 - \frac{1}{t} = 0$$

$$\Rightarrow t - \frac{1}{t} = 6$$

70. If $x^2 - 22x + 111 = 0$, then what is the value

$$\text{of } (x - 8)^2 - \frac{1}{(x - 8)^2} = ?$$

यदि $x^2 - 22x + 111 = 0$ तो $(x - 8)^2 - \frac{1}{(x - 8)^2} = ?$

☒ (a) $12\sqrt{10}$

(b) $8\sqrt{5}$

(c) $8\sqrt{3}$

(d) 18

$$\begin{aligned} t^2 - \frac{1}{t^2} &= \left(t - \frac{1}{t}\right)\left(t + \frac{1}{t}\right) \\ &= 6\sqrt{6^2 + 4} \\ &= 6 \times 2\sqrt{10} \\ &= 12\sqrt{10} \end{aligned}$$

70. If $x^2 - \underline{22}x + 111 = 0$, then what is the value

of $\underbrace{(x - 8)}_t^2 - \frac{1}{(x - 8)^2} = ?$

$$-\frac{22}{16} \\ \underline{6}$$

यदि $x^2 - 22x + 111 = 0$ तो $(x - 8)^2 - \frac{1}{(x - 8)^2} = ?$

✓ (a) $12\sqrt{10}$

(b) $8\sqrt{5}$

(c) $8\sqrt{3}$

(d) 18

$$t^2 - \frac{1}{t^2} = \left(t - \frac{1}{t}\right)\left(t + \frac{1}{t}\right)$$

$$= 6\sqrt{6^2 + 4}$$

$$= 6 \times 2\sqrt{10}$$

$$= 12\sqrt{10}$$

$$\text{let } x+8=t$$

$$x = t-8$$

$$\therefore (t-8)^2 + 13(t-8) + 39 = 0$$

$$\Rightarrow t^2 + 64 - 16t + 13t - 104 + 39 = 0$$

$$\Rightarrow t^2 - 3t + 103 - 104 = 0$$

$$\Rightarrow t^2 - 3t - 1 = 0$$

$$\Rightarrow t - 3 - \frac{1}{t} = 0$$

$$\Rightarrow t - \frac{1}{t} = 3$$

71. If $x^2 + 13x + 39 = 0$, then $(x+8)^5 - \frac{1}{(x+8)^5}$

= ?

(a) 393

(b) 396

(c) 392

(d) 394

$$\begin{aligned} t^5 - \frac{1}{t^5} &= (t^2+2)(t^3+3t)-t \\ &= 11 \times 36 - 3 \\ &= 396 - 3 \\ &= 393 \end{aligned}$$

71. If $x^2 + 13x + 39 = 0$, then $\underbrace{(x+8)}_t^5 - \frac{1}{(x+8)^5}$
 $= ?$

(a) 393

(b) 396

(c) 392

(d) 394

$$t - \frac{1}{t} = |13 - 16| = 3$$

$$\begin{aligned} t^5 - \frac{1}{t^5} &= (3^2 + 2)(3^3 + 3 \times 3) - 3 \\ &= \underline{\underline{393}} \end{aligned}$$

$$x-4=t$$

$$\therefore x=t+4$$

$$(t+4)^2 - 12(t+4) + 33 = 0$$

$$\Rightarrow t^2 + 16 + 8t - 12t - 48 + 33 = 0$$

$$\Rightarrow t^2 - 4t + 1 = 0$$

$$\Rightarrow t - 4 + \frac{1}{t} = 0$$

$$\therefore t + \frac{1}{t} = 4$$

72. If $x^2 - 12x + 33 = 0$, then what is the value

$$\text{of } (x-4)^4 + \frac{1}{(x-4)^4} = ?$$

यदि $x^2 - 12x + 33 = 0$ तो $(x-4)^4 + \frac{1}{(x-4)^4} =$

का मान क्या है?

(a) 227

(b) 326

(c) 167

(d) 194

$$\begin{aligned} t^4 + \frac{1}{t^4} &= (t^2 - 2)^2 - 2 \\ &= 196 - 2 \\ &= 194 \end{aligned}$$

$$t + \frac{1}{t} = |12 - 8| = 4$$

$$\begin{aligned} t^4 + \frac{1}{t^4} &= (t^2 - 2)^2 - 2 \\ &= (14)^2 - 2 \\ &= \underline{\underline{194}} \end{aligned}$$

72. If $x^2 - \underline{12}x + 33 = 0$, then what is the value

of $(x - 4)^4 + \frac{1}{(x - 4)^4} = ?$

यदि $x^2 - 12x + 33 = 0$ तो $(x - 4)^4 + \frac{1}{(x - 4)^4} =$

का मान क्या है?

(a) 227

(b) 326

(c) 167

(d) 194

$$t + \frac{1}{t} = |12 - 8| = 4$$

$$\begin{aligned} t^4 + \frac{1}{t^4} &= (t^2 - 2)^2 - 2 \\ &= (14)^2 - 2 \\ &= \underline{\underline{194}} \end{aligned}$$

72. If $x^2 - \underline{12}x + 33 = 0$, then what is the value

of $(x - 4)^4 + \frac{1}{(x - 4)^4} = ?$

यदि $x^2 - 12x + 33 = 0$ तो $(x - 4)^4 + \frac{1}{(x - 4)^4} =$

का मान क्या है?

(a) 227

(b) 326

(c) 167

(d) 194

$$\begin{aligned}
 x(x-b) &= 1 \\
 \Rightarrow x-b &= \frac{1}{x} \\
 \Rightarrow x - \frac{1}{x} &= 5
 \end{aligned}$$

$$\begin{aligned}
 -b+5 &= 0 \\
 5 &= b
 \end{aligned}$$

$$\begin{aligned}
 \therefore x^3 - \frac{1}{x^3} &= (n^3 + 3m) \\
 &= 125 + 15 \\
 &= \underline{\underline{140}}
 \end{aligned}$$

*** **73.** If $(x - \cancel{a})(x - b) = 1$ and $\cancel{a} - b + 5 = 0$, then

$$(x - \cancel{a})^3 + \frac{1}{(x - \cancel{a})^3} = ? \quad (\text{V.V.Imp})$$

यदि $(x - \cancel{a})(x - 5) = 1$ और तो

$$(x - \cancel{a})^3 - \frac{1}{(x - \cancel{a})^3} = ?$$

(a) 110

(c) 125

(b) 1

(d) 140

$$* x - \frac{1}{x} = 3(n)$$

$$\begin{aligned}
 * x^5 - \frac{1}{x^5} &= (n^2+2)(n^3+3n) - n \\
 &= 11 \times 36 - 3 \\
 &= 396 - 3 \\
 &= \underline{\underline{393}}
 \end{aligned}$$

$$* x^{100} - \frac{1}{x^{100}} = 3$$

$$\begin{aligned}
 * x^{500} - \frac{1}{x^{500}} &= (n^2+2)(n^3+3n) - n \\
 &= 11 \times 36 - 3 \\
 &= \underline{\underline{393}}
 \end{aligned}$$

$$x - \frac{1}{x} = \sqrt{7}$$

$$\therefore x^2 + \frac{1}{x^2} = (\sqrt{7})^2 + 2 = 9$$

$$x^4 + \frac{1}{x^4} = (9)^2 - 2 = 79$$

$$\begin{aligned} \therefore x^3 - \frac{1}{x^3} &= x^3 + 3x \\ &= 7\sqrt{7} + 3\sqrt{7} \\ &= 10\sqrt{7} \end{aligned}$$

74. If $x^{\cancel{5}^1} - \frac{1}{x^{\cancel{5}_1}} = \sqrt{7}$, then find $x^{\cancel{35}^7} - \frac{1}{x^{\cancel{35}_7}} = ?$

(a) $791\sqrt{7}$

☒ (b) $789\sqrt{7}$

(c) $790\sqrt{7}$

(d) $792\sqrt{7}$

$$x^7 - \frac{1}{x^7} = \left(x^3 - \frac{1}{x^3}\right) \left(x^4 + \frac{1}{x^4}\right) - \left(x - \frac{1}{x}\right)$$

$$= 10\sqrt{7} \times 79 - \sqrt{7}$$

$$= 790\sqrt{7} - \sqrt{7}$$

$$= 789\sqrt{7}$$

75. If $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3$, $x > 0$ then find $x^2(18x^2 - 7)$?

$$\left(3\sqrt{x} + \frac{1}{2\sqrt{x}}\right)^2 = (3)^2$$

$$9x + \frac{1}{4x} + 2 \times 3\sqrt{x} \times \frac{1}{2\sqrt{x}} = 9$$

$$\left(9x + \frac{1}{4x}\right)^2 = (6)^2$$

$$81x^2 + \frac{1}{16x^2} + 2 \times 9x \times \frac{1}{4x} = 36$$

$$\frac{9}{9} \times 81x^2 + \frac{9}{9} \times \frac{1}{16x^2} + \frac{9}{9} \times \frac{9}{9} = 36 \times \frac{9}{9}$$

$$\Rightarrow 18x^2 + \frac{1}{12x^2} + 1 = 8$$

$$\Rightarrow (18x^2 - 7) = -\frac{1}{12x^2}$$

$$x^2(18x^2 - 7) = -\frac{1}{12}$$

यदि $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3$, $x > 0$ है, तो $x^2(18x^2 - 7)$ ज्ञात

कीजिए?

(a) $-\frac{1}{36}$

(b) $-\frac{2}{63}$

(c) $-\frac{1}{72}$

(d) $-\frac{8}{81}$

$$9x + \frac{1}{4x} = 3^2 - \cancel{2}x \cancel{3} \times \frac{1}{\cancel{2}} = 6 \quad 75. \text{ If } \underline{3\sqrt{x}} + \frac{1}{2\sqrt{x}} = \textcircled{3}, x > 0 \text{ then find } x^2(18x^2 - 7)?$$

$$\frac{9}{9} \left(81x^2 + \frac{1}{16x^2} \right) = \left(6^2 - \cancel{2}x \cancel{9} \times \frac{1}{\cancel{4}2} \right) \frac{9}{9}$$

यदि $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3, x > 0$ है, तो $x^2(18x^2 - 7)$ ज्ञात

$$\Rightarrow 18x^2 + \frac{1}{12x^2} = 8 - 1$$

कीजिए?

$$\Rightarrow 18x^2 - 7 = -\frac{1}{12x^2}$$

$$\Rightarrow x^2(18x^2 - 7) = -\frac{1}{12}$$

(a) $-\frac{1}{36}$

(b) $-\frac{2}{63}$

☒ (c) $-\frac{1}{72}$

(d) $-\frac{8}{81}$